Evidence of Partial Number Word Knowledge on the Give-*N* Task

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Abstract

The most common measure of number word development is the give-*N* task. Traditionally, to receive credit for understanding a number, *N*, children must understand that *N* does not apply to other set sizes (e.g., a child who provides three when asked for "three" but also when asked for "four" would not be credited with knowing "three"). We hypothesized that such performance may reveal a transitional knowledge state that marks children who are ready to progress to the next knower level. An analysis of six previous studies ($N = 200$) revealed that two, three, and four knowers flagged as having partial knowledge of N+1 at pretest outperformed those with no such knowledge on the give-*N* task at posttest. Results support the idea of graded representations (Munakata, 2001) in number word development and suggest the traditional approach to coding the give-*N* task may not completely capture children's knowledge.

Keywords: give-*N*; cardinality; counting; partial knowledge

Introduction

Children's early understanding of number words follows a predictable but protracted development (e.g., Carey, 2009; Wynn, 1990). Initially, children fail to see any connection between their number words and the exact set sizes they represent. For example, at this point if you were to ask a child to give you "one" candy from a bowl they would likely just grab a handful for you. However, around two and a half years of age, children will begin to understand that "one" is referring to a set of exactly one item. At this point they are considered "one-knowers." Now, that same child who gave you a handful of the candies a few months before will be able to give you the one candy you requested. However, if you then go on to request "two," they will likely again grab a handful, showing they lack an understanding of exactly two. A few months after this, children will connect "two" to a set of exactly two and become "two-knowers." This developmental progression will continue until around the point at which children understand "four." At this point children begin to understand that counting can be used to determine the size of a set and become cardinality-principle knowers (e.g., Carey, 2009). This development is not trivial, as the age at which children become cardinality principle knowers uniquely predicts mathematics understanding in first grade, even after controlling for important domain-general and domain-specific factors (e.g., IQ, executive functions, preschool mathematics achievement; Geary et al., 2017).

The most common task for assessing the development of children's understanding of cardinality is the give-a-number, or give-*N* task (Wynn, 1990, 1992; see also Davidson, Eng, & Barner, 2012; Le Corre & Carey, 2007; Le Corre, Van de Walle, Brannon, & Carey, 2006; Posid & Cordes, 2015; Sarnecka & Carey, 2008; Sarnecka & Lee, 2009; Sarnecka & Wright, 2013; Shusterman, Slusser, Halberda, Odic, 2016; vanMarle, Chu, Li, & Geary, 2014). In this task, children are asked to provide a subset of items (*N*) from a larger pile as a means of testing whether the child understands *N*. For example, on a "three" trial, the child may be asked to give the experimenter "three" bananas from a pile of 15 bananas. If they are able to correctly provide three items reliably (usually defined as correct on at least 2 out of 3 trials), then they can be given credit for being a "knower" of that number word. However, what about a child who then provides three when asked for four? Are they a knower of three?

In Wynn's (1990) original coding of the give-*N* task, children were given credit for understanding a given number word if they knew both when to correctly give that amount as well as not to provide that amount for another number word. In other words, knowledge of "three" requires not only an understanding of when to provide three items, but also not to provide three items when asked for "four" or "five." For example, a child who gives three reliably when asked for "three" but also gives three when asked for "four" would be categorized as a "two knower." This "strict" coding of children's knower level may not provide the complete picture of children's number word knowledge. Indeed, it is possible that these children are farther along in their development than two knowers who just grab a handful when asked for "three."

The above disconnect stems, in part, from the traditional approach to treating children's number word development as stage-like, with there being an all-or-none understanding of a particular number word. However, this approach does not acknowledge the graded nature of children's developing knowledge (cf. Munakata, 2001). Specifically, early on in the learning process, representations exist (i.e., they are present within the cognitive system), but they do so in a relatively weak state (e.g., Munakata & McClelland, 2003; Garber, Alibali, & Goldin-Meadow, 1998). With more experience, the representations become stronger allowing them to more readily influence behavior (e.g., Alibali & Goldin-Meadow, 1993; Munakata et al., 1997; Siegler, 1976). During the strengthening process, these representations may appear as "partial" knowledge, wherein they are able to guide some kinds of behaviors but not others. Tying this idea back to scoring performance on the give-*N* task, when children are initially learning a number word, their representation of that number may be strong enough to allow them to provide the correct amount, but not strong enough to prevent them from providing that amount when asked for other, closely associated numbers. Relatedly, it may be that the representation for $N + 1$ is strong enough that it becomes activated upon hearing $N + 1$ requested. Then, when $N + 2$ is requested, and the learner, lacking any existing representation for $N + 2$, is influenced by the now latent representation for $N + 1$, leading them to provide $N + 1$ for $N + 2$.

