Unifying Consensus and Covariance Intersection for Decentralized State Estimation

Amirhossein Tamjidi
Department of Aerospace
Engineering
Texas A&M University

Suman Chakravorty
Department of Aerospace
Engineering
Texas A&M University

Dylan Shell
Department of Computer Science
and Engineering
Texas A&M University

Abstract—This paper presents a new recursive information consensus filter for decentralized dynamic-state estimation. Local estimators are assumed to have access only to local information and no structure is assumed about the topology of the communication network, which need not be connected at all times. Iterative Covariance Intersection (ICI) is used to reach consensus over priors which might become correlated, while consensus over new information is handled using weights based on a Metropolis Hastings Markov Chain (MHMC). We establish bounds for estimation performance and show that our method produces unbiased conservative estimates that are better than CI. The performance of the proposed method is evaluated and compared with competing algorithms on an atmospheric dispersion problem.

I. Introduction

This paper studies decentralized estimation using multiple robotic agents with applications to the estimation of a dynamic random field. When the field dynamics can be described by a linear, lumped-parameter model, the classical solution is the Kalman filter (KF). However, bandwidth and energy constraints may preclude the centralized implementation of such a filter and necessitate the design of a decentralized estimator. In general, a decentralized sensor network cannot achieve the estimation quality of a centralized estimator but is inherently more flexible and robust to network failure and consequently is advantageous in certain applications [1].

In decentralized estimation settings, the system comprises a set of nodes connected to each other through a communication network with some topology. Nodes are assumed to make noisy observations of a global state from which the full state of the system cannot necessarily be recovered. The goal is to design local estimators that can recursively calculate an estimate of the global state with access only to the information locally available to nodes. We desire that estimates be conservative and the estimator be consistent. No prior knowledge about the network topology is assumed.

When the topology of the network is known *a priori* and it remains connected throughout, some existing methods recover the centralized estimator's performance [2], [3] for dynamic state estimation. However, such methods are not applicable for the case where the network does not remain connected all the time.

For static state/parameter estimation, Xia, et al., introduce a method based on distributed averaging that can converge to the global state estimator provided that the infinitely occurring communication graphs are jointly connected [4]. Their method relies on the distributed averaging property of Metropolis-Hastings Markov Chains (MHMC). The advantage of it is that the network topology can be dynamically changing and it need not be connected at all times. The local estimators exchange information only with their immediate neighbors and remain agnostic about the topology of the rest of the network. The drawback is that their work is limited to static field/parameter estimation — when the network becomes disconnected for dynamic problems, the estimate priors can drift away while still having some mutual information. Performing distributed averaging on those priors is erroneous because the mutual information is counted multiple times. In order to solve this problem one would have to resort to decentralized estimators that account for the correlations between local estimates.

In [5], a Decentralized Delayed-State Extended Information Filter (DDSEIF) is described that handles the correlation between local estimates. This method only works in directed networks that do not have any loops. It is claimed that under certain assumptions local estimates would converge to the centralized estimate. However, the method requires a large amount of data communication, storage memory, and book-keeping overhead, and therefore, does not lend itself to online resource constrained recursive distributed state estimation.

Another approach to deal with the correlation of local estimates is to use Covariance Intersection (CI) methods [6] that produce conservative estimates in the absence of correlation knowledge. The work in references [7], [6], [8], [9], [10], [11] fall into this category. They propose different optimization criteria to perform CI and/or use

different iterative CI schemes for decentralized state estimation.

The downside of decentralized CI based methods is that they produce overly conservative estimates by performing the covariance intersection unnecessarily on new information (which is generally uncorrelated) at the current step. This incurs significant performance loss compared to MHMC based distributed averaging, which is a superior way to reach consensus on uncorrelated information.

In this paper we strive to bring together the best of MHMC based distributed averaging and CI. The former is suitable for reaching consensus over uncorrelated information and the later is useful for combining estimates whose correlations are unknown or difficult to keep track of. We propose a hybrid scheme that has comparable performance to MHMC consensus while being robust to network failures. The method is explained with respect to the dynamic field estimation example, but it is applicable more generally, to most decentralized estimation scenarios.

In Section II, the notation used in this paper is explained as well as assumptions and system model. Section III discusses some preliminaries on decentralized estimation which paves the way for introducing our problem objective and method. Our proposed method is presented in Section IV along with its theoretical performance analysis. We extensively evaluate our method's performance in Section V.

