Relay for Data: An Underwater Race

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***Abstract*—We show that unlike radio frequency channels, amplify-and-forward relaying can increase the data rate in non-fading underwater acoustic channels. For frequency-selective Rayleigh fading underwater acoustic channels, we focus on QPSK modulation and show that the data rate can be increased by using either decode-and-forward or amplify-and-forward relaying, with the former having a significantly higher performance. This fact indicates that rather than increase the transmit power to increase the data rate, a more power-efficient method involves increasing the number of relays and decreasing the path length between them. We also investigate power allocation methods and analyze their effect on the achievable rate. We show that distributing the power uniformly across a frequency band can match the performance of the rate-maximizing power allocation for QPSK modulation.**

I. Introduction

Long distance communication over cables relies on periodic placement of amplifying stations along the line to counteract the channel loss that increases exponentially with distance.[1] In line-of-sight wireless radio links, where free-space path loss grows with the square of distance, relaying does not provide an equal gain. Relaying in wireless radio channel is thus usually employed only in cases where there is no line-of-sight between two locations, e.g. on satellite relay links. Multi-hop relaying [1] and cooperative relaying [2] can increase the channel capacity of wireless radio channels, but this gain is not comparable to that of wired channels.

Underwater acoustic channels are not similar to either of these cases. While the acoustic energy spreads with a path loss exponent that is usually between 1 and 2 depending upon the system geometry, it also has an exponential loss associated with the absorption of sound by water. The absorption increases with frequency, and becomes a very significant part of the overall loss at higher frequencies.

[1]The exponential loss factor of cables increases with frequency.

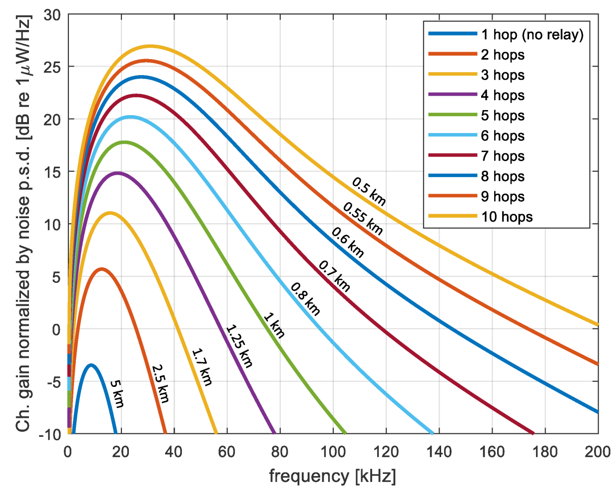

Fig. 1: Channel gain divided by the noise power spectrum density. The transmit power is in dB re 1 μW/Hz. Each curve shows the channel gain for one hop given that a certain number of relays is used to cover a distance of 5 km.

The relationship between the channel gain, carrier frequency, and distance for underwater acoustic channels is summarized in [3], where it is shown that the favorable frequency band depends on the link distance. Fig. 1 shows the channel gain to noise ratio as specified in [3].[2] This result was later used in [4] to show the benefit of relaying in underwater acoustic channels.

A comparison of relaying over underwater acoustic channels and line-of-sight wireless radio channels is shown in Fig. 2. In this figure we consider two types of relaying: amplify-and-forward, and decode-and-forward. In the amplify-and-forward case, the noise accumulates over the relays, whereas in the decode-and-forward case, the capacity for all

[2]These curves are generated with parameters w (shipping activity) and k (spreading factor, or path loss exponent) set to 0.5 and 2, respectively, which are associated with mild shipping activity and spherical spreading.