The possibility of partial number word knowledge on the give-*N* task can also be justified based on what we know about how new, ambiguous words are learned. Specifically, word learners do not suddenly develop a complete understanding of a new word (e.g., Apfelbaum & McMurray, 2017; Yurovsky, Fricker, Yu, & Smith, 2014). Consider, for example, participants in a cross-situational learning task. These participants sometimes develop partial knowledge of words before showing an understanding of the new words (e.g., Yurovsky, et al., 2014), and competition between different referents can influence learning (e.g., Apfelbaum & McMurray, 2017). In the cross-situational word learning task, individual participants are shown multiple novel items at the same time while hearing novel labels for the items. On any given trial it is not clear which label applies to which object, requiring the learner to track the statistics of the objects and labels across trials to learn each object-referent pairing. Individuals who try and fail to make the correct mappings in one block of trials are significantly more likely to make the correct mapping on a subsequent block of trials, suggesting that although learners may not display a correct understanding initially, there is something about their knowledge state that accelerates later learning (Yurovsky et al., 2014).

More specific to children's number word learning, there is also evidence that children possess knowledge beyond what is typically displayed on measures of cardinality. For example, children may display an approximate understanding of number words before they have an exact understanding (e.g., Gunderson, Spaepen, Levine, 2015). When it comes to partial knowledge of the give-N task, it has been argued that children's performance follows a pattern wherein they develop non-exact meanings for each number words and then combine this understanding for different words to show an exact understanding (Barner & Bachrach, 2010). That is, children may first develop an understanding that "two" is at least two items and "three" is at least three items. This in turn allows children to rule out "at least three" when asked for two items, resulting in an exact understanding of two. The early, non-exact understanding of number words has been suggested as the reason for why children often provide correct answers when asked for $N + 1$, despite not consistently distinguishing between $N + 1$ and $N + 2$.

Although the theories and evidence we have presented here so far support the idea that children who can provide $N + 1$ when asked for $N + 1$ may have partial knowledge of $N + 1$, it is important to note that they may not. Such performance may simply reflect random error, guessing, or a strategy to give that amount for all unknown numbers (see Barner & Bachrach, 2010 for a discussion of why such an explanation is unlikely for performance within one timepoint). If this alternative is true, then an *N*-knower who provides $N + 1$ when asked for both $N + 1$ and $N + 2$ should be no different in terms of readiness to learn than their fellow *N*-knowers who just grab a bunch when asked for $N + 1$. Researchers who do not give credit for knowing $N + 1$ when a child also gives it for $N + 2$ argue that any guesses should be lower bounded based on children's current knowledge (e.g., Sarnecka & Lee, 2009). In other words, if children have an understanding of what $N + 1$ is, then it would prevent them from providing $N + 1$ as a guess for another number word. In such a case, it may be that a child who provides three for both "three" and "four" is truly just a two-knower like any other two-knower, and so providing three items for three and four is more of a default response akin to "more than two."

The current study investigated the possibility that *N*-knowers who can give $N + 1$ reliably (even if they also give $N + 1$ for higher numbers) are in a transitional or partial knowledge state on their way to becoming "strict" $N + 1$ knowers who are able to correctly provide $N + 1$ only when it is requested. We hypothesized that these children would, therefore, be more likely to grow into strict $N + 1$ knowers than their fellow *N* knowers who do not reliably produce sets of $N + 1$. Although both groups of children would be coded as *N* knowers at pretest under the traditional approach, children who show partial knowledge of the next number word should be more likely to progress to the next knower level by posttest (after an intervention or time delay). We tested this hypothesis by pooling data across several previous studies that included a pretest assessment of children's knower level on the give-*N* task followed by some intervention and/or passage of time and then a posttest assessment of children's knower level on the give-N task.

Table 1. *Characteristics of Each Study.*

Study	Total N	Knower Level	Age M (in months)	Count Disks M (SD) of 20	% Partial $N + 1$	Time between pretest and posttest (in weeks)	N in control condition
1	24	2.08 (1.02)	41.88 (4.42)	5.42 (4.22)	38	4	0
2	44	2.82 (1.08)	50.19 (8.35)	8.14 (5.70)	34	5	16
3	63	2.43 (1.03)	54.57 (5.79)	7.95 (5.31)	33	5	23
4	38	2.11 (.95)	41.18 (3.29)	5.92 (5.43)	39	3	0
5	10	2.10 (.88)	40.51 (3.26)	8.30 (6.57)	10	2	0
6	21	2.24 (1.00)	41.37 (3.33)	7.48 (4.81)	24	4	0

