



Aerodynamic-Dynamic Interactions and Multi-Body Formulation of Flapping Wing Dynamics: Part I - Modeling

Ahmed M. Hassan* and Haithem E. Taha†

University of California, Irvine, CA 92697

Flapping flight dynamics constitutes a multi-body, nonlinear, time-varying system. The two major simplifying assumptions in the analysis of flapping flight stability are neglecting the wing inertial effects and averaging the dynamics over the flapping cycle. Relaxing the first assumption invokes a multi-body formulation of the equations of motion. In this work, the full, multi-body, equations of motion governing the longitudinal flapping flight dynamics near hover are considered. The aerodynamic loads are represented through a relatively simple analytical model that accounts for the dominant contributions; e.g., leading edge vortex and rotational effects. The dynamic and aerodynamic models are coupled together to account for mutual interactions.

I. Introduction

The flight dynamics of flapping-wing micro-air-vehicles (FWMAVs) constitutes a nonlinear, time-varying, multi-body dynamical system. It is also a multi-scale dynamical system because of the associated two time scales; the time scale of the fast flapping motion and the associated aerodynamic loads, and the relatively slow time scale of the body motion. The interaction between the periodic aerodynamic loads and the body motion may induce some interesting stabilizing mechanisms [1, 2]. All of these interesting dynamical behaviors and challenges led to a recent flurry in the research on the flight dynamics of FWMAVs.

Two major assumptions are usually adopted in the flight dynamic analysis of FWMAVs [3]. These include neglecting the wing inertial effects and averaging the dynamics over the flapping cycle. The first assumption may be justifiable because the mass of the wing is quite small when compared to that of the body (less than 5% [28]). Moreover, adopting this assumption removes the multi-body nature of the problem and yields equations of motion of exactly the same form as those governing the flight dynamics of conventional aircraft. As such, most of the previous work on dynamics and control of flapping flight have neglected the wing inertial effects [4–12]. Sun et al. [13] derived the full dynamic equations of flapping flight and showed the specific terms that are omitted by neglecting the wing inertial effects. They simulated the reduced system, quantified these eliminated terms, and showed that these terms are of a small magnitude and, hence, justified such an assumption. On the other hand, few attempts have aimed at investigating the effects of wing inertia. Orlowski and Girard [14] showed that neglecting the wing mass may be problematic in dynamics and control studies as it may lead to different dynamical behaviors. Also, Bolender [15] concluded that for proper development of control laws for FWMAVs, the wing mass should be included. Weihua and Cesnik [16] studied the dynamic stability of flexible FWMAVs including the wing inertial effects. They found that increasing the wing inertia leads to destabilizing effects in both of the longitudinal and lateral dynamics. For more details about the inertial effects of the wing motion on the dynamics of flapping flight, the reader is referred to the review article by Taha et al. [3] and the references therein.

In this work, the full, multi-body, equations of motion governing the longitudinal flight dynamics of FWMAVs near hover are considered. A relatively simple, analytical aerodynamic model that accounts for the dominant contributions (e.g., leading edge vortex and rotational effects) is adopted. Combining these two models, the nonlinear, multi-body, time-varying mechanical equations are derived. The derived equations

*PhD Student, Mechanical and Aerospace Engineering. Student Member AIAA.

†Assistant Professor, Henry Samueli Career Development Chair, Mechanical and Aerospace Engineering. Member AIAA

will then be used for trim and stability analysis of hovering FWMAVs/insects in the second part of this work.

II. Wing Kinematics

Four reference frames are required to study the flight dynamics of a rigid-wing FWMAV: an inertial reference frame $\{x_1, y_1, z_1\}$, a body-fixed reference frame $\{x_b, y_b, z_b\}$, a stroke plane reference frame $\{x_s, y_s, z_s\}$, and a wing-fixed reference frame $\{x_w, y_w, z_w\}$ for each of the flight vehicle's wings. Because only longitudinal flight is considered in this work, symmetric wing motions are assumed.

By convention, the x_b -axis points forward defining the vehicle's longitudinal axis, the y_b -axis points to starboard, and the z_b -axis completes the right-handed frame. The conventional yaw-pitch-roll (ψ - θ - ϕ) Euler angle sequence, traditionally used with fixed-wing aircraft [17], is adopted here to describe the body's inertial orientation. Because only longitudinal flight is considered, however, only the body's pitch angle θ is required.

The stroke-plane is inclined to the horizontal plane with a stroke plane angle β . That is, the stroke plane reference frame is obtained from the inertial frame through a rotation by an angle β about the y_1 -axis. The wing-fixed frame is defined such that it is aligned with the stroke plane frame at zero wing kinematic angles. The wing motion is typically described using three Euler angles: the flapping angle φ (describing back and forth motion along the stroke plane), the plunging angle ϑ (describing out of stroke plane motion), and the pitching angle η (describing rotation of the wing about a chord line). Consistent with observations of biological flyers [18, 19], the wing motion is restricted such that the plunging angle ϑ remains zero. Figure 1 shows a schematic diagram for a FWMAV.

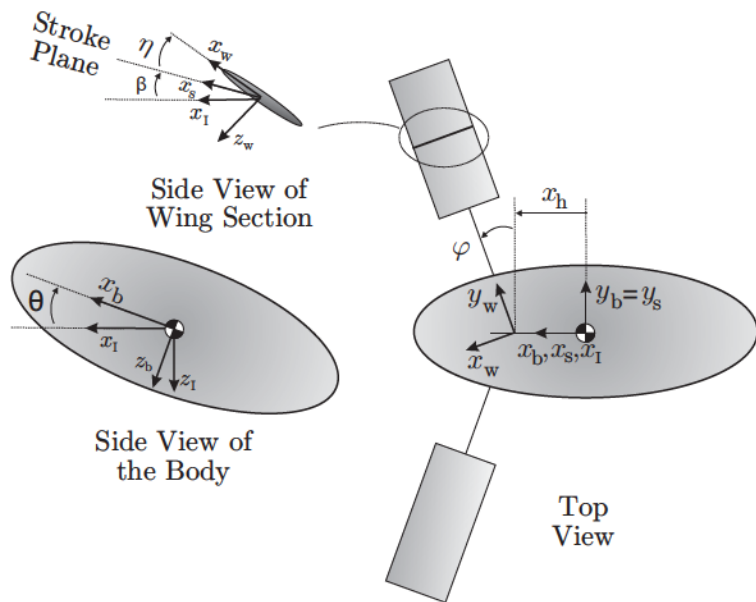


Figure 1. A schematic diagram for a FWMAV hovering in a general orientation.

Figure 1 shows a schematic diagram for a FWMAV.

III. Equations of Motion

Since the two wings move symmetrically, the equations of motion are defined in terms of five generalized coordinates: $\mathbf{q} = [x, z, \theta, \varphi, \eta]^T$, where x and z are the inertial coordinates of the body center of mass along the x_1 and z_1 axes, respectively. In the following subsections, the principle of virtual power [20] is used to derive the equations of motion

$$\sum_{i=b,w} [m_i(\dot{v}_i + \ddot{\rho}_{ci}) - \mathbf{f}_i] \cdot \frac{\partial \mathbf{v}_i}{\partial \dot{q}_j} + [\dot{\mathbf{h}}_i + m_i \rho_{ci} \times \dot{\mathbf{v}}_i - \mathbf{M}_i] \cdot \frac{\partial \boldsymbol{\omega}_i}{\partial \dot{q}_j} = 0, \quad (1)$$

where $j = \{1, \dots, 5\}$, m_i is the mass of the i^{th} rigid body, $\mathbf{v}_i$ is the inertial velocity vector of its reference point (the reference points of the body and wing frames are the body's center of gravity and the hinge root, respectively), ρ_{ci} is the vector pointing from the reference point of the i^{th} rigid body to its center of gravity, $\boldsymbol{\omega}_i$ and $\mathbf{h}_i$ are the angular velocity vector of the i^{th} rigid body with respect to the inertial frame and the corresponding angular momentum vector, respectively, and $\mathbf{f}_i$ and $\mathbf{M}_i$ are the external force and moment vectors applied on the i^{th} rigid body at its reference point.