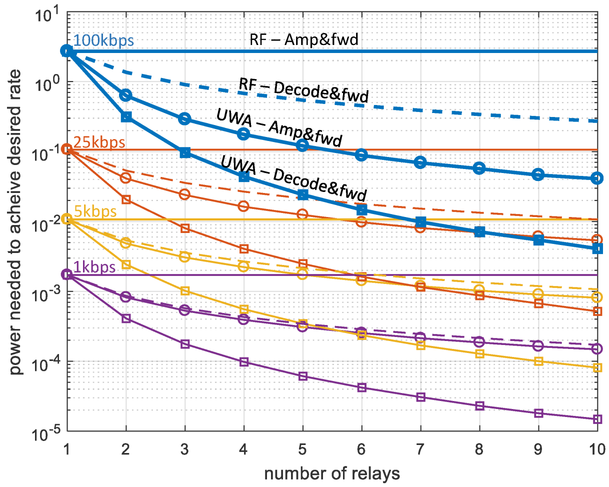

Fig. 2: Power required to achieve a certain data rate over an underwater acoustic channel at a distance of 5 km. An equivalent wireless radio channel is assumed, where same amount of power is needed to achieve the same data rate.

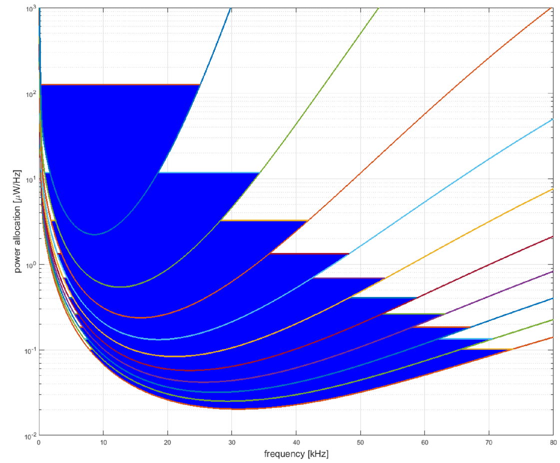

Fig. 3: Water-filling for underwater acoustic channels with relaying. This figure shows how water-filling designates power for relayed underwater acoustic channels spanning 5 km.

hops is the same and can be presented by the overall link capacity of one hop. In this figure relaying is used to conserve overall transmit power rather than increase the data rate. The simplified underwater acoustic channel is assumed to have only one path, with frequency selective gain as shown in Fig. 1. Water-filling is applied for distributing the power as shown in Fig. 3, which shows how power is conserved by relaying in underwater acoustic channels.

Note that a line-of-sight wireless radio channel does not benefit from amplify-and-forward relaying as the division of transmit power across relays and accumulation of noise compensates for lower channel loss over the shorter hops. If decode-and-forward relays are considered, however, the noise will not accumulate, and therefore the total power required to achieve the same data rate will decrease with the number of relays. In contrast, acoustic relaying returns much more significant power savings even when amplify-and-forward relaying is employed. This effect is mainly due to the frequency-dependent path-loss of the underwater acoustic channel as shown in Fig. 1. When relaying is applied, the usable range of frequencies increases, as shown in the water-filling pattern of Fig. 3, where the power is distributed over a wider frequency band when relaying is in use, and the required transmit power decreases significantly.

While the above analysis is insightful, it does not consider the effect of multipath fading on the capacity, and neither does it specify the constellation to be used. Given that fading is inevitable in most underwater acoustic channels, in what follows, we use a different channel model which includes the effect of multipath fading on the capacity. We assume QPSK modulation throughout this paper.[3]

To model the fading, we assume an orthogonal frequency division multiplexing (OFDM) communication system where the channel for each carrier experiences Rayleigh block fading. We focus on the average achievable rate as a performance measure. Note that frequency-selectivity of the transducer and effect of ICI are not included in this analysis. Analyzing the effect of ICI on channel capacity is highly dependent on the ICI-compensation techniques in use, and is omited in this analysis.

The rest of the paper is organized as follows. In Sec. II we specify the channel model and evaluate the average achievable rate. The results are summarized in Sec. III. Sec. IV contains the concluding remarks.