Method

Participants

The current study included data from six previous studies of the development of children's understanding of cardinality to examine if children have knowledge that is not captured by the traditional approach to coding the give-*N* task. All studies examined preschool-aged children's performance on the give-*N* task at two different time points separated by 2-5 weeks. Two of the studies included a control condition where children completed a print awareness intervention. These children were included in the present analyses, but excluding them does not change the pattern of results reported below (p < .05 for partial-N + 1 knowledge). There were 346 participants who completed these studies. Of these children, 39 were non-knowers and 100 were cardinality-principle-knowers (CP-knowers, further described below). To focus on the *development* of children's understanding of cardinality we used the approach of limiting our analyses to subset knowers (i.e., one-to-four knowers; Le Corre, Van de Walle, Brannon, & Carey, 2006). Of these 207 subset knowers, 7 had knower levels that were unable to be accurately determined (see note below in give-*N* section). Thus, the final sample included 200 participants (105 girls, 95 boys; M_{age} = 47.45 months, SD = 8.16).

Design

We conducted an integrative data analysis (IDA) of children's initial and later performance on the give-*N* task. By pooling participants across multiple studies, IDA allows for a more powerful test of the hypotheses (e.g., Curran & Hussong, 2009). Table 1 describes the characteristics of each study.

Measures

Give-*N* (Wynn, 1990). This task is designed to assess how far along children are in their understanding of cardinality. Children were asked to provide sets of between one and six items to a stuffed animal. For each trial, children were given a pile of 15 items. In five of the studies, the items were small rubber fruits that were being used to make a fruit salad. In one of the studies the items were small yellow counting chips. Administration followed a titration method (e.g., Wynn, 1992; Sarnecka & Carey, 2008), where children were first asked for one item. Once they had provided the item(s), the experimenter asked "Is that *one*?" If the child said yes, the trial was ended and the experimenter moved to the next trial. If the child said no, the experimenter reminded the participant of what was needed for that trial (e.g., "Zebra wanted one. Can you give Zebra *one*?"). Similar prompts were repeated after each trial. If the child correctly provided one item they were then asked for two, but if they provided an amount more than one then they were again asked for one item. This pattern continued until children consistently gave the correct amount for one set size (defined as providing the correct amount on at least two out of three trials) as well as all lower set sizes. Children were never told to count, but were allowed to do so spontaneously.

Children's knower levels were coded according to the excel spreadsheet by Negen, Sarnecka, and Lee (2012). This coding system is based off of the different patterns of responding for children at a given knower level. However, it does not allow children to be coded as five-knowers. Instead, it codes pre, one, two, three, four, and CP knowers. To qualify as a CP-knower, children needed to show evidence of understanding both "five" and "six." Though the model allows for some mistakes, as even CP-knowers will occasionally provide incorrect amounts for five or six, but under the traditional titration administration of the give-N task children would need to show reliable understanding of both five and six (at least two out of three correct on each set size). Thus, for our purposes here, we coded CP-knowers as "six-knowers" given that children could be tested on sets from one to six. Under this coding scheme, and the typical titration administration, children would generally be able to provide a "known" number for another set size on one occasion, but if they do so on multiple occasions then they would not be credited with knowing said number. In addition to a knower level, children were coded as partial-N+1 knowers if they provided $N + 1$ for $N + 1$ and for $N + 2$ (e.g., a two-knower who provides three for "three" and "four" would be considered a partial-three knower while a two-knower who provides more than three for "three" would not be considered as having partial knowledge of three). Recall that the Negen et al. coding scheme does not allow for five-knowers. Thus, there was one additional way for four-knowers to be coded as partial-*N*+1 knowers here. Specifically, four-knowers were also coded as partial-*N*+1 knowers if they reliably gave five for five but failed when asked for six. There were 10 participants who fell into this latter group, and the findings do not change if we exclude them. We did exclude seven participants in the sample whose give-*N* performance was noted as difficult to interpret because their ability to reliably give a given number when asked greatly out performed their knower level (i.e., by two or more knower levels). For example, a child who gave one, two, and three correctly but then gave two when asked for "four" should technically be coded as a one knower despite reliably producing sets of both "two" and "three" when asked. We excluded these children from the analyses below, but conclusions did not change when we included them either as partial-*N+1* knowers or as not partial-*N+1* knowers.

