A Study of Virtual Energy Storage From Thermostatically Controlled Loads Under Time-Varying Weather Conditions

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ABSTRACT

We propose a control architecture for distributed coordination of a collection of on/off TCLs (thermostatically controlled loads), such as residential air conditioners, to provide the same service to the power grid as a large battery. A key constraint is to ensure that consumers' quality of service (QoS) is maintained. Our proposal involves replacing the thermostats at the loads by a randomized controller, following recent proposals in this direction. The new local controller has a tunable parameter that serves as the control command from the balancing authority (BA). Compared to prior work in this area, our proposed architecture can handle large disturbances from the outside temperature. Weather-induced disturbance also imposes an algorithm-independent limit on the capacity of the virtual energy storage the loads can provide. This key limitation, which was ignored in prior work, is incorporated in our formulation in a principled manner.

1. INTRODUCTION

Reliable operation of the power grid requires balancing demand and supply of power at all time scales. The time variation and unpredictability of renewable energy sources such as solar and wind make it challenging. Apart from expensive batteries, a complementary and inexpensive possibility is to harness the inherent flexibility in demand of many types of loads. Loads have been used for Demand Response (DR) for a long time, which is traditionally meant as a temporary reduction in demand to help the grid. It is recognized now that loads can supply a range of services to the grid beyond DR (Makarov, Ma, Lu, & Nguyen, 2008; Barooah, Bušić, & Meyn, 2015). With appropriate control, loads can vary their demand up and down around a baseline so that the deviation from the baseline appears like the charging and discharging of a battery to the grid. We call this Virtual Energy Storage (VES) from smart loads. A key constraint in using loads to provide any kind of grid-support service is that consumer's quality of service (QoS) must not be compromised.

The topic of this paper is design of the control architecture for a collection of thermostatically controlled loads (TCLs) to provide VES while maintaining strict bounds on the consumers' QoS. TCLs include residential air conditioners (ACs), water heaters, refrigerators etc. We focus on residential ACs that are on/off type; meaning their power consumption can only take two values: zero and a positive constant. For ACs, there are at least two primary measures of consumer's QoS: indoor temperature and cycling frequency. Cycling frequency refers to the number of times a load turns on and off in a given period. Short-cycling, which refers to frequent turning on and off, is to be avoided since that can damage equipment.

The problem is challenging on many counts. First is computational complexity. A thousand on/off loads means at any instant there are 2^{10^3} possible decisions. Due to QoS constraints that make current decisions dependent on the past, the decision space is even bigger. Second, since loads are distributed geographically over a large area and QoS constraints are local, the control architecture must be non-centralized. However, loads' actions must be coordinated so that together they deliver the service that the balancing authority (BA) asks for.

There is a large and growing literature on coordination of a collections of TCLs. Several distinct heuristics have been developed to address computational complexity. One control approach is for the BA to broadcast a common thermostat set point change (Callaway, 2009; Bashash & Fathy, 2011; Kundu, Sinitsyn, Backhaus, & Hiskens, 2011; Perfumo, Kofman, Braslavsky, & Ward, 2012). The control signal is decided based on a model with set point change as input and aggregate power consumption as output. A limitation of this approach is that the control relies on extremely small set point change command, as little as 0.0025 °C. Most residential thermostats have a setpoint resolution of 0.5 °C or more, and the deadband of a thermostat is also much wider than 0.0025 °C. Large transient oscillations due to synchronization of loads is another limitation (Bashash & Fathy, 2011). The work (Perfumo et al., 2012) offered a partial amelioration of the resolution issue by dividing the loads into clusters and sending distinct, but coarse set point changes, to every cluster.

Another widely used approach is based on first modeling the distribution of temperatures of the population as a finite-state Markov chain model by dividing temperature into bins; see (Mathieu, Koch, & Callaway, 2013; Zhang, Lian, Chang, & Kalsi, 2013; Liu & Shi, 2016) and references therein. The population density model is developed assuming that the local control algorithm that operates the load remains in place. Control by the BA therefore involves computing quantities such as the fraction of loads that should turn on or off. Converting that global command to individual on/off decisions to ensure good tracking performance while satisfying loads' local constraints is challenging. Although various heuristics are developed to this end, this remains one of the weaknesses of this approach. In (Mathieu et al., 2013), division of global control command to load level decision is purely deterministic while in (Zhang et al., 2013) some randomness is introduced.

This architecture involves intelligence both at the BA (global) and the load (local). The local intelligence at the load is a randomized controller that is meant to replace the thermostat, but is designed so that the resulting behavior - from the point of view of the consumer - is indistinguishable from that of a thermostat. In this paper we adopt the architecture proposed in (Meyn, Barooah, Bušić, Chen, & Ehren, 2015). The controller at the load has a tunable parameter ζ that determines the degree of randomness in this decision making. Higher ζ increases the probability of turning on while lower ζ increases the probability of turning off. The controller at the BA computes ζ and broadcasts to every load. Randomization at the loads helps avoid synchronization, and simplifies the control computation at the BA. The latter is crucial: randomization enables approximating the complex high-dimensional non-linear dynamics of the collection by a low-order input-output system with ζ being the input signal. The control command at the BA, ζ, is computed by using a MPC (Model Predictive control) scheme.