II. System Model

We model the received signal as

$$Y_k = \sqrt{E_k} H_k X_k + z_k \tag{1}$$

[3]Extension to larger constellations is straightfowrad.

where X_k is the QPSK symbol transmitted on the k-th carrier, H_k is the channel coefficient, E_k is the energy allocated to carrier k, and z_k is the symmetric complex Gaussian noise. The noise is colored with the spectral density specified in [3], and we assume that each carrier experiences Rayleigh fading.[4] The channel is characterized by the ratio

$$A_n(f_k) = \frac{\mathrm{E}\{|H_k|^2\}}{\mathrm{E}\{|z_k|^2\}} \tag{2}$$

where $f_k = k\Delta f$, Δf is the carrier spacing, and $A_n(f)$ is shown in Fig. 1 in dB re μW/Hz.[5] Assuming that the transmitted symbol X_k has unit energy, the SNR for each carrier can be defined as

$$\mathrm{SNR}_k = E_k A_n(f_k) \tag{3}$$

We assume that the receiver knows the channel coefficients H_k, while the transmitter only knows the ratio $A_n(f_k)$ and allocates the power E_k based on this statistical information. The latter assumption is due to the fact that the feedback delay will render instantaneous channel information outdated at the transmitter (see e.g. [6])

The average BER corresponding to the k-th carrier over one hop is given in Appendix C of [7] as

$$P_{b_k,\mathrm{hop}} = \frac{1}{2}\left(1 - \frac{\mu_k}{\sqrt{2-\mu_k^2}}\right) \tag{4}$$

where μ_k is the correlation between Y_k and X_k,

$$\mu_k = \frac{1}{\sqrt{1+(E_k A_n(f_k))^{-1}}} \tag{5}$$

Here, we assume two types of relaying: regenerate-and-forward, and decode-and-forward. In the first type of relaying, QPSK symbols are detected and the OFDM symbol is regenerated. In this mode, the bit errors propagate and accumulate over the hops. In the second type, each hop uses error-correction decoding to correct the bits before regenerating the OFDM block. Assuming that the hops are independently fading, the average bit error rate after the last hop for the regenerating relay type is

$$P_{b_k,\mathrm{RF}} = \sum_{n=1, n\ \mathrm{odd}}^{N} \binom{N}{n} P_{b_k,hop}{}^{n} (1 - P_{b_k,hop})^{N-n} \tag{6}$$

The average achievable rate for the regenerating relay is dictated by this BER, while the rate of the decode-and-forward relay is dominated by single-hop BER.[6] If hard detection is used, the average achievable rate for the two relaying methods will be[7]

$$\begin{aligned}\bar{R}_{k,RF} =& 2\Delta f(1 + P_{b_k,\mathrm{RF}} \log_2 P_{b_k,\mathrm{RF}} \\ &+ (1 - P_{b_k,\mathrm{RF}}) \log_2 (1 - P_{b_k,\mathrm{RF}})) \quad (7)\\ \bar{R}_{k,DF} =& 2\Delta f(1 + P_{b_k,\mathrm{hop}} \log_2 P_{b_k,\mathrm{hop}} \\ &+ (1 - P_{b_k,\mathrm{hop}}) \log_2 (1 - P_{b_k,\mathrm{hop}})) \quad (8)\end{aligned}$$

In this analysis we ignore the overhead introduced by the cyclic prefix as each method would have the same data rate percentage decrease.

Finally, the overall achievable rate is

$$\begin{aligned}\bar{R}_{\mathrm{RF}} &= \sum_k \bar{R}_{k,\mathrm{RF}} \\ \bar{R}_{\mathrm{DF}} &= \sum_k \bar{R}_{k,\mathrm{DF}}\end{aligned} \tag{9}$$

III. Numerical Results

Fig. 4 shows the power allocation methods considered in this paper. The first method uses numerical optimization to maximize the throughput (9). The second method distributes power uniformly over a frequency band, where the frequency band selection is numerically optimized. The last two methods are water-filling, and uniform power allocation across the same frequency band as the one used for water-filling. The inverse of the average channel gain $1/\sqrt{A_n(f)}$ is also shown in this figure as a dashed line. This result shows that when Rayleigh fading and QPSK modulation are considered, water-filling is far from optimal. This is because water-filling invests in the more favorable good part of the spectrum. However, QPSK modulation saturates

[4]Note that the assumption of Rayleigh fading for each carrier is justified in light of acoustic multipath.

[5]Conversion between transmitted acoustic power spectrum in dB re μW/Hz (or dB re W/Hz) and acoustic pressure dB re μPa/$\sqrt{\mathrm{H}z}$ in underwater acoustic channels follows from the fact that 1 W acoustic power produces a pressure of about 170.9 dB re 1 μPa at distance of 1 m from the transmitter.

