

# P H M A S I A P A C I F I C 2 0 1 7

Proceedings of the  
Asia Pacific Conference of  
the Prognostics and  
Health Management Society 2017

July 12 (Wed.)~15 (Sat.), 2017  
Ramada Plaza Jeju Hotel, Jeju, Korea



# A Yield-Reliability Relation Modeling Approach based on Random Effects Degradation Models

Tao Yuan<sup>1</sup>, Xiaoyan Zhu<sup>2</sup>, and Yue Kuo<sup>3</sup>

<sup>1</sup>*Ohio University, Athens, Ohio, 45701, United States*  
yuan@ohio.edu

<sup>2</sup>*University of Chinese Academy of Sciences, Beijing, 100190, China*  
xzhu5@ucas.ac.cn

<sup>3</sup>*Texas A&M University, College Station, Texas, 77845, United States*  
yuekuo@tamu.edu

## ABSTRACT

This paper presents a unified modeling framework for yield and reliability in micro-/nano-electronics manufacturing via spatiotemporal modeling of defects. The spatial modeling and temporal modeling of defects refer to modeling of the spatial distribution of defects in manufacturing processes and modeling of the growth of defects with time when devices are subject to stresses, respectively. The defect growth process is characterized by the random-effect degradation modeling method. The presented modeling framework will allow us to use abundantly available process control data to predict the device reliability.

## 1. INTRODUCTION

Yield and reliability have been recognized as two of the most critical factors to determine the success of micro-/nano-electronics manufacturing (Kuo, 2006). Yield is often defined as the ratio between the number of usable devices at the end of production processes and the number of potentially usable devices at the beginning of production (Kuo, Chien, & Kim, 1998). Reliability of a device is usually defined as the probability that a device will perform its intended function adequately under operating conditions for a specified time duration (Meeker & Escobar, 1998).

Yield and reliability are usually modeled and evaluated separately based on different source of information and data. Yield is generally estimated based on process control data such as measurements of manufacturing defects; while reliability prediction generally relies on fitting failure-time data obtained from (accelerated) life tests and/or field operations to some parametric failure-time distributions, such as Weibull, lognormal, and exponential. It is difficult to implement such an end-of-line reliability assessment approach at early stages of a product's life cycle when the

availability of reliability data is limited. Recognizing that defects, defined as variations in quality and generated during the manufacturing processes, are responsible for both yield losses and extrinsic reliability failures, there have been growing efforts in recent years to directly estimate the device reliability from the yield information without using time-consuming and costly reliability life tests (Kim, Kuo, & Luo, 2004). Section 2 will review the existing yield-reliability relation models and state their limitations.

This paper presents a yield-reliability relation modeling approach based on spatiotemporal modeling of defects. The spatial modeling and temporal modeling of defects refer to modeling of the spatial distribution of defects in manufacturing processes and modeling of the growth of defects with time when devices are subject to stresses, respectively. The defect growth process is characterized by the random-effect degradation modeling method.

The remainder of this paper is organized as follows. Section 2 will discuss related literature and background. Section 3 will present the proposed yield-reliability relation modeling approach. Finally, Section 4 will conclude this paper and describe ongoing experimental and modeling research work.

## 2. RELATED LITERATURE AND BACKGROUND

The section reviews the related literature on yield models and yield-reliability relation models. Micro-/nano-electronic devices are highly vulnerable to defects generated in manufacturing processes. Defects can be categorized into yield defects (also called faults or fatal defects) and reliability defects. When a defect locates in a defect-sensitive area (called critical area), it is a yield defect and results in yield loss immediately. In contrast, a reliability defect does not cause immediate yield loss but may lead to early reliability failures during usage.