II. MODELING

Stochastic Field Model: We consider the case where we have noise and the system is stochastic, thus model the evolution of a time-varying, dynamic field using the following equation which relates the state at time step k to $k + 1$:

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + B\mathbf{u}(k) + w(k). \qquad (1)$$

In the above equation $\mathbf{u}(k) \in \mathbb{R}^m$ accounts for m input variables and the vector $w(k) \sim \mathcal{N}(0, Q(k))$ represents additive white noise used to model unknown perturbations.

Network Topology: Assume that we have N homogeneous agents associated with nodes of a graph. These agents can communicate with each other under a time-varying undirected network topology $G_k = \langle \mathcal{V}_k, \mathcal{E}_k \rangle$ where $\mathcal{V}_k$ and $\mathcal{E}_k$ are the set of graph nodes and edges respectively. If $(i, j) \in \mathcal{E}_k$, it means agents i and j can communicate. The node corresponding to the i-th agent is denoted by v_i. Neighbors of node v_i are defined as

$$\overline{\mathcal{N}}_i = \{\forall v_j \in \mathcal{V}, (i, j) \in \mathcal{E}\}. \qquad (2)$$

The set $\mathcal{N}_i = \overline{\mathcal{N}}_i \cup \{v_i\}$ will also be used in some of the equations. $|\mathcal{N}_i|$ is the cardinality of $\mathcal{N}_i$.

Each agent has a processor and a sensory package on-board. Sensors make observations every Δt seconds and the processors and the network are fast enough to handle calculations based on message passing among agents every δt seconds. We assume that $\delta t \ll \Delta t$. We also assume that the agents exchange their information over the communication channel which is free of both delay and error.

We assume that $\mathbf{x}(k)$ denotes the state of the field at time-step k. Each agent retains a local version of $\mathbf{x}(k)$ which is denoted by $\mathbf{x}_i(k)$. For random variables we use the following notation: $\hat{\mathbf{x}} = \mathbb{E}(\mathbf{x})$ and $P_{\mathbf{x}} = \mathbb{E}[\mathbf{x} - \hat{\mathbf{x}}]^2$ are the expected value and the covariance of the random variable $\mathbf{x}$ respectively.

Observation Model: We assume that each agent has a sensor that produces observations which are functions of the state of the field and are noisy. The observation model of the i'th sensor is

$$z_i(k) = H_i(k)\mathbf{x}(k) + v_i(k), \qquad (3)$$

$$v_i(k) \sim \mathcal{N}(0, R_i(k)). \qquad (4)$$

III. DECENTRALIZED FILTERING PRELIMINARIES

Filtering is the process of recursively computing the posterior probability of a random dynamic process $\mathbf{x}(k)$ conditioned on a sequence of measurements $Z^k = \{z(1), z(2), \dots, z(k)\}$, where $z(k)$ denotes the observation vector at the time-step k. Under the Gaussian assumption, the Kalman Filter (KF) is the optimal recursive filter for linear state space systems. We denote the predicted and estimated mean and covariance at time k by $(\hat{\mathbf{x}}^-(k), P^-(k))$ and $(\hat{\mathbf{x}}(k), P(k))$.

Centralized Kalman Filter: The KF steps are generally formulated based on the mean and covariance matrix representation of Gaussian random variables involved; however, an alternative representation, called the information form of the KF is more useful and intuitive in the development of the decentralized filter. In this representation we define

$$\mathbf{y}(k) = P_{\mathbf{x}}^{-1}(k)\mathbf{x}(k), \qquad (5a)$$

$$\mathbf{Y}(k) = P_{\mathbf{x}}^{-1}(k), \qquad (5b)$$

where $\mathbf{y}(k)$ and $\mathbf{Y}(k)$ are the information vector and information matrix respectively. The prediction step of the KF can then be written as

$$M(k) = (A^{-1})^T \mathbf{Y}(k-1) A^{-1}, \qquad (6a)$$

$$P(k) = M(k) + Q(k)^{-1}, \qquad (6b)$$

$$\mathbf{Y}^-(k) = M(k) - M(k)P(k)^{-1}M(k), \qquad (6c)$$

$$\mathbf{y}^-(k) = \mathbf{Y}^-(k)A\mathbf{Y}(k-1)\mathbf{y}(k-1). \qquad (6d)$$

The information content of an observation $z_j(k)$ is $\delta i_j(k) = H_j^T(k)R_j(k)^{-1}z_j(k)$ along with the information matrix $\delta I_j(k) = H_j^T(k)R_j(k)^{-1}H_j(k)$. Assuming

that information from all agents is available to a central processor, the update step of KF can be carried out by adding the information from different observations to the predicted values.

$$\mathbf{y}(k) = \mathbf{y}^-(k) + \sum_{j=1}^{N} \delta i_j(k) \qquad (7a)$$

$$\mathbf{Y}(k) = \mathbf{Y}^-(k) + \sum_{j=1}^{N} \delta I_j(k) \qquad (7b)$$

This formulation is called the Centralized Information Filter (CIF).

