Multi-View Network Embedding Via Graph Factorization Clustering and Co-Regularized Multi-View Agreement

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***Abstract*—Real-world social networks and digital platforms are comprised of individuals (nodes) that are linked to other individuals or entities through multiple types of relationships (links). Sub-networks of such a network based on each type of link correspond to distinct views of the underlying network. In real-world applications each node is typically linked to only a small subset of other nodes. Hence, practical approaches to problems such as node labeling have to cope with the resulting sparse networks. While low-dimensional network embeddings offer a promising approach to this problem, most of the current network embedding methods focus primarily on single view networks. We introduce a novel multi-view network embedding (MVNE) algorithm for constructing low-dimensional node embeddings from multi-view networks. MVNE adapts and extends an approach to single view node embedding using graph factorization clustering (GFC) to the multi-view setting using an objective function that maximizes the agreement between views based on both the local and global structure of the underlying multi-view graph. Our experiments with several benchmark real-world single view networks show that SVNE yields network embeddings that are competitive with or superior to those produced by the state-of-the-art single view network embedding methods when the embeddings are used for labeling unlabeled nodes in the networks. Our experiments with several multi-view networks show that MVNE substantially outperforms the single view methods on integrated view and the state-of-the-art multi-view methods. We further show that even when the goal is to predict labels of nodes within a single target view, MVNE outperforms its single-view counterpart suggesting that the MVNE is able to extract the information that is useful for labeling nodes in the target view from the all of the views.**

***Index Terms*—multi-view learning, network embedding, representation learning**

I. Introduction

Social networks e.g., Facebook, social media e.g., Flickr, and e-commerce platforms, e.g., Amazon, can be seen as very large heterogeneous networks where the nodes correspond to diverse types of entities, e.g., articles, images, videos, music, etc. In such networks, an individual can link to multiple other individuals via different types of social or other relationships e.g., friendship, co-authorship, etc [4], [12], [37]. Examples include Google+ which allows members to specify different 'circles' that correspond to different types of social relationships; DBLP which contains multiple types of relationships that link authors to articles, publication venues, institutions, etc. Such networks are naturally represented as *multi-view* networks wherein the nodes denote individuals and links denote relationships such that each *network view* corresponds to a single type of relationship, e.g., friendship, family membership, etc [2], [6], [17], [33]. Such networks present several problems of interest, e.g., recommending products, activities or membership in specific interest groups to individuals based on the attributes of individuals, the multiple relationships that link them to entities or other individuals, etc. [3], [13].

When multiple sources of data are available about entities of interest, multi-view learning offers a promising approach to integrating complementary information provided by the different data sources (views) to optimize the performance of predictive models [36], [40]. Examples of such multi-view learning algorithms include: multi-view support vector machines [7], [20], multi-view matrix (tensor) factorization [23], [24], and multi-view clustering via canonical correlation analysis [9], [11]. However, most of the existing multi-view learning algorithms are not (i) directly applicable to multi-view networks; and (ii) designed to cope with data sparsity, which is one of the key challenges in modeling real-world multi-view networks: although the number of nodes in real-world networks is often in the millions, typically each node is linked to only a small subset of other nodes. Low-dimensional network embeddings offer a promising approach to dealing with such sparse networks [10]. However, barring a few exceptions [6], [25], [31], [34], most of the work on network embedding has focused on methods for single view networks [16], [29], [37].

Against this background, the key contributions of this paper are as follows:

1) We introduce a novel multi-view network embedding (MVNE) algorithm for constructing low-dimensional embeddings of nodes in multi-view networks. MVNE exploits recently discovered connection between network adjacency matrix factorization and network embedding [30]. Specifically, we use the graph factorization

clustering (GFC) [41] algorithm to obtain single view network embedding. MVNE extends the resulting single view network node embedding algorithm (SVNE) to the multi-view setting. Inspired by [19], MVNE integrates both local and global context of nodes in networks to construct effective embeddings of multi-view networks. Specifically, MVNE uses a novel objective function that maximizes the agreement between views based on both the local and global structure of the underlying multi-view graph.