[6]Here, we assume that decoding is successful for all decode-and-forward relays.

[7]Note that this rate is based on average bit error rate. If the transmitter knows the specific channel realization, the average achievable rate will be the expected value of rate achievable averaged over all possible channel states. However, channel knowledge at the transmitter is impractical for underwater acoustic channels.

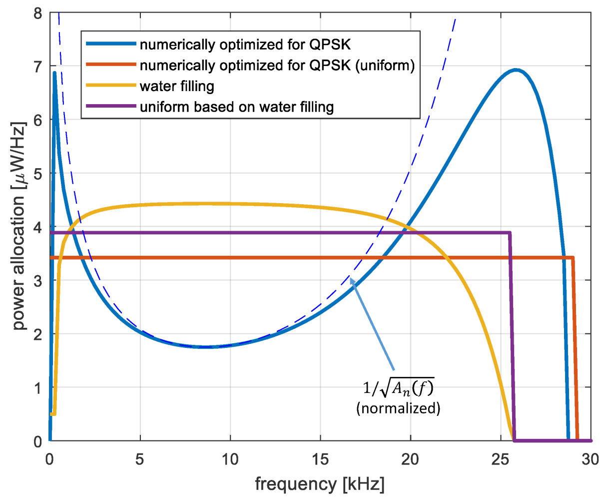

Fig. 4: Comparison of various power allocation techniques. This result is for a distance of 5 km and transmit power of 0.1 W.

at 2 bps/Hz, and therefore, it is advantageous to explore the less favorable parts of the spectrum as well. Thus, the optimal power allocation assumes the form of inverse square root water-filling. Note that in the absence of the guard interval, power spectrum allocated to the frequency bin f_k in W/Hz is the same as E_k in joules.

Figs. 5 and 6 show how the optimal power allocation changes with relaying. For decode-and-forward relaying, the bandwidth grows significantly with relaying although the transmit power per hops is reduced to keep the overall transmit power constant. Fig. 6 shows the same concept for regenerate-and-forward relaying, where the bandwidth growth is not as significant because the higher SNR for each hop controls error accumulation over the hops.

The next question to be answered is how do power allocation and relaying effect the performance in terms of achievable rate. Fig. 7 shows the achievable rate where the total transmit power is kept constant. The results for 0.01 W transmit power show that the average achievable rate grows almost linearly with the number of relays when decode-and-forward relays are in use. Regenerate-and-forward displays a smaller rate due to bit error accumulation over each hop. Furthermore, these curves demonstrate that there is a measurable performance gap between the numerically optimized power allocation and water-filling. Specifically, uniform power allocation outperforms water-filling. Finally, this figure shows that the numerically optimized uniform power allocation can match the optimal allocation. Thus, throughout the rest of the results, the performance will be shown for a uniform power allocation with numerically optimized

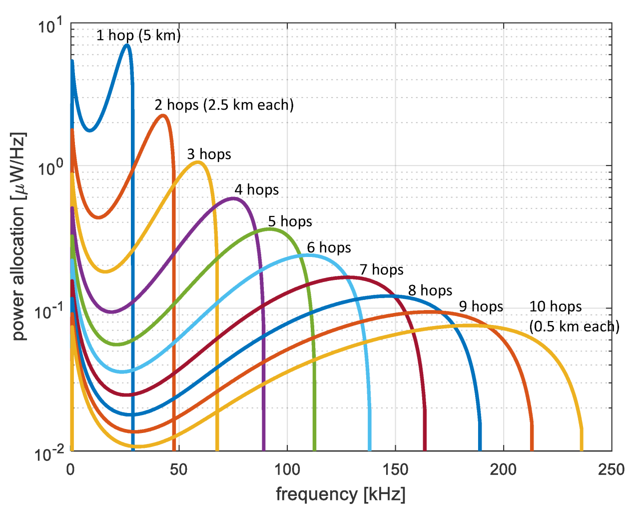

Fig. 5: Power allocation optimized for QPSK modulation and decode-and-forward relaying. The link distance is 5 km, total transmit power is 0.1 W, and 1 to 10 hops are used. The power allocation is the same for all hops.