An assumption underlying the CIF is that there is a central processor which has access to all the information available. However, when there is no central processor and each agent can only communicate with its neighbors, we want to formulate a decentralized version of the information filter. When run by all agents they should converge to the centralized estimate of the field state.

Decentralized Estimator Designs

1) Consensus Based Estimator: We start with CIF procedure outlined in previous section. Looking at Eqs. 7a–7b, one can see that

$$[\delta I, \delta i](k) \triangleq N . \frac{1}{N} \sum_{j=1}^{N} [\delta I_j, \delta i_j](k) \triangleq N[\overline{\delta I}, \overline{\delta i}](k).$$

Now if all the agents have the same prior information and if via a distributed averaging method the agents can reach a consensus over $\overline{\delta i}(k)$ and $\overline{\delta I}(k)$, they can use Eqs. 7a–7b to get a decentralized estimate whose results asymptotically converge to the centralized estimate.

Fortunately, such a method exists. The distributed averaging method of [4] makes minimal assumptions about the network topology and only relies on local information exchange between neighboring nodes of a graph to reach a consensus over the average initial value of the nodes. The method uses an iterative linear consensus filter based on the weights calculated from an MHMC. Throughout this paper, to avoid confusion, we use superscript l to indicate the consensus iterations. Consider communication graph G^l. One can use the message passing protocol of the form $\mathbf{x}^{l+1} = \sum_{j=1}^{|\mathcal{N}^l|} \gamma_{ij}^l x_j^l$ to calculate the average of the values on the graph nodes in which $d_i^l = |\overline{\mathcal{N}}_i^l|$ is the degree of the node v_i, and

$$\gamma_{ij}^l = \begin{cases} \frac{1}{1+\max\{d_i^l, d_j^l\}} & \text{if } (i,j) \in \mathcal{E}^l, \\ 1 - \sum_{(i,m)\in\mathcal{E}^l} \gamma_{im} & \text{if } i = j, \\ 0 & \text{otherwise .} \end{cases} \qquad (8)$$

Note that for each node i, γ_{ij}'s only depend on the degrees of its neighboring nodes. Also, due to the averaging property of MHMC weights, after reaching consensus, MHMC estimates converge to the centralized estimator's results. Therefore, given the ideal centralized estimate $(\hat{\mathbf{x}}^{\text{CTR}}, P_{\mathbf{x}}^{\text{CTR}})$, we have $\hat{\mathbf{x}}_i^{\text{MH}} = \hat{\mathbf{x}}^{\text{CTR}}$ and $P_{\mathbf{x}_i}^{\text{MH}} = P_{\mathbf{x}}^{\text{CTR}}$ in the limit.

In practice the priors become different as a result of network disconnection. In those cases agents have some shared information (from the time they were connected to each other) but will accumulate new information whilst disconnected from one another. Their priors will differ after reconnection so, consequently, their consensus must be handled with care.

2) Covariance Intersection Based Estimator: It follows from the above discussion that if the priors are not the same among the network nodes, distributed averaging alone will not produce consistent estimates. One way of handling such a scenario is using Covariance Intersection (CI) methods. We may use an iterative CI method to reach a consensus over the local estimates when the priors differ, either owing to disconnection or termination of the consensus process over-early. In iterative CI, the goal is to fuse different estimates of a random variable without having any knowledge about the cross covariance between such estimates. Iterative CI, iteratively solves the following optimization problem and updates local estimates accordingly until it reaches consensus.