2) We present results of experiments with several benchmark real-world data that demonstrate the effectiveness of MVNE relative to state-of-the-art network embedding methods. Specifically, we show that (i) SVNE is competitive with or superior to the state-of-the-art single view graph embedding methods when the embeddings are used for labeling unlabeled nodes in single view networks. (ii) MVNE substantially outperforms multi-view variants of the state-of-the-art multi-view networks, as well as SVNE based methods for aggregating information from multiple views, as well as SVNE based and multi-view graph clustering methods for aggregating information from multiple views, when the embeddings are used for labeling nodes in multi-view networks. (iii) MVNE is able to augment information from any target view with relevant information extracted from other views so as to improve node labeling performance on the target view in multi-view networks.

The rest of the paper is organized as follows. In Section 2, we formally define the problem of multi-view network embedding. In Section 3, we describe the proposed MVNE framework. In Section 4, we present results of experiments that compare the performance of MVNE with state-of-the-art single view network node embedding methods and their multi-view extensions. In Section 5, we conclude with a summary, discussion of related work, and some directions for further research.

II. Preliminaries

Definition 1. (Multi-view Network) A multi-view network is defined by 6-tuple $G = (V, E, T_V, T_E, \phi_V, \phi_E)$ where V is a set of nodes, E is a set of edges, T_V and T_E respectively denote sets of node and relation types, and $\phi_V : V \to \mathcal{P}(T_V)$ and $\phi_E : E \to T_E$ (where $\mathcal{P}(S)$ is the power set of set S), are functions that associate each node $v \in V$ with a subset of types in T_V and each edge $e \in E$ with their corresponding type in T_E respectively.

Note that a node can have multiple types. For example, in an academic network with nodes types authors (A), professors (R), papers (P), venues (V), organizations (O), topics (T), relation types may denote the coauthor (A-A), publish (A-P), published-in (P-V), has-expertise (R-T), and affiliation (O-A) relationships. An individual in an academic network can be an author, professor, or both.

Note that the node types are selected from the set V of nodes $|T_V|$ (potentially overlapping) subsets $V^{(1)}, V^{(2)} \cdots V^{(|T_V|)}$. Each view of a multi-view network is represented by an adjacency matrix for each type of edge $t \in T_E$. For an edge type that denotes relationships between nodes in $V^{(i)}$, the corresponding adjacency matrix $W^{(t)}$ will be of size $|V^{(i)}| \times |V^{(i)}|$. Thus, a multi-view network G can be represented by a set of single view networks $G^{(1)} \cdots G^{(|T_E|)}$ where $G^{(t)}$ is represented by the adjacency matrix $W^{(t)}$.

Definition 2. (Node label prediction problem) Suppose we are given a multi-view network G in which only some of the nodes of each node type $t \in T_V$ are assigned a finite subset of labels in L_t, where L_t is the set of possible labels for nodes of type t. Given such a network G, node label prediction entails completing the labeling of G, that is, for each node of type t that does not already have a label $l \in L_t$, specifying whether it should be labeled with l based on the information provided by the nodes and edges of the multi-view network G.

In the academic network described above, given a subset of papers that have been labeled as high impact papers, and/or review papers, node labeling might require, for example, predicting which among the rest of papers are also likely to be high impact papers and/or review papers. The link (label) prediction problem can be analogously defined.

In the case of real-world multi-view networks, because each node is typically linked to only a small subset of the other nodes, a key challenge that needs to be addressed in solving the node (and link) labeling problems has to do with the sparsity of the underlying network. A related problem has to do with the computational challenge of working with very large adjacency matrices. Network embeddings, or low-dimensional representation of each network node that summarizes the information provided about the node by the rest of the network, offers a promising approach to addressing both these problems.

Definition 3. (Multi-view Network Embedding) Given a multi-view network G, multi-view network embedding entails learning of d-dimensional latent representations $X \in \Re^{|V|d}$, where $d << |V|$ that preserve the structural and semantic relations among them adequately for performing one or more tasks, e.g., node label prediction.

The quality of specific network embeddings (and hence that of the algorithms that produce them) have to be invariably evaluated in the context of specific applications, e.g., the predictive performance of node label predictors trained using the low-dimensional representations of nodes along with their labels, evaluated on nodes that were not part of the training data.

The key challenge presented by multi-view network embedding over and above that of single view embedding has to do with integration of information from multiple views. Here, we can draw inspiration from multi-view learning [5], [36], [40], where in the simplest case, each view corresponds to a different subset of features, perhaps obtained from a different modality. Multi-view learning algorithms [22], [27] typically aim to maximize the agreement (with respect to the output of classifiers trained on each view, similarity of, or mutual information between low-dimensional latent representations of each view, etc).