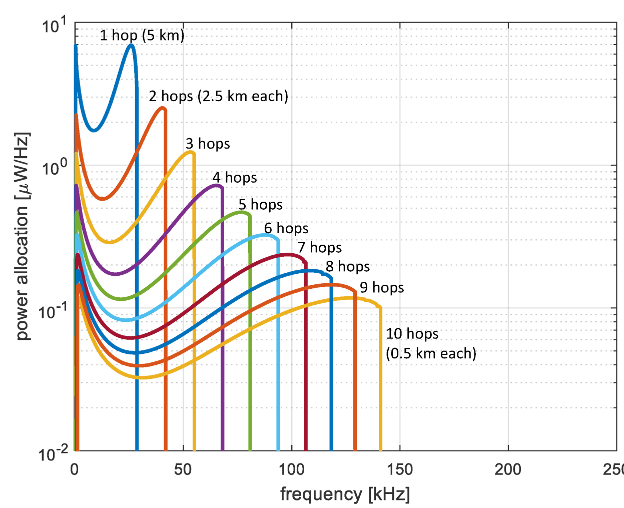

Fig. 6: Power allocation optimized for QPSK modulation and regenerate-and-forward relay. The link distance is 5 km, total transmit power is 0.1 W, and 1 to 10 hops are used. The power allocation is the same for all hops.

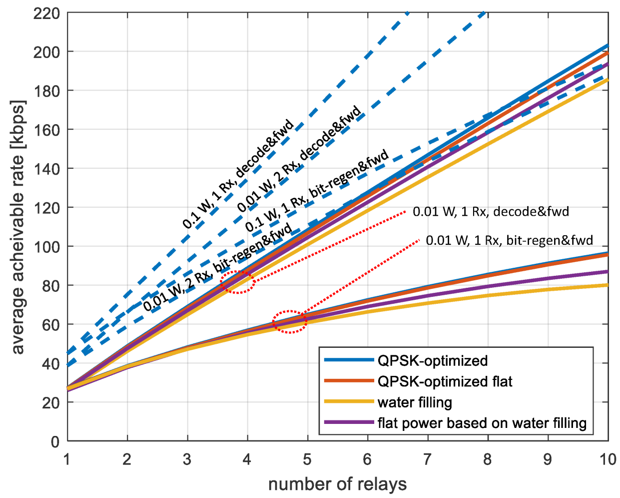

Fig. 7: Achievable rate as a function of the number of relays over a 5km acoustic channel. All of the discussed power allocation techniques are included for transmit power of 0.01 W. This result shows that optimized uniform power allocation matches the performance of optimized power allocation. The achievable rate is also shown for transmit power of 0.1 W, and for relays with array receivers. For these cases, optimized uniform power allocation is used, and the results are shown in dashed blue lines.

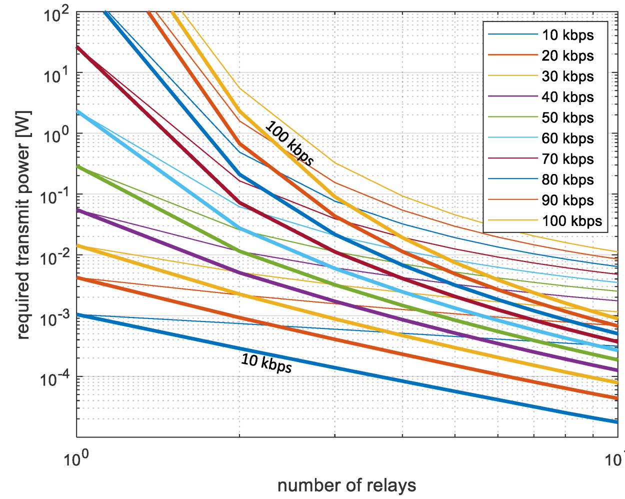

Fig. 8: Power saving of relaying when a certain data rate is desired. Bold curves show regenerate-and-forward, whereas thin curves show decode-and-forward relaying. Note that when high data rates are desired, the best solution is not to increase the power. If 70 kbps is desired over a distance of 5 km, using 2 regenerate-and-forward relays will reduce the required power 100 times, or 10 relays with decode-and-forward will reduce the total transmit power about 10^5 times.