The randomized control at the loads used here is taken from (Bušić & Meyn, 2016). Compared to (Bušić & Meyn, 2016), there are several differences. The Markov chain model of the *individual* load proposed in (Bušić & Meyn, 2016) has a finite state, obtained by binning the temperature. The disturbance from outside temperature is assumed to be small and i.i.d., so that an LTI (linear time invariant) model of the population can be derived. Extending this formulation to large (realistic) disturbance is challenging. We do so by allowing the state space of a single load to be infinite so that a physics based model determines the transition probabilities immediately.

This paper makes two key contributions to the existing literature on developing tractable methods for coordination of a collection of on/off loads. One, prior work ignores the fundamental (algorithm independent) limit of what loads can do when the range of disturbance is large and time-varying. For instance, if it is cold outside and none of the ACs are on, it is not possible to turn off an additional 10% of them to help the grid! The *capacity* of the collection is strongly dependent on the weather, and is therefore time varying. We propose a model of the weather-dependent, time-varying capacity of the collection, provide a method to estimate it, and incorporate that into the BA's decision making. Apart from improving performance of real-time control, this model and method are useful for the BA to plan for resource adequacy.

The second contribution this paper makes is implementation of a MPC control scheme at the Balancing Authority level with local randomized control. In past work (Bušić & Meyn, 2016) disturbances were assumed small so a classical dynamic compensator was enough. However, when extending the work to more realistic scenarios (large weather disturbances) it is desirable to have a control scheme that can limit the rate and bound of control inputs to avoid exciting unmodeled dynamics and pushing the system away from its linear regime. Ability of MPC to enforce explicit bounds on the control signal is useful for this purpose.

2. PROBLEM FORMULATION: NEEDS OF THE GRID

The BA needs resources to provide the *net load*, $\tilde{d}_k$, (where k denotes the discrete time index) which is the difference between the baseline demand and renewable generation. The baseline demand is the demand when the loads are not providing any grid-support service, i.e., there is no interference from the grid on power demand of the loads.

The slowly-varying component of the net load is ideally provided by conventional generators that have limited ramping ability. This component, denoted by $\tilde{d}_k^{\text{LP}}$ can be obtained by low-pass filtering the net-load. The remainder is denoted by $\tilde{d}_k^{\text{HP}} = \tilde{d}_k - \tilde{d}_k^{\text{LP}}$. This "high-pass" component is zero-mean, and can be provided by VES and actual energy storage. A variant of this methodology is used by many BAs today.

Loads providing VES can be characterized by the frequency of demand variation they can tolerate without violating QoS constraints (Lin, Barooah, Meyn, & Middelkoop, 2015; Lin, Barooah, & Mathieu, 2017; Dowling & Zavala, 2017; Meyn et al., 2015). Therefore, an appropriate reference signal for the collection of loads to track, denoted by $\tilde{P}_k^{ref}$, can be obtained by bandpass filtering the signal $\tilde{d}_k^{\text{HP}} = F_{\text{BP}}(z)\tilde{d}^{\text{HP}}(z)$, where $F_{\text{BP}}(z)$ is a bandpass filter. The reference signal $\tilde{P}_k^{ref}$ measured in Watt (or kW, MW, GW etc.). The passband of $F_{\text{BP}}(z)$ must be chosen according to the ability of the loads. To determine the appropriate passband, the BA needs knowledge of the frequency response of the collection of loads. We describe in Section 3.5 how frequency response of a collection of loads, can be identified, with ζ as input and total power consumption deviation (from the baseline) as output. The passband is then chosen to be range of frequencies in which the aggregate model has high gain and small phase lag.

Let the power consumption of the collection of loads be denoted by P_k, and P_k^* be the value of P_k in the baseline scenario (no interference from the BA). The deviation of the total power consumption of all the loads is $\tilde{P}_k := P_k - P_k^*$. *The control problem for the BA is a tracking problem: $\tilde{P}_k$ should track the reference $\tilde{P}_k^{ref}$*. Furthermore, the BA should only provide a reference that the loads *can* track.

3. LOCAL CONTROL AT THE AC'S, AND AGGREGATE BEHAVIOR

3.1 Dynamic model of indoor temperature

Consider an AC with p^{rated} being the rated electrical power consumption. A simple model of the indoor temperature T is, $\dot{\theta}(t) = -\frac{1}{RC}\theta + \frac{-q_0}{C}u(t) + w(t)$. where $w(t) := \frac{1}{RC}\theta_a(t) + \frac{1}{C}q_{\text{int}}(t)$ is a time varying disturbance with θ_a the ambient temperature and q_{int} the internal disturbance, R is the resistance to heat flow offered by the building structure and C is the thermal capacitance of the building. The term q_{int} captures both occupant induced load and solar heat gain. The q_{ac} is the heat injected into the building by the AC. The control signal $u(t)$ is binary: it can be either 1 (on) or 0 (off). Denoting by $q_0 := COPp^{\text{rated}}$ the rated thermal power consumption of the AC, COP being its coefficient of performance, $q_{ac}(t) = -q_0$ if $u(t) = 1$ and $q_{ac}(t) = 0$ if $u(t) = 0$. We now have the binary discrete control signal u_k: 1 when on and 0 when off. The power consumption of the AC (in kW) is $p^{\text{rated}}u_k$.