III. Multi-View Network Embedding

As noted already, our approach to solving multi-view network embedding problem leverages a single view network embedding (SVNE) method inspired by a graph soft clustering algorithm, namely, the graph factorization clustering (GFC) [41]. To solve the multi-view embedding problem, MVNE combines the information from the multiple views into the co-regularized factorization wherein the agreement between the multiple views is maximized using suitably designed objective function. MVNE combines the information from multiple views into the co-regularized factorization space.

A. Single view network embedding

Consider a single view network $G = (V, E)$ consisting of nodes V and edges E. Let $K(V, U, F)$ be a bipartite graph where U is a set of nodes that is disjoint from V and F contains all the edges connecting nodes in V with nodes in U. Let $B = \{b_{ip}\}$ denote the $|V| \times |U|$ adjacency matrix with $b_{ip} \geq 0$ being the weight for the edge between $v_i \in V$ and $u_j \in U$. The bipartite graph K induces a weight between v_i and v_j

$$w_{ij} = \sum_p b_{ip} b_{jp} = B\Lambda^{-1}B^T \tag{1}$$

where $\Lambda = diag(\lambda_1 \ddots \lambda_{|U|})$ with $\lambda_p = \sum_i b_{ip}$ denotes the degree of vertex $u_p \in U$. We can normalize W in Eq.(1) such that $\sum_{ij} w_{ij} = 1$ and $w_{ij} = p(v_i, v_j)$ according to the stationary probability of transition between v_i and v_j [41]. Because in a bipartite graph $K(V, U, F)$, there are no direct links between nodes in V, and all the paths from v_i to v_j must pass through nodes in U, we have:

$$p(v_i, v_j) = p(v_i|v_j)p(v_j) \tag{2}$$

We can estimate this distribution as: $\hat{p}(v_i, v_j) = \frac{w_{ij}}{\sum_{ij} w_{ij}}$. And $p(v_j)$ is given by $\frac{deg(v_j)}{\sum_{ij} w_{ij}}$ where $deg(v_j)$ represents the degree of v_j. $p(v_i|v_j) = \sum_{p=1}^{|U|} p(v_i|u_p)p(u_p|v_j))$. The transition probabilities between the graph G and the communities U (nodes of the bipartite graph) are given by $p(v_i|u_p) = \frac{b_{ip}}{\lambda_p}$ and $p(u_p|v_j) = \frac{b_{pj}}{deg(v_j)}$ where matrix B denotes the weights between graph G and U and λ_p denotes the degree of u_p. Hence, the transition probability between two nodes v_i, v_j is given by:

$$w_{ij} = \sum_{p=1}^{d} \frac{b_{ip} b_{pj}}{\lambda_p} = (B\Lambda^{-1}B^T)_{ij} \tag{3}$$

Both the local and the global information in G are thus encoded by matrix B and diagonal matrix Λ. We can optimally preserve the information in G by minimizing the objective function $\mathcal{L}(W, B\Lambda^{-1}B^T)$ where $\mathcal{L}(X, Y) = \Sigma_{ij}(x_{ij} log \frac{x_{ij}}{y_{ij}} - x_{ij} + y_{ij})$ is a variant of the K-L divergence. Replacing B by $H\Lambda$, we obatin the objective function as following:

$$\mathcal{L}(W, H\Lambda H^T) \tag{4}$$

The objective function Eq.(4) is proved to be non-increasing under the update rules Eq.(5) and Eq.(6) for H and Λ [41]:

$$\begin{gathered} \tilde{h_{ip}} \propto h_{ip} \Sigma_j log \frac{W_{ij}}{(H\Lambda H^T)_{ij}} \lambda_p h_{jp} \\ s.t. \sum_{p=1}^{d} \tilde{h_{ip}} = 1 \end{gathered} \tag{5}$$

$$\begin{gathered} \tilde{\lambda_p} \propto \lambda_p \Sigma_j log \frac{W_{ij}}{(H\Lambda H^T)_{ij}} h_{ip} h_{jp} \\ s.t. \sum_{p=1}^{d} \tilde{\lambda_p} = \sum_{ij} W_{ij} \end{gathered} \tag{6}$$

In SVNE, the factorization $H \in \mathcal{R}^{n \times d}$ corresponds to the the single view network embedding where d is the embedding dimension. Because the size of the adjacency matrix representation of the network is quadratic in the number of nodes, matrix-factorization based embedding methods typically do not scale to large networks. Hence, inspired by [15], we make use of more efficient encodings of the network structure: Instead of directly input the adjacent matrix, we use a vectorized representation of adjacency matrix to perform matrix factorization.