frequency band.[8]

Fig. 7 also shows that the solution to higher data rates is not in increasing the transmit power. If the transmit power is increased ten times, the rate will increase less than twice. This performance is matched by a two-hop decode-and-forward relay link. When all receivers are equipped with two elements, the achievable rate increases by about 50% as compared to single receiver element receivers.[9]

Finally in Fig. 8 we investigate power savings through relaying. Each curve shows how much power can be saved by relaying if the data rate is kept constant. The power saving obtained by employing 9 relays (10 hops) is shown to reduce the required transmit power by about 20 dB for 10 kbps data rate. Power savings increase with rate, reaching 50 dB at 70 kbps. The power saving for bit-regenerate relays is not as significant, but the power saving becomes significant as the desired data rate increases. This is another confirmation of the fact that the solution for higher data rates is not transmitting large amount of power, but is rather investing that power wisely, such as with relaying and receiver arrays.

IV. Conclusion

We investigated the data rates achievable using QPSK modulation over Rayleigh fading underwater acoustic channels with or without relaying. The results showed that although bandwidth efficiency saturates at 2 bps/Hz, relaying can still unlock high data rates through increasing the bandwidth, and can save power due to the exponential decay of acoustic signal at higher frequencies. We considered two types of relaying: regenerate-and-forward, and decode-and-forward. The results demonstrated that utilizing error correction codes at each relay improves the overall performance significantly, making decode-and-forward relaying the desired choice.

[8]Details of the numerical optimization of the power allocation is beyond the scope of this paper. It is noteworthy, however, that optimization of a uniform power allocation (finding the optimum frequency band) is on the order of magnitude less complex when compared to finding the optimal power allocation.

[9]The receiver elements are assumed to be sufficiently spaced such that they experience independent fading. For such receivers, the BER after maximum ratio combining is given in appendix C of [7] as $P_b = \frac{1}{2}\left(1-\rho\sum_{m=0}^{M-1}\binom{2m}{m}\left(\frac{1-\rho^2}{4}\right)^m\right)$ where M is the number of receiver elements.

The results also suggested that when the constellation is pre-selected, water-filling is ineffective, and the rate-maximizing power allocation is rather more similar to inverse water-filling as suggested in chapter 9 of Ref. [8] for fading wireless radio channels. Specifically, the bandwidth that should be utilized is wider than that suggested by water-filling.

Another conclusion was that employing a two element receivers can almost match the performance of a link with 10 times the transmit power. Therefore, the results encourage application of array receivers as well as relaying rather than focusing on increased transmit power. Specifically, when the constellation size is pre-selected, increasing the transmit power, unlike relaying and spacial diversity, will have little effect on the data rate.

There were three issues that were not addressed in this article. The first issue is the limitation of the constellation size. Here, we limited the constellation to QPSK only, while larger size constellation may play in favor of increased transmit power as the bandwidth efficiency will not saturate at 2 bps/Hz. However, we ignored the challenges of channel estimation and distortions introduced by the channel (e.g. Doppler effects), which often counter the benefits of higher order constellations. In future research we will address channel estimation issues and higher constellation sizes.

The second issue is the accumulation of delay over the relays as each relay has to wait for a sufficiently large payload and then relay the information. One possible solution to this problem is dividing the bandwidth into two sections, where odd and even relays use a different frequency band. In such relaying method, each relay can transmit the data forward while receiving the next data packet/frame. Given the performance improvements through relaying, dividing the bandwidth into two sections can be compensated by a slight increase in the number of relay stations. Such relaying will be analyzed in future work.

The last issue that remains to be addressed is that the results suggest utilization of a very wide frequency band. But what kind of a transmitter can deliver such bandwidth in underwater acoustic channels? The effective bandwidth of transducers can limit the applicability of such relaying. However, if bandwidth-to-center frequency ratio is considered as the limit, utilization of higher frequency bands will still benefit the data rates achievable over acoustic links. Furthermore, if the frequency band is in part limited by the bandwidth of transducers, the penalty of dividing the frequency band into two sections will be reduced as the limiting factor will be the transducer rather than the channel. In our future research we will include transducer models into the performance analysis.

Acknowledgments

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