3.2 Deterministic control of an AC

The control logic in a thermostat that operates a residential AC is usually based on a deadband around a user-specified set point. The AC is turned on if the measured indoor temperature exceeds the upper limit $\theta_{\max}$ and turned off if the temperature drops below the lower limit $\theta_{\min}$. When $\theta(t)$ is between $\theta_{\min}$ and $\theta_{\max}$, the previous decision (on or off) is maintained. This control strategy is *deterministic*: the on/off status of the AC at $k+1$ is a function of the temperature and on/off status at k: $u_{k+1} = u_{k+1}(\theta_k, u_k)$.

3.3 Randomized control of an AC

In randomized control (Meyn et al., 2015), the mapping $u(\theta_k, u_k) \mapsto u_{k+1}$ is no longer deterministic; it takes the two possible values (0 and 1) with certain probabilities. We first define a state space for the AC. Every element x of the state space X is a pair, $x = (u, \theta)$, where $u \in \{0,1\}$ is the "mode" (either on or off) and $\theta \in \mathbb{R}$ is the temperature of the house. For a state $x \in \mathsf{X}$, we denote by x^u the mode and x^θ the temperature θ. In the sequel, we use $\oplus$ and $\ominus$ interchangeably with 1 and 0, respectively, as it is more intuitive for on and off.

A randomized controller is a rule to determine the probabilities of x_{k+1}^u being 0 or 1 given the current state, x_k. The controller is therefore completely specified by a Markov operator $R_\zeta : \{0,1\} \times \mathbb{R} \to [0,1]$. The quantity $R_\zeta(x, y^u)$ is

the probability of the mode being y^u at the next instant $k+1$ given the state is x at the current time k. The Markov operator is parameterized by a real number ζ. *Designing a controller* is equivalent to designing the operator $R_\zeta(x,y)$ for all $x,y \in \mathsf{X}$ and all $\zeta \in \mathbb{R}$.

The evolution of the temperature is governed by the dynamic system described in section 3.1. We can represent it as another Markov operator $Q_w(x,y^\theta)$: the probability of the temperature being y^θ at the next instant $k+1$ given the state is x at the current time k. Since temperature evolution is a deterministic dynamic system, the transition probabilities $Q_w(x,y^\theta)$ will be Dirac-delta functions. The next state will depend on the disturbance w, hence the operator $Q_w(x,y^\theta)$ is parameterized by the disturbance w.

Combining the effect of the control and the disturbance, we get the total probability of transitioning from state x to y over one time period, which is denoted by $P_{\zeta,w}(x,y)$:

$$P_{\zeta,w}(x,y) = R_\zeta(x,y^u)Q_w(x,y^\theta), \quad x,y,\in \mathsf{X} \tag{1}$$

The model (1) is an extension of the finite-state Markov model of a TCL in (Bušić & Meyn, 2016); here the state space X is infinite.

The controller $R_\zeta(x,y^u)$ has to be designed in such a way that the following properties are satisfied. One, the operator $R_0(x,y^u)$ in the *baseline* case $\zeta = 0$ (corresponding to no interference from the grid) mimics the behavior of the deterministic control. Meaning, it has to turn the AC on if indoor temperature exceeds $\theta_{\max}$ and off if the indoor temperature falls below $\theta_{\min}$, and avoid short cycling. Second, if ζ is positive, the probability of turning on should increase and if ζ is negative, probability of turning off should increase. In effect, the variable ζ can be used by the BA as the control signal to manipulate the power consumption of an AC.

3.3.1 Randomized control law: To construct the baseline control law, $R_0(x,y^u)$, two switching probability functions need to be defined. To construct these functions, first consider the two random variables:

θ^+: the temperature above which the A/C turns on (after having started in the off state, or since the last off state),
θ^-: the temperature below which the A/C turns off (after having started in the on state, or since the last on state).