B. Multi-view Network Embedding

Given a multi-view network $G = \{G^{(1)}, G^{(2)}, \ldots G^{(k)}\}$, the key idea behind extending SVNE to MVNE is to design the co-regularized objective function that in addition to preserving the information in each view, seeks to maximize the agreement between the views. To accomplish this goal, we propose the following co-regularized objective function in Eq.(7) which is designed to minimizing the cost in each view:

$$\begin{gathered} \sum_{i=1}^{k} \beta_i \mathcal{L}(W^{(i)}, H^{(i)}\Lambda^{(i)}{H^{(i)}}^T) + \alpha \sum_{p,q=1}^{k} ||H^{(p)}\Lambda^{(p)} - H^{(q)}\Lambda^{(q)}||_2 \\ s.t. \sum_{i=1}^{k} \beta_i = 1 \end{gathered} \tag{7}$$

Here, $H^{(i)}$ and $\Lambda^{(i)}$ represents the matrix factorization in view i. α denotes the regularization hyper-parameter. β_i is the parameter which can be used to tune the relative importance of the different views and the role they play in maximizing the agreement between views. If we know that some views are more informative than others, one might want to set the β_i accordingly. In contrast, if we know that some views are likely to be noisy, we might want to deemphasize such views by setting the respective β_i values to be small compared to those of other views. In the absence of any information about the relative importance or reliability of the different views, we set β_i equal to $\frac{|V^{(i)}|}{\sum_{i=1}^{k} |V^{(i)}|}$.

To minimize the cost and maximize the agreement, we constrain the matrix factorization in each view to be the latent matrix factorization H and Λ. This yields the objective function shown in Eq.(8):

$$\mathcal{L}(\sum_{q=1}^{k} \beta_q W^{(q)}, H\Lambda H^T) \tag{8}$$

We find that minimizing the objective function is equivalent to minimizing $(\sum_{i=1}^{k} \beta_i W^{(i)}, H\Lambda H^T)$ (ignoring the constant term that does not impact the solution). We co-regularize the views by choosing $\tilde{W} = \sum_{i=1}^{k} \beta_i W^{(j)}$ to maximize the agreement across views. The corresponding update rules are obtained analogous to the single view case in Eq.(5) and Eq.(6) by replacing W with $\tilde{W}$

Computational Complexity

In a naive implementation of MVNE, each optimization iteration takew $O(d|V|^2)$ where $|V|$ is the total number of nodes and d is dimension of embedding space. For a given choice of d and the number of iterations, the time complexity of naive implementation of MVNE is $O(|V|^2)$. However, in typical applications, G is usually very sparse. In this case the time complexity of one optimization iteration using adjacency list based representation of the adjacency matrices [15] is $O(|V| + |E|)$ (with d assumed to be constant), where $|E|$ denotes the total number of edges across all of the views.

IV. Experimental Results

We report results of experiments designed to address the following questions:

- **Experiment 1:** How does SVNE compare to the state-of-the-art single view network embedding methods?
- **Experiment 2:** How does the MVNE algorithm introduced in this paper compare with the State-of-the-Art multi-view embedding methods?
- **Experiment 3:** Does MVNE embedding provide information that complements information provided by SVNE applied to the target view?

A. Experimental Setup

Data Sets: **Experiment 1** uses three popular single view network datasets:

- BlogCatalog [32]: A social network of the bloggers listed on the BlogCatalog website. The labels represent blogger interests inferred through the metadata provided by the bloggers.
- Protein-Protein Interactions (PPI) [8]: A subnetwork of the PPI network for Homo Sapiens where the node labels correspond to biological functions of the proteins.
- Wikipedia [26]: This is a network of words appearing in the first million bytes of the Wikipedia dump. The labels represent the Part-of-Speech (POS) tags inferred using the Stanford POS-Tagger.

TABLE I
Statical analysis of five datasets

Datasets	#nodes	#edges	#view	#label	#category
BlogCatalog	10,312	333,983	1	39	multi-label
PPI	3,890	76,584	1	50	multi-label
Wikipedia	4,777	184,812	1	40	multi-label
Last.fm	10,197	1,325,367	12	11	multi-view
Flickr	6,163	378,547	5	10	multi-view

Because each node can have multiple labels, the task entails multi-label prediction.