## 2.1. Yield Models

Yield is estimated and predicted by yield models. The Poisson and negative binomial yield models are the most frequently used models (Kuo et al., 1998). Let the random variable  $Z$  denote the size of a randomly selected defect,  $g(z)$  be the probability density function of the defect size  $Z$ , and  $A(z)$  be the critical area of a defect of size  $z$ . Then the average critical area, denoted by  $A$ , is defined by

$$A = \int_0^{\infty} A(z)g(z)dz. \quad (1)$$

The Poisson yield model assumes that the distribution of defects is random and the occurrence of a defect at any location is independent of the occurrence of any other defects. Let  $D$ , the defect density, denote the average number of defects per unit area (Hansen & Thyregod, 1996). Then  $\mu=BD$  is the average number of defects on a device, where  $B$  is the total area on the device that a defect can fall. For a given  $\mu$ , the random number of defects on a device,  $N$ , follows the Poisson distribution. That is, the probability that a device contains  $k$  defects is given by

$$\Pr(N = k) = \frac{e^{-\mu} \mu^k}{k!}. \quad (2)$$

Let  $\Phi$  represent the fault probability, i.e., the probability that a random defect is a yield defect. It can be shown that  $\Phi = A/B$ . Then the Poisson yield is defined as the probability that there is no yield defect on a device, which is given by (Kuo et al., 1998).

$$Y = e^{-\Phi BD} = e^{-AD}. \quad (3)$$

If the defect density,  $D$ , is not a constant, instead is a random variable with a probability density function denoted by  $f(D)$ , then the yield is expressed by

$$Y = \int_0^{\infty} e^{-AD} f(D)dD. \quad (4)$$

Different assumptions of the  $f(D)$  function lead to different compound Poisson yield models. If the defect density follows the Gamma distribution, i.e.,

$$f(D) = \frac{D^{\alpha-1} e^{-D/\beta}}{\Gamma(\alpha)\beta^{\alpha}}, \quad (5)$$

where  $\alpha$  and  $\beta$  are the shape parameter and scale parameter, respectively, then the number of defects on a device follows the negative binomial distribution and the negative binomial yield has the form of

$$Y = \left(1 + \frac{AD}{\alpha}\right)^{-\alpha}, \quad (6)$$

where  $\bar{D} = E[D] = \alpha\beta$  is the average defect density, and  $\alpha$  is often called the clustering factor.

Recently other yield models that consider the defect clustering effect on the yield have been proposed. Bae,

Hwang, and Kuo (2007) proposed yield models using Poisson regression, zero-inflated Poisson regression, and negative binomial regression. They used the spatial locations of devices on a wafer as covariates for corresponding defect counts. Yuan, Ramadan, and Bae (2011) later extended that approach using a hierarchical Bayesian modeling framework.

The work by Bae et al. (2007) and Yuan et al. (2011) estimated the yield considering the spatial clustering of defect counts. Hwang, Kuo, and Ha (2011) proposed a different yield modeling approach by directly accounting for the spatial distribution and clustering of defects. The spatial distribution of defects on a wafer was modeled by a spatial nonhomogeneous Poisson process, whose intensity function varies by the spatial locations.

## 2.2. Yield-Reliability Relation Models

In assessing device reliability, engineers usually plan and conduct accelerated life tests to collect failure-time data, and then fit the failure-time data to some failure-time models. For example, the Weibull distribution and the lognormal distribution have been widely applied to model the dielectric breakdown and electromigration failure-time distributions, respectively. For each failure phenomenon, there are generally two failure modes – intrinsic mode and extrinsic mode. Intrinsic failures due to material wearout occur during the aging period of device life; and extrinsic failures are caused by the growth of manufacturing defects and occur during the early lifetime (Kim & Kuo, 1999). The early extrinsic failures are the key target for reliability improvement and design-for-reliability activities (Xuan, Singh, & Chatterjee, 2006).