Consider the usual heating (left) and cooling (right) scenario when the AC has been off (on) and temperature is increasing (decreasing). Then, at time k, the probability of switching on/off (left/right) is

$$\begin{aligned}
p^{\oplus}(k) &:= \mathsf{P}(X^u_k = 1 | X^u_{k-1} = 0, X^u_{k-2} = 0, \dots) \\
&= \mathsf{P}(\theta^+ \le \theta_k | \theta^+ > \theta_{k-1}) \\
&= \frac{\mathsf{P}(\theta_{k-1} < \theta^+ \le \theta_k)}{\mathsf{P}(\theta^+ > \theta_{k-1})} \\
&= \frac{F^{\oplus}(\theta_k) - F^{\oplus}(\theta_{k-1})}{1 - F^{\oplus}(\theta_{k-1})}
\end{aligned}
\qquad
\begin{aligned}
p^{\ominus}(k) &:= \mathsf{P}(X^u_k = 0 | X^u_{k-1} = 1, X^u_{k-2} = 1, \dots) \\
&= \mathsf{P}(\theta^- \ge \theta_k | \theta^- \le \theta_{k-1}) \\
&= \frac{\mathsf{P}(\theta_k < \theta^- \le \theta_{k-1})}{\mathsf{P}(\theta^- \le \theta_{k-1})} \\
&= \frac{F^{\ominus}(\theta_{k-1}) - F^{\ominus}(\theta_k)}{F^{\ominus}(\theta_{k-1})}
\end{aligned} \tag{2}$$

where $F^{\oplus}/F^{\ominus}$ is the CDF of the r.v. θ^+/θ^-. Notice that in the second line left (right) column we have assumed that temperature has been increasing (decreasing), so that $\theta_{k-1} < \theta_k$ ($\theta_k < \theta_{k-1}$). To ensure positivity of probabilities under all scenarios, e.g., if temperature decreases (increases) when AC is off (on), we use

$$p^{\oplus}(k) = \frac{[F^{\oplus}(\theta_k) - F^{\oplus}(\theta_{k-1})]_+}{1 - F^{\oplus}(\theta_{k-1})} \qquad p^{\ominus}(k) = \frac{[F^{\ominus}(\theta_{k-1}) - F^{\ominus}(\theta_k)]_+}{F^{\ominus}(\theta_{k-1})}. \tag{3}$$

where $[x]_+ = \max(x,0)$. The two CDF's appearing in (3) are additional design choices and are taken from (Bušić & Meyn, 2016) as,

$$F^{\oplus}(z) = \exp(-(\theta_{\max} - z)^\rho / (2\sigma^\rho)) \qquad F^{\ominus}(z) = 1 - F^{\oplus}(\theta_{\max} + \theta_{\min} - z) \tag{4}$$

where ρ and σ are design parameters. In this paper we use $\rho = 0.75$ and $\sigma = 0.1$.

With the previous developments the baseline (nominal) control law, $R_0(x,y^u)$, can be stated explicitly as:

$$\begin{aligned} R_0(x,1) &= \mathsf{P}\big(X_k^u = 1 | X_{k-1} = x\big) \qquad R_0(x,0) = \mathsf{P}\big(X_k^u = 0 | X_{k-1} = x\big) \\ &= \begin{cases} p^{\oplus}(x^\theta), & x^u = \ominus \\ 1 - p^{\ominus}(x^\theta), & x^u = \oplus \end{cases} \qquad = \begin{cases} p^{\ominus}(x^\theta), & x^u = \oplus \\ 1 - p^{\oplus}(x^\theta), & x^u = \ominus \end{cases} \end{aligned} \tag{5}$$

We choose the control law (when $\zeta \neq 0$) in this paper to be the *myopic policy* of (Bušić & Meyn, 2016):

$$R_\zeta(x,y^u) := R_0(x,y^u) \exp(\zeta \mathscr{U}(y^u) - \Lambda_\zeta(x)) \tag{6}$$

where $\mathscr{U}(\cdot)$ is the *utility function*:

$$\mathscr{U}(x) = \mathscr{U}(x^u) = \begin{cases} 1 & x^u = \oplus \\ 0 & x^u = \ominus \end{cases} \tag{7}$$

and $\Lambda_\zeta(x)$ is the normalization constant to make the probabilities sum to 1.

3.4 Aggregate behavior of N ACs with local randomized control

Let μ_k be the pdf (defined over the state space X) of the state of an AC when it operates according to the randomized control law described in Section 3.3. The pdf (left) and output (right) evolve according to

$$\mu_{k+1} = \mu_k P_{\zeta_k, w_k} \quad \gamma_k := \mathsf{P}(x_k^u = \oplus) = \int_{R^\oplus} \mu_k dx, \tag{8}$$

With γ_k being the probability, at time k, of the state being in the "on" mode. As a system with inputs ζ, w and output γ, (8) is an infinite dimensional dynamic system that is linear in the state but nonlinear in the inputs. To make the connection between the individual and the aggregate, consider N homogeneous loads operating under the same randomized policy and subjected to the same disturbance. Define the fraction of loads that are on at k:

$$y_k^{(N)} := \frac{1}{N} \sum_{i=1}^{N} \mathscr{U}(x_k^{(i)}) \tag{9}$$

We assume a mean-filed limit holds. That is, if N homogeneous loads are subjected to the same inputs ζ and w, as $N \to \infty$ the fraction of loads that are on at time k approaches the probability of a single load being on at k:

$$\lim_{N\to\infty} y_k^{(N)} = \gamma_k \tag{10}$$

In the sequel, for simplicity we drop the superscript N from all quantities that contain an average over N.