A. Body

The linear velocity of the reference point of the body axis system (the body's center of gravity) and the corresponding angular velocity are written as

$$\mathbf{v}_b = \dot{x}\mathbf{i}_I + \dot{z}\mathbf{k}_I \quad \text{and} \quad \boldsymbol{\omega}_b = \dot{\theta}\mathbf{j}_b = \dot{\theta}\mathbf{j}_I,$$

where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors along the x , y , and z directions in the axis-system indicated by the subscript. Thus

$$\begin{aligned} \frac{\partial \mathbf{v}_b}{\partial \dot{x}} &= \mathbf{i}_I & \frac{\partial \mathbf{v}_b}{\partial \dot{z}} &= \mathbf{k}_I & \frac{\partial \mathbf{v}_b}{\partial \dot{\theta}} &= \mathbf{0} & \frac{\partial \mathbf{v}_b}{\partial \dot{\varphi}} &= \mathbf{0} & \frac{\partial \mathbf{v}_b}{\partial \dot{\eta}} &= \mathbf{0} \\ \frac{\partial \boldsymbol{\omega}_b}{\partial \dot{x}} &= \mathbf{0} & \frac{\partial \boldsymbol{\omega}_b}{\partial \dot{z}} &= \mathbf{0} & \frac{\partial \boldsymbol{\omega}_b}{\partial \dot{\theta}} &= \mathbf{j}_I & \frac{\partial \boldsymbol{\omega}_b}{\partial \dot{\varphi}} &= \mathbf{0} & \frac{\partial \boldsymbol{\omega}_b}{\partial \dot{\eta}} &= \mathbf{0} \end{aligned},$$

and

$$\dot{\mathbf{v}}_b = \ddot{x}\mathbf{i}_I + \ddot{z}\mathbf{k}_I.$$

The angular momentum vector of the body about the body center of gravity and its inertial derivative are given by

$$\mathbf{h}_b = I_{yb}\dot{\theta}\mathbf{j}_I, \quad \dot{\mathbf{h}}_b = I_{yb}\ddot{\theta}\mathbf{j}_I.$$

The aerodynamic contribution of the body is neglected and, hence, the body exhibits gravitational forces only with no moments. Thus, the body force in the inertial frame is written as

$$\mathbf{f}_b^{(I)} = [0, \ 0, \ m_b g]^T.$$

B. Wing

The linear velocity of the reference point of the wing frame (the hinge root) and its angular velocity are written as

$$\mathbf{v}_w = (\dot{x} - x_h\dot{\theta}\sin\theta)\mathbf{i}_I + (\dot{z} - x_h\dot{\theta}\cos\theta)\mathbf{k}_I, \quad \boldsymbol{\omega}_w = \dot{\theta}\mathbf{j}_b - \dot{\varphi}\mathbf{k}_s + \dot{\eta}\mathbf{j}_w.$$

Thus

$$\begin{aligned} \frac{\partial \mathbf{v}_w}{\partial \dot{x}} &= \mathbf{i}_I & \frac{\partial \mathbf{v}_w}{\partial \dot{z}} &= \mathbf{k}_I & \frac{\partial \mathbf{v}_w}{\partial \dot{\theta}} &= -x_h(\sin\theta\mathbf{i}_I + \cos\theta\mathbf{k}_I) & \frac{\partial \mathbf{v}_w}{\partial \dot{\varphi}} &= \mathbf{0} & \frac{\partial \mathbf{v}_w}{\partial \dot{\eta}} &= \mathbf{0} \\ \frac{\partial \boldsymbol{\omega}_w}{\partial \dot{x}} &= \mathbf{0} & \frac{\partial \boldsymbol{\omega}_w}{\partial \dot{z}} &= \mathbf{0} & \frac{\partial \boldsymbol{\omega}_w}{\partial \dot{\theta}} &= \mathbf{j}_I & \frac{\partial \boldsymbol{\omega}_w}{\partial \dot{\varphi}} &= -\mathbf{k}_b & \frac{\partial \boldsymbol{\omega}_w}{\partial \dot{\eta}} &= -\mathbf{j}_w \end{aligned},$$

and

$$\dot{\mathbf{v}}_w = [\ddot{x} - x_h\ddot{\theta}\sin\theta - x_h\dot{\theta}^2\cos\theta]\mathbf{i}_I + [\ddot{z} - x_h\ddot{\theta}\cos\theta + x_h\dot{\theta}^2\sin\theta]\mathbf{k}_I.$$

The rotation matrices from the the stroke plane frame to the wing frame are

$$\mathbf{R}_\varphi = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{R}_\eta = \begin{bmatrix} \cos\eta & 0 & -\sin\eta \\ 0 & 1 & 0 \\ \sin\eta & 0 & \cos\eta \end{bmatrix},$$

and

$$\mathbf{R}_{ws} = \mathbf{R}_\eta\mathbf{R}_\varphi.$$

The wing angular velocity vector in the wing frame is

$$\boldsymbol{\omega}_w^{(w)} = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \mathbf{R}_{ws} \begin{pmatrix} 0 \\ \dot{\theta} \\ -\dot{\varphi} \end{pmatrix} + \begin{pmatrix} 0 \\ \dot{\eta} \\ 0 \end{pmatrix} = \begin{pmatrix} \dot{\varphi}\sin\eta - \dot{\theta}\cos\eta\sin\varphi \\ \dot{\theta}\cos\varphi + \dot{\eta} \\ -\dot{\varphi}\cos\eta - \dot{\theta}\sin\eta\sin\varphi \end{pmatrix}.$$

The position vector pointing from the hinge root to the wing center of gravity is $\rho_{c_w} = -d\mathbf{i}_w + r_{cg}\mathbf{j}_w$ where d and r_{cg} are the distances between the wing root hinge point and the wing center of gravity along the negative x_w -axis and the y_w -axis, respectively. Thus, the inertial acceleration is obtained as

$$\ddot{\rho}_{c_w}^{(w)} = \begin{pmatrix} \ddot{\rho}_1 \\ \ddot{\rho}_2 \\ \ddot{\rho}_3 \end{pmatrix} = \begin{pmatrix} d(\omega_2^2 + \omega_3^2) - r_{cg}(\dot{\omega}_3 - \omega_1\omega_2) \\ -d(\dot{\omega}_3 + \omega_1\omega_2) - r_{cg}(\omega_1^2 + \omega_3^2) \\ d(\dot{\omega}_2 - \omega_1\omega_3) + r_{cg}(\dot{\omega}_1 + \omega_2\omega_3) \end{pmatrix}.$$

Assuming the wing reference frame is fixed in the wing principal axes, the inertial time derivative of the angular momentum vector represented in the wing frame is written as

$$\dot{h}_w^{(w)} = \begin{pmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \end{pmatrix} = \begin{pmatrix} I_x\dot{\omega}_1 + (I_z - I_y)\omega_2\omega_3 \\ I_y\dot{\omega}_y + (I_x - I_z)\omega_1\omega_3 \\ I_z\dot{\omega}_3 + (I_y - I_x)\omega_1\omega_2 \end{pmatrix}.$$