Because yield and extrinsic reliability are both determined by the manufacturing defects, there have been attempts in recent years to estimate device reliability directly from yield without using the time-consuming reliability life tests. Huston and Clarke (1992) presented a yield-reliability relation by  $R = Y^{A_r/A_y}$ , where  $R$  is the Poisson probability of no reliability defects,  $Y$  denotes the Poisson probability of no yield defects, and  $A_r$  and  $A_y$  are the reliability critical areas and the yield critical areas, respectively. Another yield-relation model has the form of  $R = (Y/\delta)^{D_r/D_y}$  (Kuper, van der Pol, Ooms, Johnson, Wijburg, Koster, & Johnston, 1996), where  $\delta$  is a parameter for clustering effects and edge exclusions, and  $D_y$  and  $D_r$  are the yield defect density and the reliability defect density, respectively. These two yield-reliability relation models, however, ignore the time-dependence of reliability.

Kim and Kuo (1994) and Kim et al. (2004) proposed a time-dependent yield-reliability relation model of the form  $R(t) = Y^{\gamma(t)}$ , where  $\gamma(t)$  is the ratio of the probability that a defect is not a yield defect but causes device failure by time  $t$  to the probability that a defect is a yield defect. Kim and Kuo

(1994) and Kim et al. (2004) applied this model to the dielectric reliability and the defect growth was described by the “effective thinning” mechanism (Moazzami & Hu, 1990). Hwang (2004) built an extrinsic reliability model based on the yield model developed by Hwang et al. (2011) considering the defect growth. All those defect-growth reliability models assumed that the defect growth is constant without considering the defect-to-defect variation.

### 3. PROPOSED YIELD-RELIABILITY RELATION MODELS

Consider a device with zero yield defect and  $k$  reliability defects, which are observed at the end of the manufacturing processes. The extrinsic failure of this device under a constant set of operating and environmental conditions is effectively due to the growth of the manufacturing defects. For the  $i$ th reliability defect, let  $\mathbf{x}_i$  and  $z_i^0$  denote its location and its size at time 0 (i.e., at the end of the manufacturing processes). At time  $t$ , the defect grows to size  $z_i^t$  according to some physics-based random defect-growth mechanism. Then the probability that the  $i$ th reliability defect does not cause the device to fail by time  $t$  is given by

$$R_i(t) = \Pr(T_i > t) = \Pr[\mathbf{x}_i \notin A(z_i^t)], \quad (7)$$

where  $R(t)$  is the reliability function and  $T_i$  is the time-to-failure caused by the  $i$ th reliability defects.

The random defect growth is described by a random-effect degradation model, namely

$$z^t = \eta(t; z^0, \boldsymbol{\beta}), \quad (8)$$

where the random degradation-coefficient vector  $\boldsymbol{\beta}$  is assumed to follow a multivariate distribution  $f(\boldsymbol{\beta} | \boldsymbol{\theta})$  with a parameter vector  $\boldsymbol{\theta}$  to account for the defect-to-defect variation. Then  $R_i(t)$  given by Eq. (7) can be evaluated by

$$R_i(t) = \int_{\boldsymbol{\beta}} \Pr\{x_i \notin A[\eta(t; z^0, \boldsymbol{\beta})]\} f(\boldsymbol{\beta} | \boldsymbol{\theta}) d\boldsymbol{\beta}. \quad (9)$$

The failure-time distribution of the device with  $k$  reliability defects is predicted as

$$R(t) = \Pr[\min(T_1, T_2, \dots, T_k) > t] = \prod_{i=1}^k R_i(t), \quad (10)$$

if the  $k$  reliability defects on the device grow and cause the device failure independently.

#### 3.1. Poisson yield and reliability models

The Poisson yield model given by Eq. (3) can be extended to build a reliability prediction model. The number of defects (both yield and reliability defects) on a device follow the Poisson distribution with mean  $\mu = BD$ . The fault probability at time zero is given by

$$\Phi^0 = \left[ \int_0^\infty A(z^0) g(z^0) dz^0 \right] / B, \quad (11)$$

and the Poisson yield is the probability that no defects cause device failure at time zero,

$$\begin{aligned} Y &= \sum_{k=0}^{\infty} (1 - \Phi^0)^k P(N = k) \\ &= \sum_{k=0}^{\infty} (1 - \Phi^0)^k e^{-\mu} \mu^k / k! = e^{-\Phi^0 BD}. \end{aligned} \quad (12)$$