Experiments 2-3 use two multi-view network data, namely, Last.fm and Flickr [6]:

- Last.fm: The Last.fm dataset was collected from the music network[1] with the nodes representing the users and the edges corresponding to different relationships between Last.fm users and other entities. In each view, two users are connected by an edge if they share similar interests in artists, events, etc. [6] yielding 12 views: ArtistView (2118 nodes, 149495 links), EventView (7240 nodes, 177000 links), NeighborView (5320 nodes, 8387 links), ShoutView (7488 nodes, 14486 links), Release-View (4132 nodes, 129167 links), TagView (1024 nodes, 118770 links), TopAlbumView (4122 nodes, 128865 links), TopArtistView (6436 nodes, 124731 links), Top-TagView (1296 nodes, 136104 links), TopTrackView (6164 nodes, 87491 links), TrackView (2680 nodes, 93358 links), and UserView (10197 nodes, 38743 links).
- Flickr: The Flickr data are collected from the photo sharing website[2]. Here, the views correspond to different aspects of Flickr (photos, comments, etc.) and edges denote shared interests between users. For example, in the comment view, there is a link between 2 users if they have both commented on the same set of 5 or more photos. The resulting dataset has five views: CommentView (2358 nodes, 13789 links), FavoriteView (2724 nodes, 30757 links), PhotoView (4061 nodes, 91329 links), TagView (1341 nodes, 154620 links), and UserView (6163 nodes, 88052 links).

Some basic statistics about the datasets described above are summarized in Table I. The results of our analyses of Last.fm and Flickr data suggest that their node degree distributions obey the power law, a desirable property, for the application of skip-gram based models [29].

Parameter Tuning: SVNE (and MVNE) are compared with other single view methods (and their multi-view extensions) using the code provided by the authors of the respective methods (with the relevant parameters set or tuned as specified in the respective papers). We explored several different settings for d, the dimension of the embedding space (64, 128, 256, 512) for all the methods. We used grid search over

[1]https://www.last.fm/
[2]https://www.flickr.com/

$\gamma \in \{40, 80\}$ for Deepwalk and $p, q \in \{0.25, 0.50, 1, 2, 4\}$ for node2vec.

Performance Evaluation: In experiments 1-2, we measure the performance on the node label prediction task using different fractions of the available data (10% to 90% in increments of 10%) for training and the remaining for testing the predictors.

In experiment 3, we use 50% of the nodes in each view for training and the rest for testing. We repeat this procedure 10 times, and report the performance (as measured by Micro F1 and Macro F1) averaged across the 10 runs.

In each case, the embeddings are evaluated with respect to the performance of a standard one-versus-rest L2-regularized sparse logistic regression classifiers [14] trained to perform node label prediction.

B. Exp. 1: Single view methods compared

Experiment compares SVNE with three state-of-the-art single view embedding methods on three standard single view benchmark datasets mentioned above (Note that MVNE applied to a single view dataset yields a single view embedding):

- **Deepwalk** which constructs a network embedding such that two nodes are close in the embedding if the short random walks originating in the nodes are similar (i.e., generated by similar language models) [29].
- **LINE** which constructs a network embedding such that two nodes are close in the embedding space if their first and second order network neighborhoods are similar [37].
- **Node2Vec** which constructs a network embedding that maximizes the likelihood of preserving network neighborhoods of nodes using a biased random walk procedure to efficiently explores diverse neighborhoods [16].

Results: The results of comparison of SVNE with Deepwalk, LINE, and Node2Vec are shown in Figure 1. In the case of LINE, we report results for LINE(1st+2nd) (which uses 1st and 2nd order neighborhoods), in our experiments, the best performing of the 3 variants of LINE, with $d = 256$. In the case of Deepwalk, we report the best results obtained with $\gamma = 40$, $w = 10$, $t = 40$ and $d = 128$. For node2vec, we report the best results obtained with $p, q = 1$. For SVNE, we report the results with optimal d, which was found to be 128 for Blogcatalog, PPI and Wikipedia. The results summarized in Figure 1 show that on Blogcatalog data, SVNE consistently outperforms Node2vec and LINE and is competitive with Deepwalk. On PPI data, SVNE outperforms all other methods in terms of Micro-F1 score and in terms of Macro-F1 when more than 50% of the nodes are labeled. On wikipedia data, SVNE performs better than LINE(1st+2nd) and Deepwalk methods and is competitive with Node2vec.