The wing is subject to aerodynamic and gravitational forces. Noting that the y_b -components of the aerodynamic force on each wing are equal and opposite, the force vector applied on the wing is written as

$$\mathbf{f}_w = \begin{pmatrix} F_x \\ 0 \\ F_z \end{pmatrix}^{(w)} + \begin{pmatrix} 0 \\ 0 \\ m_w g \end{pmatrix}^{(I)},$$

where F_x and F_z are the aerodynamic loads in the x_w and z_w directions, respectively. The moment vector comprises three contributions: aerodynamic, gravitational, and the control torque. The aerodynamic contribution M_{a_w} is determined by integrating the radial distributions of the forces F_x and F_z over the wing. That is, $M_{a_w} = M_x\mathbf{i}_w + M_y\mathbf{j}_w + M_z\mathbf{k}_w$, where

$$M_x = 2 \int_0^R F'_z(r)rdr, \quad M_y = 2 \int_0^R F'_z(r)d_{ac}(r)dr, \quad \text{and} \quad M_z = -2 \int_0^R F'_x(r)rdr,$$

where $F'_x(r)$ and $F'_z(r)$ are the two-dimensional aerodynamic loads on an airfoil that is at distance r from the wing root and $d_{ac}(r)$ is the distance between the hinge line and the quarter-chord line at each airfoil section along x_w direction. It should be noted that M_x and M_z will not contribute to the body dynamics because they cancel out due to symmetry of both wings. However, they will contribute to the wing dynamics. Clearly, the actuating torques have to overcome these hinge moments. The gravitational contribution is written as $M_{g_w} = (-d\mathbf{i}_w + r_{cg}\mathbf{j}_w) \times m_w g \mathbf{k}_I$. The last contribution (the control torque) is written as $M_{c_w} = -\tau_\varphi \mathbf{k}_s + \tau_\eta \mathbf{j}_w$, where τ_φ and τ_η are the actuating torque in the flapping and pitching directions, respectively.

Constructing all the required terms to apply the principle of virtual power (1), the equations of motion are obtained as

$$\begin{aligned} & m_w \left(\ddot{\rho}_1 (\cos \beta \cos \eta \cos \varphi - \sin \beta \sin \eta) + \ddot{\rho}_3 (\cos \beta \sin \eta \cos \varphi + \sin \beta \cos \eta) + \ddot{\rho}_2 \cos \beta \sin \varphi + \right. \\ & \left. - x_h \ddot{\theta} \sin \theta - x_h \dot{\theta}^2 \cos \theta \right) + m_v \dot{u} = F_x (\cos \beta \cos \eta \cos \varphi - \sin \beta \sin \eta) + F_z (\cos \beta \sin \eta \cos \varphi + \sin \beta \cos \eta) \end{aligned} \quad (2)$$

$$\begin{aligned} & - m_w \left(\ddot{\rho}_1 (\sin \beta \cos \eta \cos \varphi + \cos \beta \sin \eta) + \ddot{\rho}_3 (\sin \beta \sin \eta \cos \varphi - \cos \beta \cos \eta) + \ddot{\rho}_2 \sin \beta \sin \varphi + \right. \\ & \left. + x_h \ddot{\theta} \cos \theta - x_h \dot{\theta}^2 \sin \theta \right) + m_v (\dot{w} - g) = -F_x (\sin \beta \cos \eta \cos \varphi + \cos \beta \sin \eta) + \\ & - F_z (\sin \beta \sin \eta \cos \varphi - \cos \beta \cos \eta) \end{aligned} \quad (3)$$

$$\begin{aligned}
& m_w \left[-x_h \left(\ddot{\theta} d(\cos \beta \cos \eta \cos \theta \cos \varphi + \sin \beta \cos \eta \sin \theta \cos \varphi - \sin \beta \sin \eta \cos \theta + \cos \beta \sin \eta \sin \theta) + \right. \right. \\
& - \ddot{\theta} r_{cg} \sin \varphi \cos(\beta - \theta) + \dot{\theta}^2 (-r_{cg} \sin \varphi \sin(\beta - \theta) + d \cos \eta \cos \varphi \sin(\beta - \theta) + d \sin \eta \cos(\beta - \theta)) \Big) + \\
& + \dot{u} (-\sin \beta r_{cg} \sin \varphi + d \sin \beta \cos \eta \cos \varphi + d \cos \beta \sin \eta) + \\
& + \dot{w} (-\cos \beta r_{cg} \sin \varphi + d \cos \beta \cos \eta \cos \varphi - d \sin \beta \sin \eta) \Big] + \\
& + I_{y_b} \ddot{\theta} + x_h m_w \left[\ddot{\rho}_3 \sin \beta \sin \eta \cos \theta \cos \varphi + \ddot{\rho}_1 (\cos \eta \cos \varphi \sin(\beta - \theta) + \sin \eta \cos(\beta - \theta)) \right. \\
& - \ddot{\rho}_3 (\cos \beta \sin \eta \sin \theta \cos \varphi - \cos \beta \cos \eta \cos \theta - \sin \beta \cos \eta \sin \theta) + \ddot{\rho}_2 \sin \varphi \sin(\beta - \theta) + g \cos \theta + \\
& + x_h \ddot{\theta} - \dot{u} \sin \theta - \dot{w} \cos \theta \Big] - \dot{h}_3 \sin \eta \sin \varphi - \dot{h}_1 \cos \eta \sin \varphi + \dot{h}_2 \cos \varphi = \\
& = \tau_\eta \cos \varphi - F_z x_h \left(\sin \beta (\cos \eta \sin \theta - \sin \eta \cos \theta \cos \varphi) + \cos \beta (\sin \eta \sin \theta \cos \varphi + \cos \eta \cos \theta) \right) + \\
& + F_x x_h (\cos \eta \cos \varphi \sin(\beta - \theta) + \sin \eta \cos(\beta - \theta)) - M_x \cos \eta \sin \varphi + M_y \cos \varphi - M_z \sin \eta \sin \varphi
\end{aligned} \tag{4}$$

$$\begin{aligned}
& r_{cg} x_h m_w \cos \varphi \left(\ddot{\theta} \cos \theta \sin \beta - \ddot{\theta} \sin \theta \cos \beta - 2\dot{\theta}^2 \cos \theta \cos \beta - 2\dot{\theta}^2 \sin \theta \sin \beta \right) + \\
& + r_{cg} m_w \cos \varphi \left(\dot{u} \cos \beta + u \dot{\theta} \sin \beta - \dot{w} \sin \beta + w \dot{\theta} \cos \beta \right) + \\
& + d x_h m_w \cos \eta \sin \varphi \left(\ddot{\theta} \cos \theta \sin \beta - \ddot{\theta} \sin \theta \cos \beta - 2\dot{\theta}^2 \cos \theta \cos \beta - 2\dot{\theta}^2 \sin \theta \sin \beta \right) + \\
& + d m_w \cos \eta \sin \varphi \left(\dot{u} \cos \beta + u \dot{\theta} \sin \beta - \dot{w} \sin \beta + w \dot{\theta} \cos \beta \right) + \dot{h}_1 \sin \eta - \dot{h}_3 \cos \eta = \\
& = \tau_\varphi + M_x \sin \eta - M_z \cos \eta
\end{aligned} \tag{5}$$