Iterative CI (ICI): At initial iteration $l = 0$, for each agent, assign the local estimate, $[\mathcal{Y}_i^0, y_i^0]$, to be

$$\mathcal{Y}_i^0 \triangleq \mathbf{Y}_i(t_0) + \delta I_i(t_0), \quad y_i^0 \triangleq \mathbf{y}_i(t_0) + \delta i_i(t_0).$$

Then for each iteration afterward solve for w^* such that

$$\omega^* = \underset{\omega}{\arg\min} \mathcal{J}\big([\sum_{j\in\mathcal{N}_i^l} \omega_j \mathcal{Y}_j^l]^{-1}\big),$$
$$s.t. \quad \sum_{j=1}^{|\mathcal{N}_i^l|} \omega_j = 1, \quad \forall j \quad \omega_j \geq 0, \qquad (9)$$

where $\mathcal{J}(\cdot)$ is an optimization objective function; we consider $trace(\cdot)$ or $\log\det(\cdot)$. Local estimates are then updated for the next iteration

$$[\mathcal{Y}_i^{l+1}, y_i^{l+1}] = \sum_{j\in\mathcal{N}_i^l} \omega_j^* [\mathcal{Y}_j^l, y_j^l]. \qquad (10)$$

As discussed in [6], CI and consequently iterative CI (ICI) generate conservative estimates which means that $\mathbb{E}[\mathbf{x} - \hat{\mathbf{x}}^{\text{ICI}}] = \mathbb{E}[\mathbf{x} - \hat{\mathbf{x}}^{\text{CTR}}] = 0$ and $P_{\mathbf{x}_i}^{\text{ICI}} \geq P_{\mathbf{x}}^{\text{CTR}}$ for the local estimates and the consensus value. The disadvantage of CI is that it generates overly conservative estimates by continually neglecting the cross correlation information.

Problem Objective

Our goal is to design a network agnostic recursive decentralized estimator to calculate the local estimate $\mathbf{x}_i^{\text{HYB}}$ along with an associated covariance $P_{\mathbf{x}_i}^{\text{HYB}}$ such that the following properties hold:

$$\mathbb{E}[\mathbf{x} - \hat{\mathbf{x}}_i^{\text{ICI}}] = \mathbb{E}[\mathbf{x} - \hat{\mathbf{x}}_i^{\text{HYB}}] = \mathbb{E}[\mathbf{x} - \hat{\mathbf{x}}^{\text{CTR}}] = 0,$$
$$\mathcal{J}(P_{\mathbf{x}}^{\text{CTR}}) \leq \mathcal{J}(P_{\mathbf{x}_i}^{\text{HYB}}) \leq \mathcal{J}(P_{\mathbf{x}_i}^{\text{ICI}}), \qquad (11)$$

i.e., we are looking for an unbiased estimate whose covariance is less than that of CI.

Algorithm 1: Hybrid Method

Input : $[\mathbf{y}_j(t_0), \mathbf{Y}_j(t_0)]$

1 Use Eqs. 6c – 6d to calculate predicted values $[\mathbf{y}_j^-(t_1), \mathbf{Y}_j^-(t_1)]$ given $[\mathbf{y}_j(t_0), \mathbf{Y}_j(t_0)]$

2 Collect local observation $z_j(t_1)$ and calculate jacobian and noise covariance $[H_j(t_1), R_j(t_1)]$

3 Calculate the local information update

$$\delta i_j(t_1) = H_j^T(t_1) R_j^{-1}(t_1) z_j(t_1)$$
$$\delta I_j(t_1) = H_j^T(t_1) R_j^{-1}(t_1) H_j(t_1)$$

4 Initialize consensus variables $(l = 0)$

$$[y_j^0, \mathcal{Y}_j^0] = [\mathbf{y}_j^-, Y_j^-](t_1) \quad [\overline{\delta i}_j^0, \overline{\delta I}_j^0] = [\delta i_j, \delta I_j](t_1)$$

5 **while** *NOT CONVERGED* **do**

6   BROADCAST$[y_j^l, \mathcal{Y}_j^l, \overline{\delta i}_j^l, \overline{\delta I}_j^l]$

7   RECEIVE$[y_k^l, \mathcal{Y}_k^l, \overline{\delta i}_k^l, \overline{\delta I}_k^l] \quad \forall k \in \overline{\mathcal{N}}_j^l$

8   Collect received data

$$\mathcal{C}_j^l = \{y_{k \in \mathcal{N}_j^l}, \mathcal{Y}_{k \in \mathcal{N}_j^l}\} \quad \mathcal{M}_j^l = \{\overline{\delta i}_{k \in \mathcal{N}_j^l}, \overline{\delta I}_{k \in \mathcal{N}_j^l}\}$$