3.5 Control oriented modeling of the aggregate

A model relating the two inputs (ζ_k, w_k) to the output $\tilde{\gamma}_k$ of the Markovian model of the AC is needed for computing the control signal ζ_k by the BA. Because of the mean field limit, this model will predict the change in the fraction of ACs on, $\tilde{y}_k$, over baseline due to ζ_k.

A linear approximation of the dynamic model (8) can be obtained by linearizing around an equilibrium point, as was done in (Meyn et al., 2015; Bušić & Meyn, 2016). There are two difficulties. In contrast to the Markov models in (Meyn et al., 2015; Bušić & Meyn, 2016) which had finite state spaces, the model in this paper has an infinite state space. So Jacobian linearization will lead to an infinite dimensional LTI (linear time invariant) system. To make the control design and implementation tractable, a *finite dimensional linear approximation* is needed. This was also the approach adopted in (Bušić, Hashmi, & Meyn, 2017). Second, we need to choose an appropriate equilibrium point (ζ^*, w^*) to linearize around, and the model is likely to be accurate only for ζ and w close to that equilibrium point. In this work we use a LTI (linear time invariant) approximation of the Markovian model around the equilibrium point $(\zeta^* = 0, w^*(T_a = 27^\circ C))$:

$$\tilde{x}_{k+1} = A\tilde{x}_k + B\zeta_k, \qquad \tilde{y}_k = C\tilde{x}_k \tag{11}$$

where $x_k \in \mathbb{R}^n$ is a (fictitious) state, and its dimension n is a design choice. In this paper we fit discrete time transfer functions to empirically estimated transfer functions (ETFE) for system identification (Ljung, 1999). The transfer function obtained from identification can be seen in Figure 2. The state space model (A,B,C) (11), is a minimal realization of the shown transfer function ($n = 3$).

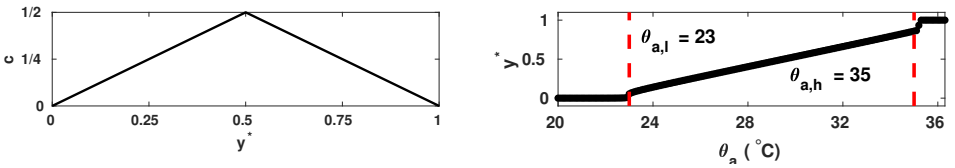

Figure 1: Left: Proposed capacity model. Right: y^* (baseline fraction of loads on) versus ambient temperature θ_a estimated from simulation of 20000 loads.

4. DECISION MAKING AT THE BA

The control problem faced by the BA was discussed in Section 2. We now pose the problem more precisely and connect it to the mean filed limit.

4.1 The tracking problem

When $\zeta_k \equiv 0$ and $w_k \equiv w$, then γ_k reaches a constant steady state value as $k \to \infty$, which we denote by $\gamma^*(w)$. In other words, $\gamma^*(w)$ is the fraction of the loads that are on at steady state for a constant disturbance w under baseline randomized control. We now define the deviation signal $\tilde{\gamma}_k := \gamma_k - \gamma^*(w_k)$.

When the system is operating in baseline conditions, i.e., $\zeta_k \equiv 0$, but with a time-varying disturbance, then $\gamma_k = \gamma^*(w_k)$ at every k if the response of the aggregate to the disturbance is instantaneous. Assuming such a speedy response, we have $\tilde{\gamma}_k \equiv 0$ for baseline operation. When $\zeta_k \equiv \zeta > 0$ then $\tilde{\gamma}_k$ converges to a positive value that is larger for larger ζ. This follows from the design of the controlled transition probability operator R_ζ that was described in Section 3.3. Similarly, if $\zeta_k \equiv \zeta < 0$ then $\tilde{\gamma}_k$ converges to a negative value that is smaller for smaller ζ.

Recall y_k defined in (9); the empirical counterpart to γ_k. We similarly define the empirical counterparts to $\gamma^*(w)$ and $\tilde{\gamma}_k$, and call them $y^*(w)$ and $\tilde{y}_k$. Therefore,

$$\tilde{y} = y_k - y_k^*. \tag{12}$$

Assuming the mean field limit holds, $y^*(w) \to \gamma^*(w)$ and $\tilde{y}_k \to \tilde{\gamma}_k$ as $N \to \infty$.

Recall the tracking objective from Section 2: the power consumption deviation of the collection $\tilde{P}_k$ should track an exogenous reference signal $\tilde{P}_k^{ref}$. When the loads are homogeneous, $\tilde{P}_k = P_k - P_k^* = Np^{\text{rated}}(y_k - y_k^*) = Np^{\text{rated}}\tilde{y}_k$. The problem is then equivalent to $\tilde{y}_k$ tracking the *normalized reference* $r_k := \frac{1}{Np^{\text{rated}}}\tilde{P}_k^{ref}$. Because of the mean field limit, $\tilde{y}_k$ tracking r_k is (approximately) equivalent to $\tilde{\gamma}_k$ tracking r_k. For the BA, the control design problem can be posed in terms of a single load: choose the control command ζ_k so that the deviation probability $\tilde{\gamma}_k$ of a load tracks the normalized reference r_k.