$$\begin{aligned}
& d x_h m_w \left(\ddot{\theta} \sin \eta \cos \theta \cos \varphi \sin \beta - \ddot{\theta} \sin \eta \sin \theta \cos \varphi \cos \beta - \ddot{\theta} \cos \eta \cos \theta \cos \beta - \ddot{\theta} \cos \eta \sin \theta \sin \beta + \right. \\
& - 2\dot{\theta}^2 \sin \eta \cos \theta \cos \varphi \cos \beta - 2\dot{\theta}^2 \sin \eta \sin \theta \cos \varphi \sin \beta - 2\dot{\theta}^2 \cos \eta \cos \theta \sin \beta + 2\dot{\theta}^2 \cos \eta \sin \theta \cos \beta \Big) + \\
& + d m_w \left(\dot{u} \sin \eta \cos \varphi \cos \beta + \dot{u} \cos \eta \sin \beta + u \dot{\theta} \sin \eta \cos \varphi \sin \beta - u \dot{\theta} \cos \eta \cos \beta - \sin \eta \cos \varphi \sin \beta \dot{w} + \right. \\
& + \dot{w} \cos \eta \cos \beta + w \dot{\theta} \sin \eta \cos \varphi \cos \beta + w \dot{\theta} \cos \eta \sin \beta \Big) + \dot{h}_2 = \tau_\eta + M_y,
\end{aligned} \tag{6}$$

where $m_v = m_b + m_w$

IV. Aerodynamic Model

We use an aerodynamic model similar to that developed in an earlier work [21, 22], which is based on [23–25]. However, the derivation here is clearer, more rigorous, and more general. This model accounts for the dominant contributions (i.e., leading edge vortex and rotational contributions) using a quasi-steady, strip theory formulation.

A two-dimensional airfoil undergoing a translational motion with velocity components V_x and V_z in the wing frame and a rotational pitching motion ω_y is subjected to the following forces [23–25]

$$\begin{aligned}
F'_x &= \pi \rho c \Delta x V_z \omega_y \\
F'_z &= -\frac{1}{2} \rho a_0 c V^2 \sin \alpha - \pi \rho c \Delta x V_x \omega_y,
\end{aligned} \tag{7}$$

where c is the chord length, Δx is the distance between the pitching axis (hinge line) and the three-quarter chord point, a_0 is the two-dimensional lift curve slope that will be replaced by the three-dimensional lift curve slope when integrating over the whole wing, α is the angle of attack that is given by $\alpha = \tan^{-1} \frac{V_z}{V_x}$, and $V^2 = V_x^2 + V_z^2$.

Using the kinematics described above, the velocity of a wing section that is a distance r from the wing root is written as

$$\mathbf{V}^{(w)}(r) = [\mathbf{R}_{ws}][\mathbf{R}_\beta] \mathbf{v}_w^{(I)} + \boldsymbol{\omega}_w^{(w)} \times r \mathbf{j}_w, \quad (8)$$

where the rotation matrix $\mathbf{R}_\beta$ is written as

$$\mathbf{R}_\beta = \begin{bmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{bmatrix}.$$

We use the following approximation for $V^2 \sin \alpha$ in order to be represented in terms of the state variables

$$V^2 \sin \alpha = V^2 \frac{V_z}{|V|} = |V| V_z = (|V| V_z) \Big|_0 + \sum_{i=1}^3 \frac{\partial(|V| V_z)}{\partial x_i} \Big|_{x'_i s=0} \Delta x_i, \quad (9)$$

where $x'_i s$ are the body state variables; u, w , and $\dot{\theta}$. As such, we obtain

$$\begin{aligned} V^2 \sin \alpha &= r^2 \sin \eta \dot{\varphi} |\dot{\varphi}| + u \left(2 r \cos \beta \sin \eta \cos \varphi |\dot{\varphi}| + r \sin \beta \cos \eta |\dot{\varphi}| \right) + \\ &+ w \left(r \cos \beta \cos \eta |\dot{\varphi}| - 2 r \sin \beta \sin \eta \cos \varphi |\dot{\varphi}| \right) + \\ &+ r \dot{\theta} |\dot{\varphi}| \left(-r \cos \eta \sin \varphi + 2 x_h \sin \beta \sin \eta \cos \theta \cos \varphi - 2 x_h \cos \beta \sin \eta \sin \theta \cos \varphi + \right. \\ &\left. - x_h \cos \beta \cos \eta \cos \theta - x_h \sin \beta \cos \eta \sin \theta \right). \end{aligned} \quad (10)$$

The Aerodynamic model, accounting for the nonlinear dependence of the non-autonomous aerodynamic loads on the aerodynamic state variables (u, w, q), can be written as follows

$$\begin{pmatrix} \frac{1}{m_w} X(x, t) \\ \frac{1}{m_w} Z(x, t) \\ \frac{1}{I_x} \mathcal{L}(x, t) \\ \frac{1}{I_y} M(x, t) \\ \frac{1}{I_z} N(x, t) \end{pmatrix} = \begin{pmatrix} X_0(t) \\ Z_0(t) \\ \mathcal{L}_0(t) \\ M_0(t) \\ N_0(t) \end{pmatrix} + \begin{bmatrix} X_u(t) & X_w(t) & X_q(t) \\ Z_u(t) & Z_w(t) & Z_q(t) \\ \mathcal{L}_u(t) & \mathcal{L}_w(t) & \mathcal{L}_q(t) \\ M_u(t) & M_w(t) & M_q(t) \\ N_u(t) & N_w(t) & N_q(t) \end{bmatrix} \begin{pmatrix} u \\ w \\ \dot{\theta} \end{pmatrix} + \begin{pmatrix} X_{nl}(t) \\ Z_{nl}(t) \\ \mathcal{L}_{nl}(t) \\ M_{nl}(t) \\ N_{nl}(t) \end{pmatrix}, \quad (11)$$

where $X(\cdot), Z(\cdot), \mathcal{L}(\cdot), M(\cdot)$, and $N(\cdot)$ are the forces and moments in the wing frame. It should be noted that in this model, we assume a general stroke plane angle β , and a general variation in the wing pitch angle η . As such, we obtain the following forces and moments derivatives

$$\begin{aligned} X_0 &= \rho \pi \left(k I_{11} - \frac{1}{4} I_{12} \right) \sin \eta \dot{\eta} \dot{\varphi} \quad , \quad Z_0 = -\frac{1}{2} \rho C_{L_\alpha} I_{21} \sin \eta \dot{\varphi} |\dot{\varphi}| - \rho \pi \left(k I_{11} - \frac{1}{4} I_{12} \right) \cos \eta \dot{\eta} \dot{\varphi} \\ \mathcal{L}_0 &= -\frac{1}{2} \rho C_{L_\alpha} I_{31} \sin \eta \dot{\varphi} |\dot{\varphi}| - \rho \pi \left(k I_{21} - \frac{1}{4} I_{22} \right) \cos \eta \dot{\eta} \dot{\varphi} \\ M_0 &= \frac{3}{4} \left(-\frac{1}{2} \rho C_{L_\alpha} I_{22} \sin \eta \dot{\varphi} |\dot{\varphi}| - \rho \pi \left(k I_{12} - \frac{1}{4} I_{13} \right) \cos \eta \dot{\eta} \dot{\varphi} \right) - k Z_0 \quad , \quad N_0 = -\rho \pi \left(k I_{21} - \frac{1}{4} I_{22} \right) \sin \eta \dot{\eta} \dot{\varphi} \\ X_u &= \rho \pi \left(k I_{01} - \frac{1}{4} I_{02} \right) (\cos \beta \sin \eta \cos \varphi + \sin \beta \cos \eta) \dot{\eta} \\ X_w &= -\rho \pi \left(k I_{01} - \frac{1}{4} I_{02} \right) (\sin \beta \sin \eta \cos \varphi - \cos \beta \cos \eta) \dot{\eta} \\ X_q &= \rho \pi \left(k I_{11} - \frac{1}{4} I_{12} \right) \sin \eta \cos \varphi \dot{\varphi} - \rho \pi \left(x_h \left(k I_{01} - \frac{1}{4} I_{02} \right) (\cos \eta \cos (\beta - \theta) - \sin \eta \cos \varphi \sin (\beta - \theta)) + \right. \\ &\quad \left. + \cos \eta \sin \varphi \left(k I_{11} - \frac{1}{4} I_{12} \right) \right) \dot{\eta} \end{aligned}$$