9   Do one iteration of CI on consensus variables for local prior information $\mathcal{C}_j^l$

$$[y_j^{l+1}, \mathcal{Y}_j^{l+1}] = \text{CI}(\mathcal{C}_j^l)$$

10   Do one iteration of MHMC on consensus variables for new information $\mathcal{C}_j^l$

$$[\overline{\delta i}_j^{l+1}, \overline{\delta I}_j^{l+1}] = \text{MHMC}(\mathcal{M}_j^l)$$

11 $l \leftarrow l + 1$

12 Calculate the posteriors according to:

$$\mathbf{Y}_j(t_1) = \mathcal{Y}_j^l + n_{CG} \overline{\delta I}_j^l$$
$$\mathbf{y}_j(t_1) = y_j^l + n_{CG} \overline{\delta i}_j^l$$

return $[\mathbf{Y}_j(t_1), \mathbf{y}_j(t_1)]$

IV. HYBRID CI CONSENSUS

We propose a hybrid approach that uses ICI to reach consensus over priors and the MHMC based consensus filter for distributed averaging of local information updates. Our method is summarized in Algorithm 1. We explain the flow of the proposed method using a simple scenario with two agents. Generalization to more than two agents is straightforward and follows similar steps.

Imagine a scenario consisting of two agents, observing a dynamic field with state vector $\mathbf{x}$, that are communicating with each other through a time-varying network topology. At time t_0, the agents start with priors $[\mathbf{y}_1^-(t_0), \mathbf{Y}_1^-(t_0)]$ and $[\mathbf{y}_2^-(t_0), \mathbf{Y}_2^-(t_0)]$ respectively.

At time t_1 the field evolves to the new state $\mathbf{x}(t_1)$ and agents calculate their own local prediction (line 1 in the algorithm). Then they make observations $z_1(t_1)$ and $z_2(t_1)$, respectively, and compute the local information updates $[\delta i_1(t_1), \delta I_1(t_1)]$ and $[\delta i_2(t_1), \delta I_2(t_1)]$ (lines 2 and 3 of the algorithm).

The two agents, if performing ICI, would find a fused estimate

$$Y^{\text{ICI}} = w^{\text{ICI}}(\mathbf{Y}_1^- + \delta I_1) + (1 - w^{\text{ICI}})(\mathbf{Y}_2^- + \delta I_2),$$

where w^{ICI} is obtained from solving the optimization problem in Eq. 9. Note that doing MHMC alone is not possible here since $\mathbf{Y}_1^-$ and $\mathbf{Y}_2^-$ are different. In our hybrid method we do the following:

$$Y^{\text{HYB}} = \underbrace{w^{\text{HYB}} \mathbf{Y}_1^- + (1 - w^{\text{HYB}}) \mathbf{Y}_2^-}_{\substack{\text{CI to reach} \\ \text{consensus over priors}}} + \underbrace{\delta I_1 + \delta I_2}_{\substack{\text{consensus over} \\ \text{the incremental} \\ \text{information}}}.$$

It can be seen that $\delta I_1 + \delta I_2 \geq w^{\text{CI}} \delta I_1 + (1 - w^{\text{CI}}) \delta I_2$ and $\mathcal{J}(w^{\text{HYB}} \mathbf{Y}_1^- + (1 - w^{\text{HYB}}) \mathbf{Y}_2^-) \geq \mathcal{J}(w^{\text{CI}} \mathbf{Y}_1^- + (1 - w^{\text{CI}}) \mathbf{Y}_2^-)$ due to the fact that the optimization problem for $\mathbf{Y}_2^-$ and $\mathbf{Y}_2^-$ has the optima w^{HYB}. If $\mathcal{J}(\cdot)$ has the property that if $\mathcal{J}(\mathcal{Y}_1) \geq \mathcal{J}(\mathcal{Y}_2)$ and $\mathcal{I}_1 \geq \mathcal{I}_2$ then $\mathcal{J}(\mathcal{Y}_1 + \mathcal{I}_1) \geq \mathcal{J}(\mathcal{Y}_2 + \mathcal{I}_2)$, then our method is guaranteed to outperform CI.