4.2 Modified tracking problem due to capacity of the aggregate

There is a fundamental (algorithm-independent) limit to capacity. Consider a constant disturbance w and no interference from the grid, so that the fraction of loads that are on is a constant (at steady state). If this fraction is only 10% (i.e., $y^*(w) = 0.1$), it is impossible for an additional 20% of the loads to be turned *off*, though an additional 90% of the loads can be turned *on*. Therefore, the largest amplitude of a sinusoidal reference that $\tilde{y}_k$ can track now, irrespective of how that tracking is achieved, is 0.1. If the disturbance is larger (warmer outside temperature), $y^*(w)$ will increase and so will the capacity. When $y^*(w)$ exceeds 50%, the trend will reverse. Now a large fraction of loads (greater than 50%) can be turned off, but the maximum fraction that can be turned on is smaller than 50%. The capacity therefore is a non-monotonic function of the disturbance, and achieves a theoretical maximum value of 0.5.

We therefore propose a capacity model - capacity as a function of the disturbance - as a piecewise linear function that is shown in Figure 1(left). Since disturbance is time varying, so is capacity. The BA can predict the capacity of a collection of loads as follows:

1. Determine the function $y^*(w)$ by running a simulation of a large number of loads operating with $\zeta_k \equiv 0$ and $w_k \equiv w$, estimate the corresponding y^*, and then repeat for a range of w values.

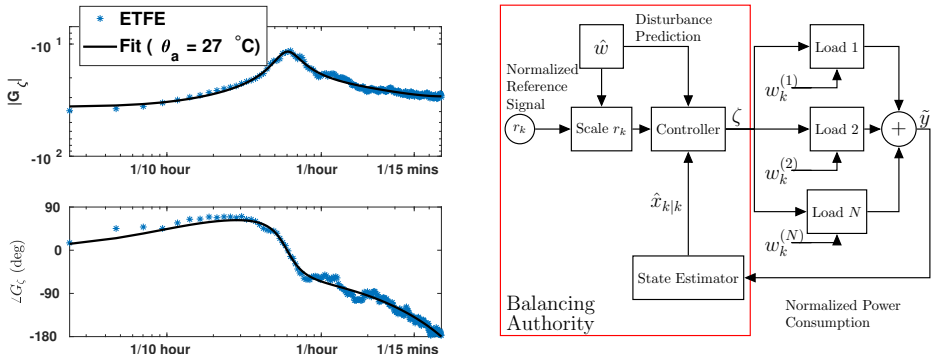

Figure 2: (Left): Bode plot of the finite dimensional LTI system that approximates the aggregate model (Right): Control Architecture.

2. Use weather prediction to predict w_k from $w_k := \frac{1}{RC}\theta_{a,k} + \frac{1}{C}q_{int,k}$ (assuming $q_{\text{int}}(t) \equiv 0$), and use the function $c(w)$ shown in Figure 1(left) to predict the capacity at k.

Figure 1(right) shows an example of the curve $y^*(w)$, obtained by using this method on a collection of 20000 ACs. Section 5.1 describes details of simulation parameters. Note that for outside temperature below the minimum threshold $\theta_{a,l}$, all the ACs turn off, so $y^* = 0$. Conversely, $y^* = 1$ for outside temperature above the maximum threshold $\theta_{a,h}$.

The capacity of the collection must be respected when specifying a reference signal to be tracked by a collection of loads. Recall from Section 4.1 that the nominal reference signal r_k varies between -1 and 1, and is obtained without considering the time-varying capacity of the collection. This must be scaled by the capacity c_k to determine the actual reference that $\tilde{y}_k$ needs to track. We define $\tilde{r}_k := c_k r_k$ the *scaled reference*, which is what the collection of TCLs are asked to track. It is assumed that the BA uses other resources for the difference $r_k - \tilde{r}_k$.

4.3 Computing the control signal ζ

We propose to use MPC at the BA level to compute the control signal ζ_k. For large realistic disturbances it is desirable to use a control scheme that can ensure bounds and rate constraints on the control input, so that the the collection operates near its linear regime. At time k the BA solves the following optimization problem with the decision variable $\zeta := [\zeta_k, \zeta_{k+1}, \zeta_{k+k_{plan}-1}]^T \in \mathbb{R}^{k_{plan}}$, where k_{plan} is the planning horizon:

$$\begin{aligned} \min_{\zeta \in \mathbb{R}^{k_{plan}}} \quad & \sum_{t=k}^{k+k_{plan}-1} (r_t c_t - C\tilde{x}_t)^2 \\ & \text{s.t., (for } k \le t \le k+k_{plan}-1\text{)} \\ & \underline{\zeta} \le \zeta_t \le \overline{\zeta}, \quad \underline{r} \le \zeta_{t+1} - \zeta_t \le \overline{r} \\ & \tilde{x}_{t+1} = A\tilde{x}_t + B\zeta_t, \quad \tilde{x}_k = \hat{x}_{k|k} \end{aligned} \tag{13}$$

where $\hat{x}_{k|k}$ is the estimate of the starting state $\tilde{x}_k$ of the LTI model (11) obtained by using all measurements collected up to k. A Kalman filter is used for this purpose. The BA broadcasts the sample of the control signal for the current time instant, i.e., ζ_k, and discards the future samples $\zeta_{k+1}, \ldots, \zeta_{k_{plan}-1}$. The process repeats at $k+1$.