$$\begin{aligned} X_{nl} &= \rho\pi \cos \varphi \dot{\theta} \left(kI_{01} - \frac{1}{4}I_{02} \right) \left(u (\cos \beta \sin \eta \cos \varphi + \sin \beta \cos \eta) + w (\cos \beta \cos \eta - \sin \beta \sin \eta \cos \varphi) \right) + \\ &- \rho\pi \left(x_h \cos \varphi \left(kI_{01} - \frac{1}{4}I_{02} \right) (\cos \eta \cos (\beta - \theta) - \sin \eta \cos \varphi \sin (\beta - \theta)) + \cos \eta \sin \varphi \left(kI_{11} - \frac{1}{4}I_{12} \right) \right) \dot{\theta}^2 \end{aligned}$$

$$Z_u = -\frac{1}{2}\rho C_{L_\alpha} I_{11} (2 \cos \beta \sin \eta \cos \varphi + \sin \beta \cos \eta) |\dot{\varphi}| - \rho\pi \left(kI_{01} - \frac{1}{4}I_{02} \right) (\cos \beta \cos \eta \cos \varphi - \sin \beta \sin \eta) \dot{\eta}$$

$$Z_w = \rho\pi \left(kI_{01} - \frac{1}{4}I_{02} \right) (\sin \beta \cos \eta \cos \varphi + \cos \beta \sin \eta) \dot{\eta} - \frac{1}{2}\rho C_{L_\alpha} I_{11} (\cos \beta \cos \eta - 2 \sin \beta \sin \eta \cos \varphi) |\dot{\varphi}|$$

$$\begin{aligned} Z_q &= \frac{1}{2}\rho C_{L_\alpha} I_{21} \cos \eta \sin \varphi |\dot{\varphi}| + \\ &+ \rho C_{L_\alpha} I_{11} |\dot{\varphi}| \left(x_h \sin \beta \sin \eta \cos \theta \cos \varphi + \frac{1}{2} \cos \beta (2x_h \sin \eta \sin \theta \cos \varphi + x_h \cos \eta \cos \theta) + \right. \\ &+ \left. \frac{1}{2} x_h \sin \beta \cos \eta \sin \theta \right) + \rho\pi x_h \cos \eta \cos \varphi \sin (\beta - \theta) \left(\frac{1}{4}I_{02} - kI_{01} \right) \dot{\eta} + \\ &+ \rho\pi x_h \sin \eta \cos (\beta - \theta) \left(\frac{1}{4}I_{02} - kI_{01} \right) \dot{\eta} - \rho\pi \sin \eta \sin \varphi \left(kI_{11} - \frac{1}{4}I_{12} \right) \dot{\eta} - \rho\pi \cos \eta \cos \varphi \left(kI_{11} - \frac{1}{4}I_{12} \right) \dot{\varphi} \end{aligned}$$

$$\begin{aligned} Z_{nl} &= -\rho\pi \cos \varphi \dot{\theta} \left(kI_{01} - \frac{1}{4}I_{02} \right) \left(u (\cos \beta \cos \eta \cos \varphi - \sin \beta \sin \eta) + w (\sin \beta \cos \eta \cos \varphi + \cos \beta \sin \eta) \right) + \\ &+ \dot{\theta}^2 \left(2\pi \rho x_h \cos \varphi \left(\frac{1}{4}I_{02} - kI_{01} \right) (\cos \eta \cos \varphi \sin (\beta - \theta) + \sin \eta \cos (\beta - \theta)) + \right. \\ &- \left. 2\pi \rho \sin \eta \sin \varphi \cos \varphi \left(kI_{11} - \frac{1}{4}I_{12} \right) \right) \end{aligned}$$

$$\mathcal{L}_u = -\frac{1}{2}\rho C_{L_\alpha} I_{21} (2 \cos \beta \sin \eta \cos \varphi + \sin \beta \cos \eta) |\dot{\varphi}| - \rho\pi \left(kI_{11} - \frac{1}{4}I_{12} \right) (\cos \beta \cos \eta \cos \varphi - \sin \beta \sin \eta) \dot{\eta}$$

$$\mathcal{L}_w = \rho\pi \left(kI_{11} - \frac{1}{4}I_{12} \right) (\sin \beta \cos \eta \cos \varphi + \cos \beta \sin \eta) \dot{\eta} - \frac{1}{2}\rho C_{L_\alpha} I_{21} (\cos \beta \cos \eta - 2 \sin \beta \sin \eta \cos \varphi) |\dot{\varphi}|$$

$$\begin{aligned} \mathcal{L}_q &= \frac{1}{2}\rho C_{L_\alpha} I_{31} \cos \eta \sin \varphi |\dot{\varphi}| + \rho C_{L_\alpha} I_{21} |\dot{\varphi}| \left(x_h \sin \beta \sin \eta \cos \theta \cos \varphi + \right. \\ &+ \left. \frac{1}{2} \cos \beta (2x_h \sin \eta \sin \theta \cos \varphi + x_h \cos \eta \cos \theta) + \frac{1}{2} x_h \sin \beta \cos \eta \sin \theta \right) + \\ &+ \rho\pi x_h \left(\frac{1}{4}I_{12} - kI_{11} \right) \dot{\eta} \left(\cos \eta \cos \varphi \sin (\beta - \theta) + \sin \eta \cos (\beta - \theta) \right) + \\ &- \rho\pi \left(kI_{21} - \frac{1}{4}I_{22} \right) \left(\sin \eta \sin \varphi \dot{\eta} - \cos \eta \cos \varphi \dot{\varphi} \right) \end{aligned}$$

$$\begin{aligned}\mathcal{L}_{nl} = & -\rho\pi \cos\varphi \left(kI_{11} - \frac{1}{4}I_{12}\right) (\cos\beta \cos\eta \cos\varphi - \sin\beta \sin\eta) \dot{\theta} u + \\ & + \rho\pi \cos\varphi \left(kI_{11} - \frac{1}{4}I_{12}\right) (\sin\beta \cos\eta \cos\varphi + \cos\beta \sin\eta) \dot{\theta} w + \\ & + \dot{\theta}^2 \left(2\rho\pi x_h \cos\varphi \left(\frac{1}{4}I_{12} - kI_{11}\right) (\cos\eta \cos\varphi \sin(\beta - \theta) + \sin\eta \cos(\beta - \theta)) + \right. \\ & \left. - 2\rho\pi \sin\eta \sin\varphi \cos\varphi \left(kI_{21} - \frac{1}{4}I_{22}\right) \right)\end{aligned}$$