For an N-agent system with the i'th agent having prior Y_i^-, the ICI approach is used to find a consensus over the priors using Eq. 9 recursively. The MHMC approach is used to form the consensus over the new information, i.e., $\sum_{j=1}^N \delta I_j$ (Eq. 8). In line 12 of the algorithm, n_{CG} is the number of agents that form a connected group, and it can be determined by assigning unique IDs to the agents and passing these IDs along with the consensus variables. Each agent keeps track of unique IDs it receives and passes them to its neighbors. The following propositions hold. (They are stated here without proof due to paucity of space. The reader is referred to the technical report for details.[1])

Proposition 1. *The ICI process is guaranteed to reach a consensus over the priors, i.e., $\exists \mathcal{Y}_\star$, such that $\forall i$ $\lim_{l \to \infty} \mathcal{Y}_i^l = \mathcal{Y}_\star$. The same result holds for the information vector as well.*

Remark 1. *It is straightforward to show that ICI and our method both produce unbiased estimates. Regarding the second part of Eq. 11, our extensive experiments on the system considered in this paper and various other systems show that the inequality holds for $\mathcal{J} = \log \det(\cdot)$. The existence of a theoretical proof is currently being investigated and will be the subject of future work.*

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[1] http://edplab.org/wp-content/uploads/2016/02/technical_report.pdf

V. Experiments

We report two sets of experiments performed on an atmospheric dispersion problem to show the effectiveness of our method and evaluate its performance during disconnection and after reconnection. This is a three dimensional problem and after proper discretizing of its Partial Differential Equation (PDE), we get a system in the form of Eq. 1.

For our experiments after discretization, the dimension of the state is 80. We assume that there are 10 sources emitting pollutant Zinc (referred to as Zn from now on) into atmosphere. There are also 9 receptors making noisy measurements of the concentrations of Zn around their location in space. We assume that receptors can communicate to each other through a time varying network which does not remain connected at all times. Receptors receive information only from their immediate neighbors. They all have access to the sources' locations and the source emission is modeled as a white noise process with known covariance.

1) The effect of disconnection on estimation performance: In this experiment we intend to evaluate the performance of the proposed method during the phase where some receptors become disconnected from the rest of the group and are reconnected again after some interval. The topology of the network takes one of the forms depicted in Fig. 1. Initially the network is fully connected but beginning with timestep 3, receptors 7, 8 and 9 become isolated and remain in this situation for 2 steps, until they are connected back to the rest of the receptors. Similarly, disconnection happens in intervals $[17 - 20]$ and $[23 - 30]$.

In order to make a comparison we obtain the estimation result using ICI, our method and also a hypothetical god's-eye-view centralized estimator to see how much of its performance can be recovered. Note that the MHMC consensus cannot be done here due to disconnection. The results are summarized visually in Fig. 2.

We use three measures to evaluate the estimates.

1) The Bhattacharyya distance [12] between the estimation results and the centralized estimator. The Bhattacharyya distance can be used to evaluate the similarity of two continuous probability distribution function. For Gaussian distributions parametrized as (μ_1, Σ_1), and (μ_2, Σ_2), $0 \leq D_B(p_1, p_2) \leq 1$ and is defined:

$$D_B(p_1, p_2) = e^{-D(p_1, p_2)} \quad (12)$$
$$D(p_1, p_2) = \tfrac{1}{8}(\mu_1 - \mu_2)^T \Sigma(\mu_1 - \mu_2) +$$
$$\tfrac{1}{2} \ln \frac{\det \Sigma}{\sqrt{\det \Sigma_1 \det \Sigma_2}}, \quad \Sigma = \frac{\Sigma_1 + \Sigma_2}{2}$$

Here $D_B(p_1, p_2) = 1$ means complete similarity and $D_B(p_1, p_2) = 0$ means complete dissimilarity.
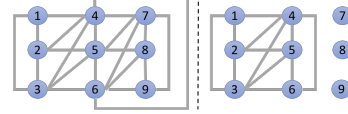

Fig. 1. Topology of the Network when all receptors are connected (left) and when receptors 7,8 and 9 get disconnected from the rest of the group (right).
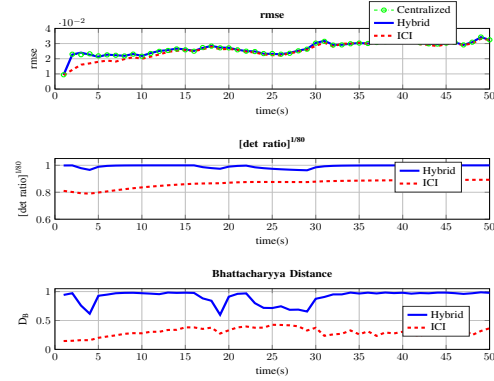

Fig. 2. Comparison of the estimation results using centralized Kalman filter, ICI, and the proposed method. (Note that the lower two figures are measures normalized so that the centralized solution = 1.0.)