The bound and rate constraints on ζ_k in Problem (13) are not due to physical or actuator based limitations, but rather are needed as a consequence of the problem formulation and are a design choice. Since the LTI model is a linear approximation around $\zeta = 0$, it is accurate only for small ζ. Additionally, the rate of change of ζ is constrained so as to avoid exciting unmodeled dynamics (Rawlings & Mayne, 2009).

The architecture of the overall control system, including the controller at the BA, and the loads, is shown in Figure 2. The BA needs the LTI model of the collection, and the design variables such as rate and bound constraints on ζ. In real time it will need predictions of the nominal reference r_k, capacity c_k, and disturbance w_k, from now (k) up to k_{plan} time instants into the future. Accurate prediction of the reference signal is assumed reasonable as it is composed

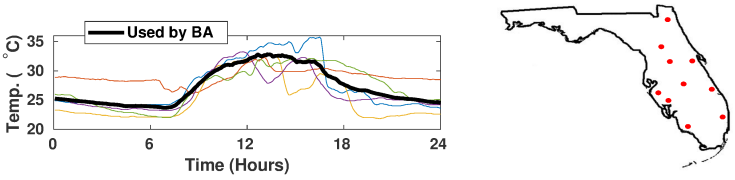

Figure 3: (left): Temperatures seen by five weather stations and temperature prediction used by the BA (right): Locations of weather stations (red dots).

of quantities already available to the BA, see Section 4.1. Disturbance and capacity prediction can be obtained from weather predictions (Section 4.2). At time k the BA will need measurements of $\tilde{y}_k$. Additionally, if no reference scaling is performed this is numerically equivalent to setting the value of c_k to 1.

We assume that the feedback signal $\tilde{y}_k$ can be measured by the BA. From this measurement, the fraction of loads that are on at time k, the BA can estimate $\tilde{y}_k$ from (12). The fraction of loads that are on at time k can be measured exactly if each load communicates back to the grid their current on/off state. Thus the BA has all the required information to determine the control signal ζ_k by solving (13). The optimization problem (13) is a QP (quadratic program): the objective function is quadratic in the decision variables and the constraints are linear (Luenberger, 2003). Thus, as long as the problem is feasible, its solution can be computed efficiently with standard NLP solvers. The computations reported in this paper were done in MATLAB©, and problem (13) was solved using `cvx` (Grant & Boyd, 2011).

5. PERFORMANCE EVALUATION

5.1 Simulation set up

The BA performs all calculations, including the off-line system identification, assuming the loads are homogeneous. However, during control each load receives a different disturbance value. This is illustrated in Figure 2 (right) and 5 sample paths for temperature are provided in Figure 3 (left). The parameters for the homogeneous loads are described in Table 1. The MPC calculations are performed by using a planning period of 1800 seconds ($k_{plan} = 6$).

Table 1: Parameters

TCL parameters	Symbol	Value	Units	Parameters for BA	Symbol	Value	Units
Resistance of TCL model	R	1.5	K/kW	lower rate bound	$\underline{r}$	-5	NA
Capacitance of TCL model	C	7.2×10^6	J/K	upper rate bound	$\overline{r}$	5	NA
Rated elec. power	p^{rated}	2.8	kW	upper bound	$\overline{\zeta}$	7	NA
Coeff. of Perf.	COP	3.5	NA	lower bound	$\underline{\zeta}$	-7	NA
Temperature set point	θ_{set}	22	°C	number of loads	$N_{devices}$	20000	NA
Temperature deadband	δ	1	°C	Sample time	T_s	300	sec.

Geographically, we imagine the BA is located in the state of Florida in the southeastern United States, and its territory covers almost the entire state; see Figure 3 (right). The ambient temperature prediction it uses in all of its calculations is shown as the thick black line in Figure 3 (left). This signal is obtained by averaging the temperature measurements collected from ten weather stations in Florida during two days in June 2017. The stations are shown in Figure 3. The BA predicts the disturbance signal from weather prediction by using the R,C values assumed and assuming that the occupant-induced heat-gain q_{int} is 0 at every load and every instant. We also envision a future for Florida, which is called the Sunshine State, in which it has a high penetration of solar. To mimic such a scenario, the reference signal r_k is chosen by filtering BPA's balancing reserves data as described in Section 2. A Butterworth filter with passband $[\frac{1}{6}, \frac{1}{2}]$ (1/hour) was used. The choice of the passband is informed by the identified model, since the LTI model observed high gain in this region (see Figure 2 (left)).