$$\begin{aligned}M_u = & \frac{3}{4} \left(-\frac{1}{2}\rho C_{L_\alpha} I_{12} (2\cos\beta \sin\eta \cos\varphi + \sin\beta \cos\eta) |\dot{\varphi}| - \rho\pi \left(kI_{02} - \frac{1}{4}I_{03}\right) (\cos\beta \cos\eta \cos\varphi - \sin\beta \sin\eta) \dot{\eta} \right) + \\ & - k Z_u\end{aligned}$$

$$\begin{aligned}M_w = & \frac{3}{4} \left(\rho\pi \left(kI_{02} - \frac{1}{4}I_{03}\right) (\sin\beta \cos\eta \cos\varphi + \cos\beta \sin\eta) \dot{\eta} - \frac{1}{2}\rho C_{L_\alpha} I_{12} (\cos\beta \cos\eta - 2\sin\beta \sin\eta \cos\varphi) |\dot{\varphi}| \right) + \\ & - k Z_w\end{aligned}$$

$$\begin{aligned}M_q = & \frac{3}{4} \left(\frac{1}{2}\rho C_{L_\alpha} I_{22} \cos\eta \sin\varphi |\dot{\varphi}| + \rho C_{L_\alpha} I_{12} |\dot{\varphi}| \left[x_h \sin\beta \sin\eta \cos\theta \cos\varphi + \right. \right. \\ & \left. \left. + \frac{1}{2}\cos\beta (2x_h \sin\eta \sin\theta \cos\varphi + x_h \cos\eta \cos\theta) + \frac{1}{2}x_h \sin\beta \cos\eta \sin\theta \right] + \right. \\ & + \rho\pi x_h \cos\eta \cos\varphi \sin(\beta - \theta) \left(\frac{1}{4}I_{03} - kI_{02}\right) \dot{\eta} + \rho\pi x_h \sin\eta \cos(\beta - \theta) \left(\frac{1}{4}I_{03} - kI_{02}\right) \dot{\eta} + \\ & \left. - \rho\pi \sin\eta \sin\varphi \left(kI_{12} - \frac{1}{4}I_{13}\right) \dot{\eta} - \rho\pi \cos\eta \cos\varphi \left(kI_{12} - \frac{1}{4}I_{13}\right) \dot{\varphi} \right) - k Z_q\end{aligned}$$

$$\begin{aligned}M_{nl} = & \frac{3}{4} \left(-\rho\pi \left(kI_{02} - \frac{1}{4}I_{03}\right) \cos\varphi \dot{\theta} \left[u (\cos\beta \cos\eta \cos\varphi - \sin\beta \sin\eta) - w (\sin\beta \cos\eta \cos\varphi + \cos\beta \sin\eta) \right] + \right. \\ & + \dot{\theta}^2 \left[2\rho\pi x_h \left(\frac{1}{4}I_{03} - kI_{02}\right) \cos\varphi (\cos\eta \cos\varphi \sin(\beta - \theta) + \sin\eta \cos(\beta - \theta)) + \right. \\ & \left. \left. - 2\rho\pi \left(kI_{12} - \frac{1}{4}I_{13}\right) \sin\eta \sin\varphi \cos\varphi \right] \right) - k Z_{nl}\end{aligned}$$

$$N_u = -\rho\pi \left(kI_{11} - \frac{1}{4}I_{12}\right) (\cos\beta \sin\eta \cos\varphi + \sin\beta \cos\eta) \dot{\eta}$$

$$N_w = \rho\pi \left(kI_{11} - \frac{1}{4}I_{12}\right) (\sin\beta \sin\eta \cos\varphi - \cos\beta \cos\eta) \dot{\eta}$$

$$N_q = \rho\pi \left(x_h \left(kI_{11} - \frac{1}{4}I_{12} \right) (\cos \eta \cos (\beta - \theta) - \sin \eta \cos \varphi \sin (\beta - \theta)) + \cos \eta \sin \varphi \left(kI_{21} - \frac{1}{4}I_{22} \right) \right) \dot{\eta} + \\ - \rho\pi \left(kI_{21} - \frac{1}{4}I_{22} \right) \sin \eta \cos \varphi \dot{\varphi}$$

$$N_{nl} = -\rho\pi \left(kI_{11} - \frac{1}{4}I_{12} \right) \cos \varphi \dot{\theta} \left(u (\cos \beta \sin \eta \cos \varphi + \sin \beta \cos \eta) + w (\cos \beta \cos \eta - \sin \beta \sin \eta \cos \varphi) \right) + \\ + \rho\pi \dot{\theta}^2 \left(x_h \cos \varphi \left(kI_{11} - \frac{1}{4}I_{12} \right) (\cos \eta \cos (\beta - \theta) - \sin \eta \cos \varphi \sin (\beta - \theta)) + \cos \eta \sin \varphi \left(kI_{21} - \frac{1}{4}I_{22} \right) \right),$$

where $k = c_r(1 - x_{or})$, c_r is the root chord, x_{or} is the position of the hinge point along x_w normalized by the root chord, and x_h is the distance from the vehicle center of mass to the root of the wing hinge line (i.e., the intersection of the hinge line with the x_b -axis). Also, ρ is the air density, C_{L_α} is the three-dimensional lift curve slope of the wing, $c(r)$ is the spanwise chord distribution, R is the wing radius, $I_{mn} = 2 \int_0^R r^m c^n(r) dr$.

V. Aerodynamic-Dynamic Interaction

As explained in Sec. III and Sec. IV, the aerodynamic loads generated by the wing oscillatory motion are represented as a function of the wing variables; φ and η , as well as the body variables; u , w , and θ , in addition to the stroke plane angle β . As such, the interaction between the body motion and the generated aerodynamic loads by the wing can be revealed. This interaction is explained in Fig. V. This allows for a more accurate trim and stability analysis. More precisely; the intuitive notion of achieving averaged lift over the flapping cycle equals to the vehicle weight is sufficient to achieve balance at hover has been refuted by Taha et al. [26] by revealing a negative lifting mechanism emanating from an aerodynamic-dynamic interaction source. As such, using the above developed model for a rigorous trim and stability analysis allows for a better scrutiny of the complex behavior of such a system which is considered in the second part of this work.

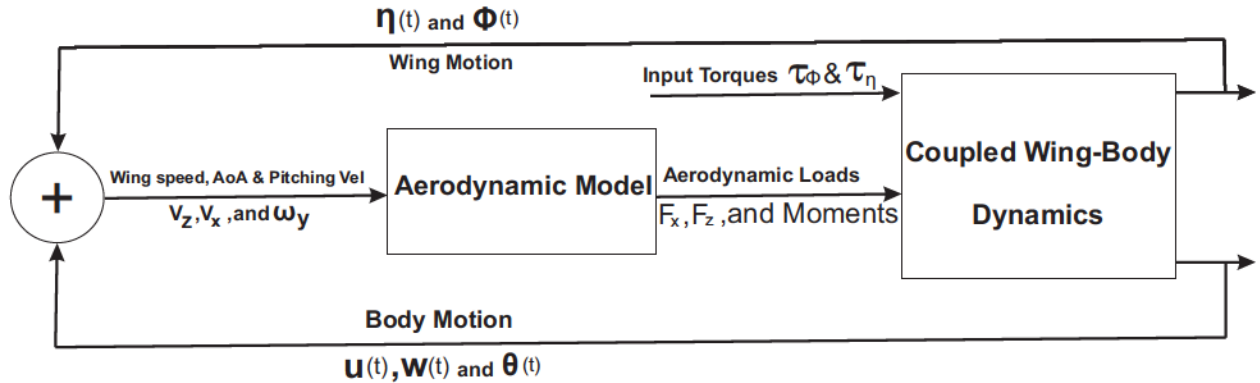


Figure 2. A schematic diagram for the aerodynamic-dynamic interaction in a FWMAV.