2) $\left[\frac{\det P_{cen}}{\det P_{CI/Hyb}} \right]^{\frac{1}{n_d}}$ which is a non-dimensionalized measure of covariance volume ratio in which n_d is the dimensionality of the state vector.

3) $rmse$ estimation error.

As it can be seen, the proposed method outperforms ICI as expected and is able to get the performance very close to centralized estimator results after reconnection. Based on Bhattacharyya distance, closeness between centralized and decentralized estimates for drops during disconnection interval as expected since receptors do not have access to all the information available to the centralized estimator. While the proposed method is able to immediately recover after reconnection, ICI continues to have lower performance even after reconnection due to the fact that it ignores the correlations.

Fig. 3 takes a closer look at the performance of the proposed method and compares the estimation results of receptor 5 and 8 during two different time steps. The horizontal axes represent consensus steps not time. Based on Fig. 1, receptor 5 remains in a group of size 6 during disconnection period whereas receptor 8 is totally isolated for that duration. The higher difference between centralized and decentralized estimate for this receptor can be explained based on the fact that it has less information at its disposal. However, after reconnection both receptors are able to converge to the same value which is very close to the centralized estimator.

2) Performance analysis and robustness to link failure: In this experiment we evaluate the performance of our
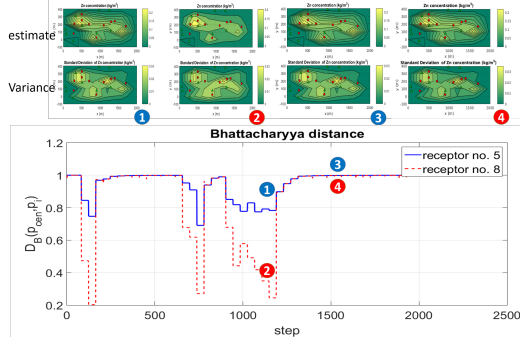
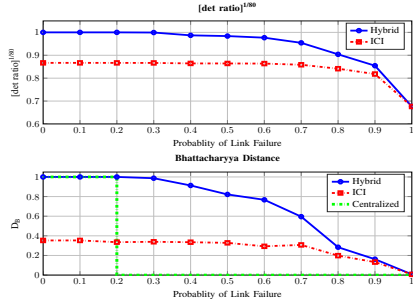Fig. 3. Estimation performance comparison between receptors.

Fig. 4. Composite diagram for performance comparison for different probablity of link failure.

method in a systematic way to establish its robustness and usefulness in networks with high-likelihood of link failure. We consider the same system as in the first experiment and simulate it for 50 time steps. At the beginning of each step a 4 regular graph with 9 nodes is generated and, given a probability of failure for each link, some links in the graph will randomly be disconnected. The graph still remains connected some percentage of times, but this depends on the degree, and probably of failure. However, if the regularity degree goes down or the probability of failure increases, more often than not, the graph becomes disconnected.

In practice, for $p \geq 0.2$, consensus methods are no longer guaranteed to succeed since the network almost always suffers disconnection at some point in time.

We ran our method for 50 steps for each probability of link failure and compared its performance with the ideal centralized result (which is obtained by assuming full connectivity at all times). The performance is evaluated by calculating the average value for Bhattacharyya distance and determinant ratio measure at all steps and for all receptors. Based on Fig. 4, for the case considered in this experiment, our decentralized estimator performs very similarly to the ideal centralized one for $p \in [0.0, 0.4]$ while drastically outperforming ICI all the time. This means that in the case considered here, our

method can perform almost as well as the ideal estimator for an unreliable network. Obviously, the performance can vary from one system to another and under different network topologies. However, our method can recover the performance of the centralized method when the network is unreliable and substantially outperforms ICI always as it has already been established theoretically.

VI. Conclusion

This paper proposes a decentralized estimator for dynamic systems in networks with changing topology and those that do not remain connected all the time. Separating the process of consensus for the correlated and uncorrelated information was the key to achieve a better performance compared to ICI alone. Evaluating the proposed method on an 80-dimensional estimation problem showed substantial performance improvement compared to CI and also the ability to recover after a disconnection interval occurs.

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