5.2 Simulation Results

Figures 4-5 show the closed loop tracking results for the system under MPC control with two scenarios: scaled reference (Figure 5) and unscaled reference (Figure 4). Scaled reference refers to the reference signal being $\tilde{r}_k$, while

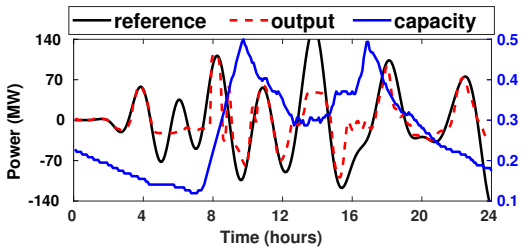

Figure 4: Non-scaled reference: reference r_k, output (power deviation), and capacity c_k.

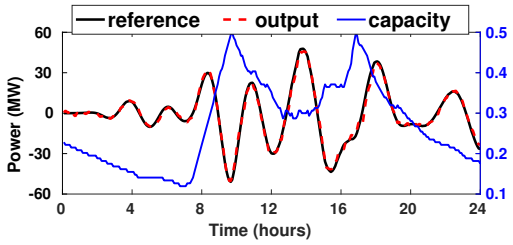

Figure 5: Scaled reference: reference $\tilde{r}_k$, output (power deviation), and capacity c_k.

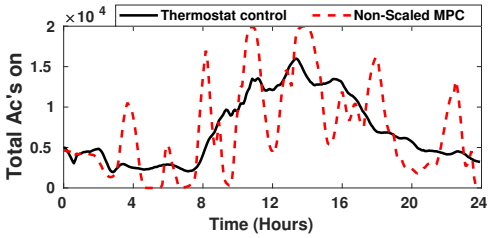

Figure 6: Non-scaled reference: Total number of loads on for thermostat and randomized control.

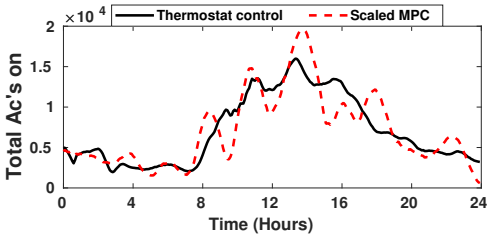

Figure 7: Scaled reference: Total number of loads on for thermostat and randomized control.

unscaled refers to the the reference signal being r_k. These comparisons are provided to show the need for using the scaled the reference that considers the capacity of the collection of TCLs.

5.2.1 Reference Tracking Performance: The tracking results for the non-scaled scenario are shown in Figure 4. Observe that there are periods where the capacity is too small for the BA supplied reference to be tracked by the collection of ACs. As a result, the reference tracking performance is poor.

The tracking results for the scaled reference scenario are shown in Figure 5. The problem encountered in the previous scenario vanishes, and good tracking performance is obtained. The tracking error was reduced by $\approx 95\%$ compared to the unscaled scenario.

5.2.2 QoS Performance: There are two key QoS signals: indoor temperature and cycling of ACs. Recall that the temperature QoS is preserved through design: the local randomized controller is designed to behave identically to the deterministic thermostat controller when the temperature goes beyond the deadband.

To examine the cycling QoS, we compare cycling of the ACs when providing VES to the cycling incurred when the ACs operate with the deterministic thermostat controller and not providing any VES service. The fraction of TCLs on vs. time is shown in Figure 6-7. We see from the figures that when the ACs are providing VES with the proposed control strategy (randomized control at ACs, MPC at the BA, and capacity-scale reference), the fraction of loads that are on at any given time does not deviate significantly from the baseline value when the ACs are not providing VES but merely operating under thermostat control. Another important observation from the figures is that when the reference is computed without taking into account the capacity of the collection, cycling QoS is not maintained.

6. CONCLUSION

Randomized control at the loads addresses many of the limitations of previous approaches to distributed coordination of TCLs. It provides a global control signal for the BA to manipulate states of the TCLs, while ensuring that local constraints are never violated. In this paper we leveraged the randomized control philosophy espoused in (Meyn et al., 2015; Bušić & Meyn, 2016; Bušić et al., 2017) and extended it to handle realistic weather induced disturbance on

the TCLs. We proposed a principled method for determining the algorithm-independent capacity of the collection and using it to improve the control problem formulation.

Although the capacity model proposed here is based on a heuristic, it is shown to dramatically improve tracking results. In future work a more refined capacity model will need to be determined. This model needs to answer how much power/energy a collection of AC's can deliver, and how this depends on the consumers desired QoS level.

Another natural extension for future work is to consider other operating conditions for the outside temperature. In this work, we have considered a "summer" scenario. In future work, scenarios where T_a is consistently below or in the TCL deadband will be studied. The challenge here is that these scenarios contain periods of zero capacity; reference generation must be taken with even more care so as to avoid rapid changes to and from zero reference value.

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