VI. Simulation

In this section, an open-loop time simulation is performed for the full flight dynamic model. The control input torques are assumed to be sinusoidal as follows

$$\tau_\varphi = U_\varphi \cos \omega t \\ \tau_\eta = U_\eta \sin \omega t$$

where U_φ and U_η are the flapping and pitching torques' amplitudes respectively. In this simulation, the Hawkmoth morphological parameters, given in Appendix A, are adopted with zero-stroke plane angle, i.e.,

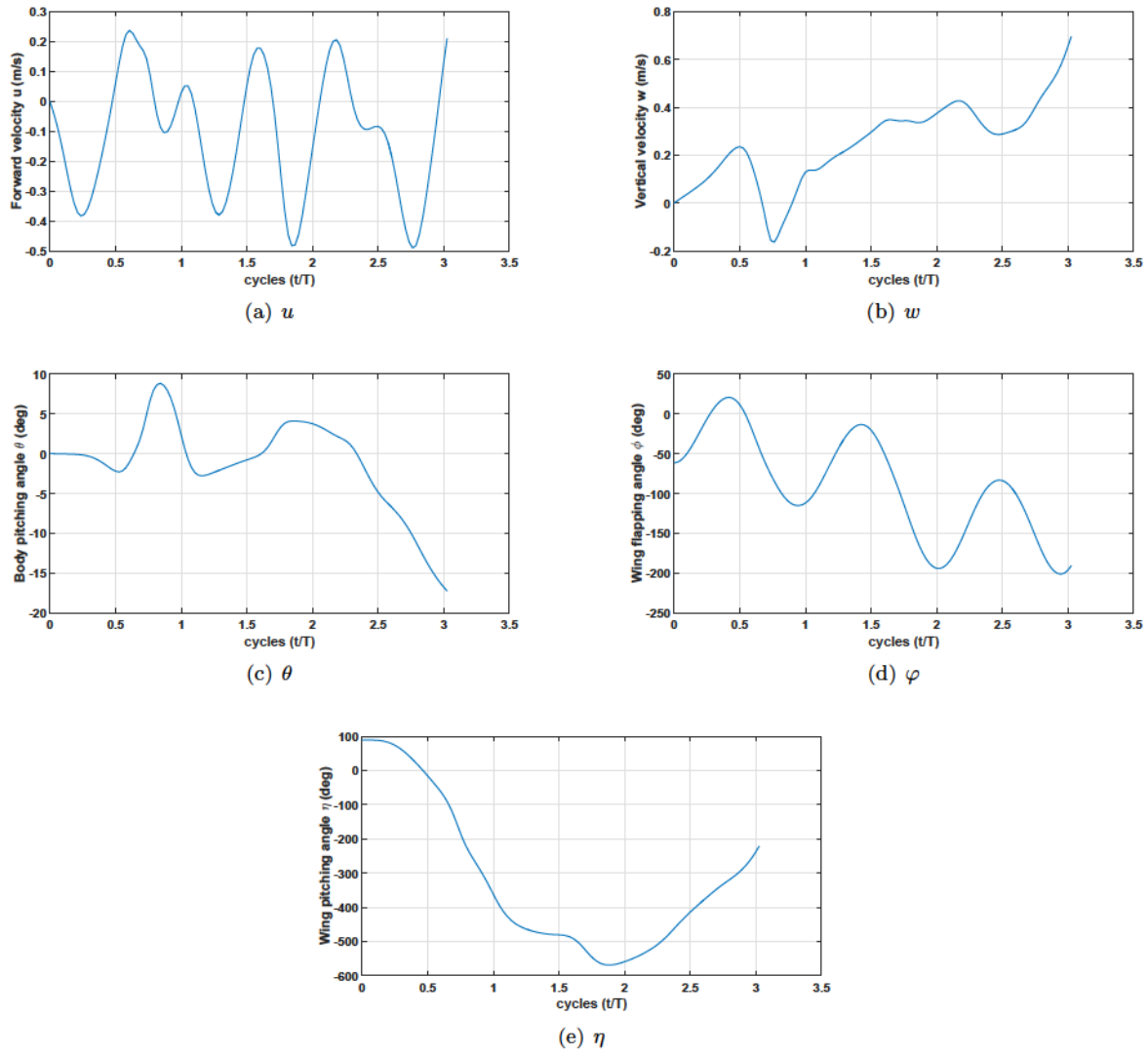


Figure 3. Full flight dynamic model simulation using the Hawkmoth parameters over three flapping cycles.

$\beta = 0$. Figure 3 shows a time simulation for the full flight dynamic model developed earlier and using the Hawkmoth morphological parameters. It is noted from the simulation that the open-loop dynamics is unstable especially the body pitching and wing pitching modes. The flight stability of a reduced-order model is scrutinized in the second part of this work.

VII. Conclusion

The longitudinal flight dynamics of flapping-wing micro-air-vehicles (FWMAVs)/insects is considered. The equations of motion of the multi-body problem including the wing inertial effects are derived. A simple, analytical aerodynamic model that captures the dominant contributions (leading edge vortex and rotational contributions) is adopted. In the second part of this effort, The flight dynamic model is then represented in a mechanical control framework in order to rigorously analyze the balance problem and assess the longitudinal flight stability near hover.

Appendix

A. Hawkmoth Morphological Parameters

The morphological parameters and the wing planform for the Hawkmoth, as given in [13] and [27], are

$$\begin{aligned} R &= 51.9\text{mm}, S = 947.8\text{mm}^2, \bar{c} = 18.3\text{mm}, \\ \hat{r}_1 &= 0.44, \hat{r}_2 = 0.525, f = 26.3\text{Hz}, \Phi = 60.5^\circ, \\ \alpha_m &= 30^\circ, m_b = 1.648\text{gm}, \text{ and } I_{yb} = 2080\text{mg.cm}^2, \end{aligned}$$

where R is the semi-span of the wing, S is the area of one wing, $\bar{c}$ is the mean chord, f is the flapping frequency, Φ is the flapping angle amplitude, m_b is the mass of the body, and I_{yb} is the body moment of inertia around the body y -axis. The moments of the wing chord distribution $\hat{r}_1$ and $\hat{r}_2$ are defined as

$$I_{k1} = 2 \int_0^R r^k c(r) dr = 2SR^k \hat{r}_k^k.$$

As for the wing planform, the method of moments used by Ellington [27] is adopted here to obtain a chord distribution for the insect that matches the documented first two moments $\hat{r}_1$ and $\hat{r}_2$; that is,

$$c(r) = \frac{\bar{c}}{\beta} \left(\frac{r}{R} \right)^{\lambda-1} \left(1 - \frac{r}{R} \right)^{\gamma-1},$$

where

$$\begin{aligned} \lambda &= \hat{r}_1 \left[\frac{\hat{r}_1(1-\hat{r}_1)}{\hat{r}_2^2 - \hat{r}_1^2} - 1 \right], \quad \gamma = (1 - \hat{r}_1) \left[\frac{\hat{r}_1(1-\hat{r}_1)}{\hat{r}_2^2 - \hat{r}_1^2} - 1 \right], \\ \text{and } \beta &= \int_0^1 \hat{r}^{\lambda-1} (1 - \hat{r})^{\gamma-1} d\hat{r}. \end{aligned}$$

The mass of one wing is taken as 5.7% of the body mass according to Wu et al. [28] and is assumed uniform with an areal mass distribution m' . The inertial properties of the wing are then estimated as

$$\begin{aligned} I_x &= 2 \int_0^R m' r^2 c(r) dr, \quad I_y = 2 \int_0^R m' \hat{d}^2 c^3(r) dr \\ I_z &= I_x + I_y, \quad \text{and } r_{cg} = \frac{2 \int_0^R m' r c(r) dr}{m_w} = \frac{I_{11}}{2S}, \end{aligned}$$

where $\hat{d}$ is the chord-normalized distance from the wing hinge line to the center of gravity line.