C. Exp. 2: MVNE Compared with the State-of-the-Art Multi-View Methods

We first compare MVNE with traditional network embeddings methods such as Deepwalk, LINE and node2vec on two multi-view datasets Last.fm and Flickr. Since the methods are designed to work with single view networks, we combine multiple views to obtain an integrated view such that each pair of nodes is linked by an edge in the integrated view if the corresponding pair is linked by an edge in at least one of the constituent views.

We next compare MVNE with three other baseline multi-view learning methods:

- **Co-RegSC** which constructs a representation of the multi-view network using co-regularized eigenvectors of the graph Laplacians of each view [18]
- **MultiNMF** which constructs a latent representation of the multi-view network where in the common subspace is obtained by regularized joint matrix factorization of each of the views [21]
- **MVWE** which constructs a multi-view network embedding by combining the single view embeddings using a weighted voting scheme [31]

Similar to the previous works [31], in our experiments, we use the centroid eigenvectors produced by Co-RegSC and consensus matrix produced by MultiNMF respectively as the multi-view network embedding. We explored several different settings for d, the dimension of the embedding space (64, 128, 256) for the three baseline methods.

Results: The results of comparison of MVNE with other methods are shown in Tables II and III. MVNE consistently, and often substantially, outperforms both (i) the state-of-the-art single view methods on the integrated view and (ii) Co-RegSC, MultiNMF, MVWE.

We observe that the performance of MVWE deteriorates as the views become increasingly incomplete (i.e., large fractions of the nodes appear in only small subsets of the views). In contrast, MVNE copes with incomplete views through co-regularization of nodes that are missing in each of the views.

D. Exp. 3: MVNE compared with SVNE on Node Labeling in a Single Target View

Experiment 3 investigates whether MVNE outperforms SVNE on node label prediction on any single target view by leveraging information from the all of the views. Considering each view of the Last.fm and Flickr data as the target view, we compare the node labeling performance using embeddings obtained using SVNE applied to the target view alone with MVNE that integrates information from all of the views.

Results: Because of space constraints, we show only the results of comparison of MVNE with SVNE when each of the 5 views of the Flickr dataset and each of the 6 views (1 with the most nodes (Userview), one with the most edges (Event), two with most edges per node (TagView, TopTagView), and two with the fewest edges per node(NeighborView, ShoutView)) selected from the 12 views of the Last.fm dataset are designated as the target view. The results summarized in Figure 2 show that MVNE consistently outperforms SVNE on each target view. We conclude that even when the goal is to predict the labels of nodes in a single target view, MVNE is able to leverage information from all of the views to outperform SVNE applied only to the target view, by 10% points or better. Similar results were observed with MVNE relative to SVNE

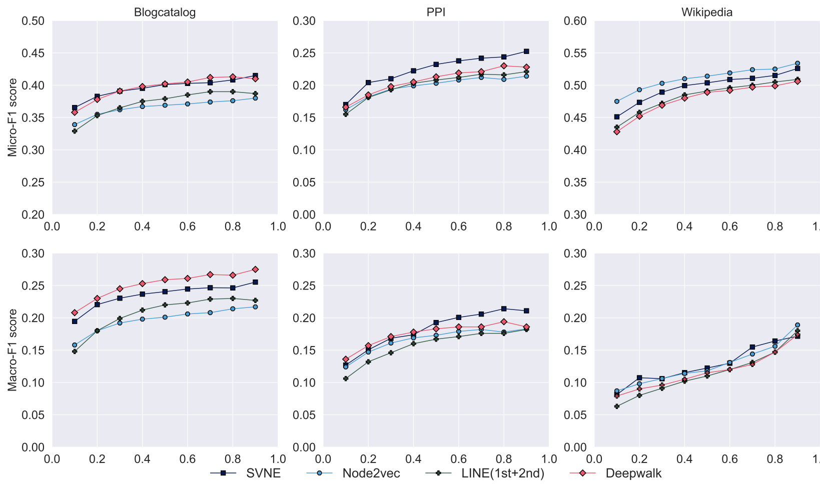
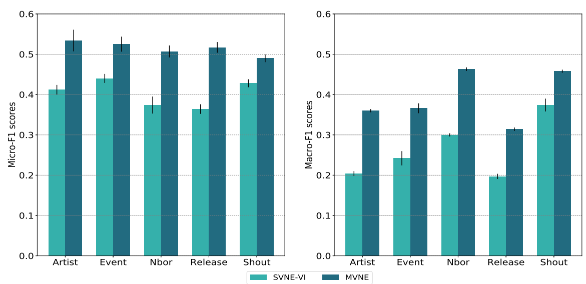
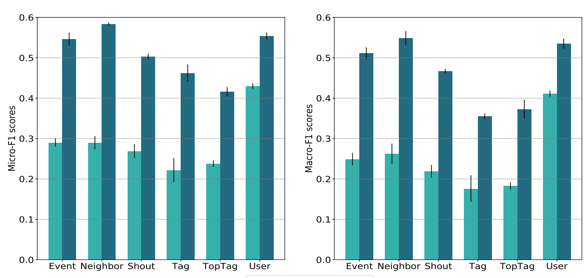