Based on this yield model, the extrinsic reliability at time  $t$  may be defined as the probability that no defects cause device failure by time  $t$ ,

$$R_E(t) = \sum_{k=0}^{\infty} (1 - \Phi^t)^k P(N = k), \quad (13)$$

where the fault probability at time  $t$  is given by

$$\Phi^t = \left[ \int_0^\infty \int_{\boldsymbol{\beta}} A[\eta(t; z^0, \boldsymbol{\beta})] f(\boldsymbol{\beta} | \boldsymbol{\theta}) g(z^0) d\boldsymbol{\beta} dz^0 \right] / B. \quad (14)$$

The yield and reliability are predicted based on the same information related to the defects distribution in the manufacturing processes. The yield is the reliability at time zero, i.e.,  $R_E(0) = Y$ . Moreover,  $R_E(\infty) = e^{-\mu} = P(N = 0)$  because devices without defects do not have the defect-growth caused extrinsic failure mode.

Therefore, to be consistent with traditional definition of reliability, the extrinsic reliability needs to be defined as the probability that no defects cause device failure by time  $t$  conditioning on that no defects have caused failure at time 0 for devices with non-zero defects,

$$R_E(t) = \frac{1}{1 - \Pr(N = 0)} \sum_{k=1}^{\infty} \left( \frac{1 - \Phi^t}{1 - \Phi^0} \right)^k P(N = k). \quad (15)$$

It can be shown that  $R_E(0) = 1$  and  $R_E(\infty) = 1$  for the reliability function given by Eq. (15).

#### 3.2. Negative Binomial Yield and Reliability Models

Similarly, the negative binomial yield model given by Eq. (6) can be extended to build a negative binomial reliability model. The probability  $P(N=k)$  in Eq. (15) needs to be changed from the Poisson probability mass function to the negative binomial probability mass function

$$\Pr(N = k) = \frac{\Gamma(\alpha + k)}{\Gamma(\alpha) \Gamma(k + 1)} \left( 1 + \frac{BD}{\alpha} \right)^{-\alpha} \left( \frac{BD}{\alpha + BD} \right)^k, \quad (16)$$

for  $k=0, 1, 2, \dots$

### 4. CONCLUSION AND ONGOING WORK

This paper presented a unified framework for modeling the yield and reliability of micro-/nano-electronic devices via the random-effect modeling of the defect-growth process. The models link the defect data collected in process control to reliability prediction. By integrating the yield and reliability modeling in a unified framework, the reliability can be predicted without time-consuming reliability life tests, which is valuable because of the scarcity of reliability data,

the ever-decreasing time-to-market of electronic device, and the urgent need to assess product reliability and life-cycle cost at early stages of produce life cycle.

Ongoing research focuses on the following two directions. First, accelerated degradation testing and analysis are being conducted to derive and validate the physics-based random-effect degradation model given by Eq. (8). Electromigration for ultra-narrow copper interconnects is employed as a testbed for the proposed methodology. Second, reliability models are being constructed for other yield models developed by Bae et al. (2007), Yuan et al. (2011), and Hwang et al. (2011).

# ACKNOWLEDGEMENT

This work is partially supported by the National Sciences Foundation projects 1633500 and 1633580.