Acknowledgments

The authors gratefully acknowledge the support of the National Science Foundation grants CMMI-1435484 and CMMI-1709746, and program manager Dr. Jordan Berg.

References

- ¹ Taha, H. E., Nayfeh, A. H., and Hajj, M. R., “Effect of the aerodynamic-induced parametric excitation on the longitudinal stability of hovering MAVs/insects,” *Nonlinear Dynamics*, Vol. 78, No. 4, 2014, pp. 2399–2408.
- ² Taha, H. E., Tahmasian, S., Woolsey, C. A., Nayfeh, A. H., and Hajj, M. R., “The need for higher-order averaging in the stability analysis of hovering, flapping-wing flight,” *Bioinspiration & biomimetics*, Vol. 10, No. 1, 2015, pp. 016002.
- ³ Taha, H. E., Hajj, M. R., and Nayfeh, A. H., “Flight Dynamics and Control of Flapping-Wing MAVs: A Review,” *Nonlinear Dynamics*, Vol. 70, No. 2, 2012, pp. 907–939.
- ⁴ Thomas, A. L. R. and Taylor, G. K., “Animal flight dynamics I. Stability in gliding flight,” *Journal of Theoretical Biology*, Vol. 212, No. 1, 2001, pp. 399–424.
- ⁵ Taylor, G. K. and Thomas, A. L. R., “Animal Flight Dynamics II. Longitudinal Stability in Flapping Flight,” *Journal of Theoretical Biology*, Vol. 214, 2002.

- ⁶ Taylor, G. K. and Thomas, A. L. R., "Dynamic Flight Stability in the Desert Locust," *Journal of Theoretical Biology*, Vol. 206, 2003, pp. 2803–2829.
- ⁷ Taylor, G. K. and Zbikowski, R., "Nonlinear Time Periodic Models of the Longitudinal Flight Dynamics of Desert Locusts," *J. Roy. Soc. Interface*, Vol. 1, 2005.
- ⁸ Khan, Z. A. and Agrawal, S. K., "Control of Longitudinal Flight Dynamics of a Flapping Wing Micro Air Vehicle Using Time Averaged Model and Differential Flatness Based Controller," IEEE American Control Conference, 2007, pp. 5284–5289.
- ⁹ Sun, M. and Xiong, Y., "Dynamic flight stability of a hovering bumblebee." *Journal of Experimental Biology*, Vol. 208, No. 3, 2005, pp. 447–459.
- ¹⁰ Xiong, Y. and Sun, M., "Dynamic Flight Stability of a Bumble Bee in Forward Flight," *Acta Mechanica Sinica*, Vol. 24, No. 3, 2008, pp. 25–36.
- ¹¹ Dietl, J. M. and Garcia, E., "Stability in Ornithopter Longitudinal Flight Dynamics," *Journal of Guidance, Control and Dynamics*, Vol. 31, No. 4, 2008, pp. 1157–1162.
- ¹² Oppenheimer, M. W., Doman, D. B., and Sigthorsson, D. O., "Dynamics and Control of a Biomimetic Vehicle Using Biased Wingbeat Forcing Functions," *Journal Guidance, Control and Dynamics*, Vol. 34, No. 1, 2011, pp. 204–217.
- ¹³ Sun, M., Wang, J., and Xiong, Y., "Dynamic Flight Stability of Hovering Insects," *Acta Mechanica Sinica*, Vol. 23, No. 3, 2007, pp. 231–246.
- ¹⁴ Orlowski, C. T. and Girard, A. R., "Modeling and Simulation of the Nonlinear Dynamics of Flapping Wing MAVs," *AIAA Journal*, Vol. 49, No. 5, 2011, pp. 969–981.
- ¹⁵ Bolender, M. A., "Rigid Multi-Body Equations-of-Motion for Flapping Wing MAVs Using Kanes Equations," AIAA Guidance, Navigation, and Control Conference, Chicago, IL, 2009.
- ¹⁶ Su, W. and Cesnik, C. E. S., "Flight Dynamic Stability of a Flapping Wing MAV in Hover," AIAA-Paper 2011-2009, Apr 2011.
- ¹⁷ Nelson, R. C., *Flight Stability and Automatic Control*, McGraw-Hill, 1989.
- ¹⁸ Weis-Fogh, T., "Quick Estimates of Flight Fitness in Hovering Animals, Including Novel Mechanisms for Lift Production," *Journal of Experimental Biology*, Vol. 59, No. 1, 1973, pp. 169–230.
- ¹⁹ Ellington, C. P., "The aerodynamics of hovering insect flight: III. Kinematics," *Philosophical Transactions Royal Society London Series B*, Vol. 305, 1984, pp. 41–78.
- ²⁰ Greenwood, D. T., *Advanced Dynamics*, Cambridge University Press, The Edinburgh Building, Cambridge CB2 2RU, UK, 2003.
- ²¹ Taha, H. E., Hajj, M. R., and Nayfeh, A. H., "On the Longitudinal Flight Dynamics of Hovering MAVs/Insects," *Journal of Guidance Control and Dynamics*, In Press.
- ²² Taha, H. E., Nayfeh, A. H., and Hajj, M. R., "Aerodynamic-Dynamic Interaction and Longitudinal Stability of Hovering MAVs/Insects," No. AIAA 2013-1707, 54th Structural Dynamics and Materials Conference, Boston, Apr 2013.
- ²³ Taha, H. E., Hajj, M. R., and Beran, P. S., "Unsteady Nonlinear Aerodynamics of Hovering MAVs/Insects," AIAA-Paper 2013-0504, Jan 2013.
- ²⁴ Taha, H. E., Hajj, M. R., and Beran, P. S., "State-space representation of the unsteady aerodynamics of flapping flight," *Aerospace Science and Technology*, Vol. 34, 2014, pp. 1–11.
- ²⁵ Berman, G. J. and Wang, Z. J., "Energy-minimizing kinematics in hovering insect flight," *Journal of Fluid Mechanics*, Vol. 582, No. 1, 2007, pp. 153,168.

- ²⁶ Taha, H. E., Woolsey, C. A., and Hajj, M. R., “Geometric Control Approach to Longitudinal Stability of Flapping Flight,” *Journal of Guidance, Control, and Dynamics*, 2015, pp. 1–13.
- ²⁷ Ellington, C. P., “The aerodynamics of hovering insect flight: II. Morphological parameters,” *Philosophical Transactions Royal Society London Series B*, Vol. 305, 1984, pp. 17–40.
- ²⁸ Wu, J. H., Zhang, Y. L., and Sun, M., “Hovering of model insects: simulation by coupling equations of motion with Navier-Stokes equations,” *The Journal of Experimental Biology*, Vol. 212, No. 20, 2009, pp. 3313–3329.