Fig. 1. **SVNE compared with Deepwalk, LINE, and Node2Vec on Single View Data**. The fraction of labeled data are plotted along the x-axis. The Micro-F1 (Top) and Macro F1 (Bottom) scores are along the y-axis.



(a) Flickr



(b) Last.fm

Fig. 2. MVNE compared with SVNE on Flickr dataset (a) and selected six views on the Last.fm dataset (b). The view names are shown along the x-axis and Micro-F1 (Left) and Macro-F1 (Right) scores are plotted on the y-axis

when the rest of the views of last.fm data (results not shown). Furthermore, similar trends were observed for all the multi-view embedding methods considered in the paper relative to their single view counterparts (results not shown).

V. Summary and Discussion

We have introduced MVNE, a novel Multi-View Network Embedding (MVNE) algorithm for constructing low-dimensional embeddings of multi-view networks. MVNE uses a novel objective function that maximizes the agreement between views based on both the local and global structure of the underlying multi-view network. We have shown that (i) SVNE, the single view version of MVNE, is competitive with or superior to the state-of-the-art single view network embedding methods when the embeddings are used for labeling unlabeled nodes in the networks; (ii) MVNE substantially outperforms single view methods on integrated view, as well as state-of-the-art multi-view graph methods for aggregating information from multiple views, when the embeddings are used for labeling nodes in multi-view networks; and (iii) MVNE outperforms SVNE, when used to predict node labels in any target view, suggesting that it is able to effectively integrate from all of the views, information that is useful for labeling nodes in the target view.

A. *Related work*

There is a growing body of recent works on multi-view learning algorithms, e.g., [21], [25], [39], that attempt to integrate information across the multiple views to optimize the predictive performance of the classifier (see [36], [40]). Some multi-view learning methods seek to maximize the agreement between views using regularization [18], [35] where as others seek to optimally select subsets of features from different views for each prediction task [21], [23] However, these methods were not designed for network embedding. Most of the existing multi-view learning algorithms are either not directly applicable to multi-view networks or are not designed to cope with high degrees of data sparsity, a key challenge in modeling real-world multi-view networks.

Network embedding methods aim to produce information preserving low-dimensional embeddings of nodes in large networks. State-of-the-art network embedding methods include Deepwalk [29], LINE [37] and node2vec [16] are limited to single view networks, i.e, networks with a single type of links.

TABLE II
PERFORMANCE EVALUATION ON FLICKR NETWORK. BOLD VALUE INDICATES THE BEST METHODS BASED ON PAIRED T-TEST ($p < 0.01$)

Metric	Algorithm	10%	20%	30%	40%	50%	60%	70%	80%	90%
Micro-F1	MVNE	**0.490**	**0.508**	**0.517**	**0.510**	**0.514**	**0.517**	**0.530**	**0.537**	**0.536**
	Deepwalk	0.236	0.260	0.271	0.280	0.287	0.291	0.298	0.306	0.308
	LINE(1st+2nd)	0.467	0.482	0.492	0.497	0.505	0.507	0.507	0.514	0.518
	node2vec	0.467	0.477	0.483	0.488	0.487	0.488	0.488	0.485	0.484
	MVWE	0.449	0.480	0.492	0.499	0.506	0.507	0.513	0.513	0.512
	MultiNMF	0.178	0.181	0.182	0.186	0.185	0.185	0.184	0.183	0.176
	Co-RegSC	0.153	0.160	0.160	0.160	0.160	0.159	0.159	0.158	0.157
Macro-F1	MVNE	**0.450**	**0.475**	**0.484**	**0.491**	**0.495**	**0.501**	**0.503**	**0.510**	**0.507**
	Deepwalk	0.212	0.232	0.242	0.250	0.259	0.261	0.262	0.268	0.270
	LINE(1st+2nd)	0.425	0.447	0.458	0.463	0.471	0.474	0.474	0.479	0.483
	node2vec	0.398	0.421	0.430	0.440	0.438	0.440	0.441	0.438	0.437
	MVWE	0.376	0.433	0.450	0.460	0.468	0.471	0.475	0.478	0.479
	MultiNMF	0.075	0.078	0.079	0.082	0.082	0.083	0.083	0.082	0.079
	Co-RegSC	0.033	0.029	0.029	0.029	0.029	0.029	0.029	0.029	0.028