# REFERENCES

- Bae, S. J., Hwang, J. Y., & Kuo, W. (2007). Yield prediction via spatial modeling of clustered defects counts across a wafer map. *IIE Transactions*, vol. 39, pp. 1073-1083. doi: 10.1080/07408170701275335
- Hansen, C. K., & Thyregod, P. (1996). Modeling and estimation of wafer yields and defect densities from microelectronics test structure data. *Quality and Reliability Engineering International*, vol. 12, pp. 9-18.
- Huston, H. H., & Clarke, C. P. (1992). Reliability defect detection and screening during processing – theory and implementation. *Proceedings of the International Reliability Physics Symposium*, March 31-April 2. doi: 10.1109/RELPHY.1992.187656
- Hwang, J. Y. (2004). *Spatial stochastic processes for yield and reliability management with applications to nano electronics*. Doctoral dissertation. Texas A&M University, College Station. <http://oaktrust.library.tamu.edu/bitstream/handle/1969.1/1500/etd-tamu-2004C-INEN-Hwang.pdf>
- Hwang, J. Y., Kuo, W., & Ha, C. (2011). Modeling of integrated circuit yield using a spatial nonhomogeneous Poisson process. *IEEE Transactions on Semiconductor Manufacturing*, vol. 24, pp. 377-384. doi: 10.1109/TSM.2011.2143733
- Kim, K. K., Kuo, W., & Luo, W. (2004). A relation model of gate oxide yield and reliability. *Microelectronics Reliability*, vol. 44, pp. 425-434. doi: 10.1016/j.microrel.2003.09.009
- Kim, T., & Kuo, W. (1999). Modeling manufacturing yield and reliability. *IEEE Transactions on Semiconductor Manufacturing*, vol. 12, pp. 485-492. doi: 10.1109/66.806126
- Kuo, W. (2006). Challenges related to reliability in nano electronics. *IEEE Transactions on Reliability*, vol. 55, pp. 569-570. doi:10.1109/TR.2006.884585

- Kuo, W., Chien, W.-T. K., & Kim, T. (1998). *Reliability, yield, and stress burn-in*. Boston, MA: Kluwer Academic Publishers.
- Kuper, F., van der Pol, J., Ooms, E., Johnson, T., Wijburg, R., Koster, W., & Johnston, D. (1996). Relation between yield and reliability of integrated circuits: experimental results and application to continuous early failure rate deduction programs. *Proceedings of the IEEE International Reliability Physics Symposium*, April 30-May 2. doi: 10.1109/RELPHY.1996.492055
- Meeker, W. Q., & Escobar, L. A. (1998). *Statistical methods for reliability data*. New York, NY: John Wiley & Sons, Inc.
- Moazzami, R. M., & Hu, C. (1990). Projecting gate oxide reliability and optimizing reliability screens. *IEEE Transactions on Electron Devices*, vol. 37, pp. 1643-1650. doi: 10.1109/16.55751
- Xuan, X., Singh, A. D., & Chatterjee, A. (2006). Lifetime prediction and design-for-reliability of IC interconnections with electromigration induced degradation in the presence of manufacturing defects. *Journal of Electronics Testing: Theory and Applications*, vol. 22, pp. 471-482. doi: 10.1007/s10836-006-9498-2
- Yuan, T., Ramadan, S. Z., & Bae, S. J. (2011). Yield prediction for integrated circuits manufacturing through hierarchical Bayesian modeling of spatial defects. *IEEE Transactions on Reliability*, vol. 60, pp. 729-741. doi: 10.1109/TR.2011.2161698

# BIOGRAPHIES

**Tao Yuan** is an Associate Professor in Industrial and Systems Engineering at Ohio University. He received his Ph.D. in Industrial Engineering from the University of Tennessee, Knoxville, in 2007. His research interests are mainly in reliability testing, modeling, and analysis.

**Xiaoyan Zhu** is a Professor in the School of Economics and Management at University of Chinese Academy of Sciences, Beijing, China. She obtained Ph.D. in Industrial Engineering from Texas A&M University, College Station in 2005. Her research interests are in system reliability optimization, importance measures, network and integer programming.

**Yue Kuo** is the Dow Professor of Chemical Engineering, Electrical Engineering, and Materials Science and Engineering at Texas A&M University, College Station. He received Ph.D. from Columbia University in 1979. Before joining Texas A&M University in 1998, he worked in industry (IBM T. J. Watson Research Center, Yorktown Heights, NY, and Semiconductor Division, Data General Corp., Sunnyvale, CA). He has authored more than 200 papers, and edited 10 textbooks, proceedings, and journals. He has delivered numerous talks, and organized international conferences. Dr. Kuo is an IEEE Fellow, Electrochemical Society (ECS) Fellow, and American Vacuum Society Fellow.