TABLE III
PERFORMANCE EVALUATION ON LAST.FM NETWORK. BOLD VALUE INDICATES THE BEST METHODS BASED ON PAIRED T-TEST ($p < 0.01$)

Metric	Algorithm	10%	20%	30%	40%	50%	60%	70%	80%	90%
Micro-F1	MVNE	**0.565**	**0.582**	**0.588**	**0.593**	**0.596**	**0.598**	**0.604**	**0.602**	**0.602**
	Deepwalk	0.260	0.298	0.318	0.330	0.340	0.347	0.351	0.354	0.355
	LINE(1st+2nd)	0.534	0.552	0.559	0.563	0.569	0.573	0.576	0.578	0.584
	node2vec	0.511	0.520	0.525	0.530	0.534	0.534	0.536	0.533	0.536
	MVWE	0.542	0.563	0.570	0.573	0.576	0.576	0.578	0.577	0.574
	MultiNMF	0.218	0.221	0.224	0.225	0.226	0.225	0.226	0.226	0.222
	Co-RegSC	0.131	0.143	0.146	0.148	0.148	0.15	0.149	0.150	0.145
Macro-F1	MVNE	**0.561**	**0.579**	**0.586**	**0.590**	**0.593**	**0.596**	**0.596**	**0.597**	**0.594**
	Deepwalk	0.244	0.279	0.296	0.310	0.317	0.325	0.330	0.332	0.329
	LINE(1st+2nd)	0.515	0.531	0.539	0.543	0.543	0.549	0.552	0.556	0.562
	node2vec	0.475	0.485	0.491	0.499	0.503	0.503	0.505	0.501	0.503
	MVWE	0.499	0.529	0.540	0.545	0.549	0.551	0.552	0.550	0.549
	MultiNMF	0.150	0.153	0.153	0.153	0.157	0.155	0.156	0.156	0.154
	Co-RegSC	0.057	0.063	0.066	0.068	0.069	0.071	0.07	0.071	0.07

However, most real-world networks are comprised of multiple types of nodes and links [4], [12], [37] wherein each type of link induces a view. Hence, there is a growing interest in network embedding methods for multi-view networks [2], [6], [17], [33]. Some multi-view network embedding methods use canonical correlation analysis (CCA) [1], [3], [38] to integrate information from multiple views. Others construct multi-view embeddings by integrating embeddings obtained from the individual views. Examples include MVWE [31] which uses a weighted voting mechanism to combine information from multiple views; MVE2vec [34] which attempts to balance the preservation of unique information provided by specific views against information that is shared by multiple views; and DMNE [28] which uses a co-regularized cost function to combine information from different views. MVWE, MVE2vec, and DMNE use deep neural network models at their core. Specifically, MVWE and MVE2vec are based on a skip-gram model and DMNE is based on an AutoEncoder.

In contrast to the existing multi-view network embedding methods, MVNE exploits a recently discovered connection between network adjacency matrix factorization and network embedding [30] to utilize GFC [41], a graph factorization method, to perform single view network embedding. MVNE extends the resulting single view network embedding algorithm to the multi-view setting. Inspired by [19], MVNE uses a novel objective function that maximizes the agreement between views while combining information derived from the local as well as the global structure of the underlying multi-view networks. Like DMNE [28], MVNE uses a co-regularized objective function to maximize the agreement in the embedding space and to control the embedding dimension. Unlike DMNE which requires on computationally expensive training of a deep neural network, MVNE is considerably more efficient and hence scalable to large networks.

B. Future Directions

Work in progress is aimed at extending MVNE (i) to cope with dynamic update of graphs e.g., using asynchronous stochastic gradient descent (SGD) to update the latent space with the only newly added or deleted edges or nodes; and (ii) work with multi-modal networks that include richly structured digital objects (text, images, videos, etc).

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