Count Disks (Mix et al., 2012). This task is designed to assess children's counting skill. Children are shown 20, one-inch disks placed an inch apart on a foam board. The disks are arranged in a straight line and alternate between blue and green. The experimenter asked for the child's help to count the disks. The highest number the child was able to count to while maintaining a stable-sequence (counting in the correct order) and one-to-one correspondence (counting each disk in order and only once) was coded as their highest count. The task was administered twice in each session for five of the studies, but only once for one of the studies (Study 1). To equate coding across the studies, the first successful attempt that children completed was used for the analyses (i.e., if the child refused to count the disks the first time in the sessions with two attempts, the second attempt was used). There were no refusals in Study 1.

Results

To test whether partial-*N*+1 knowledge predicts posttest knower level, we conducted an ANCOVA with posttest knower level as the dependent variable, partial-*N+1* knowledge (yes or no), pretest knower level, and the interaction as fixed factors. Study was included as a random factor, as was the interaction between study and partial $N + 1$ knowledge, allowing for a test of whether the effects of partial $N + 1$ knowledge differed between studies. Count disks performance at pretest was included as a covariate. There was a significant main effect of partial-*N*+1 knowledge, $F(1, 26.788) = 9.225$, $p = .005$, $\eta_p^2 = .256$, as well as an effect of pretest knower level, $F(3, 181) = 32.545$, $p < .001$, $\eta_p^2 = .350$. The effect of pretest count disks performance was not statistically or practically significant, $F(1, 181) = 3.776$, $p = .054$, $\eta_p^2 = .020$. However, the main effect of partial $N + 1$ knowledge was qualified by an interaction with pretest knower level $F(3, 181) = 3.127$, $p = .027$, $\eta_p^2 = .049$. For the group of one-knowers, there was not an effect of partial $N + 1$ knowledge $F(1, 16.256) = .034$, $p = .857$, $\eta_p^2 = .002$. For the group of two, three, or four knowers there was a significant effect of partial $N + 1$ knowledge, $F(1, 6.941) = 15.554$, $p = .006$, $\eta_p^2 = .691$. Thus, children who are two, three, or four knowers who show partial knowledge for $N + 1$ at pretest show greater performance on the give-*N* measure at posttest than those who do not show such knowledge.

Although knower levels are often analyzed using ANCOVAs or regression models, they may better be conceptualized as ordinal rather than continuous. To ensure that the way the data was analyzed did not influence our interpretations of the effect of partial $N + 1$ knowledge for the two, three, and four knowers we also conducted an ordinal logistic regression with knower level at posttest as the dependent variable, partial $N + 1$ knowledge (yes or no) and study as factors, and count disks performance and knower level at pretest as covariates. Similar to the ANCOVA, partial $N + 1$ knowledge predicted posttest knower level $\hat{\beta} = 1.24$, $Wald(1, N = 154) = 4.06$, OR = 3.47, $p < .001$. See Figure 1 for information about the proportion of children improving by knower level.

Note that age was not included as a covariate in the analyses. Although a child's age is often included as a covariate in analyses of knower level (e.g. Sarnecka & Lee, 2009), we chose not to include age here because of the existence of the pretest measure. When pretest is included as a covariate, the benefits of including additional covariates depend on their ability to predict growth from pretest to posttest. Although age is often associated with knower level within a given time point (e.g., Sarnecka & Carey, 2007), initial analyses of the data showed age was unrelated to growth ($p = .313$), so it was not included in the final model. However, conclusions are the same when age is included as an additional covariate.

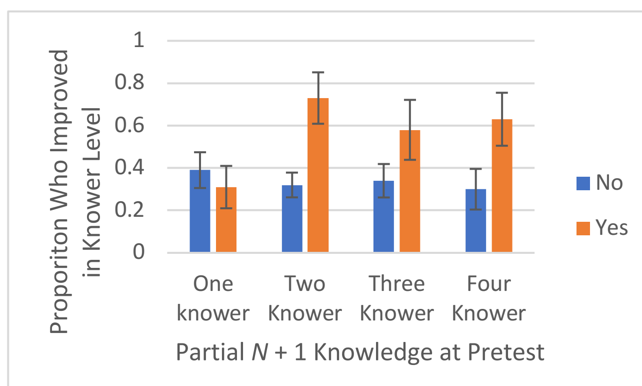

Figure 1. Proportion of children who improved in knower level by pretest knower level and whether they exhibited partial knowledge of $N + 1$ at pretest. Error bars reflect +/- 1 standard error.

Discussion

This study examined the possibility that children may have partial knowledge of number words that is not reflected in how the give-*N* task is traditionally coded. In partial support of our hypothesis, two, three, or four knowers who gave $N + 1$ for both $N + 1$ and $N + 2$ progressed to a higher knower level, on average, when compared to their fellow N knowers who did not reliably give $N + 1$ when asked for $N + 1$. It is important to note here that this pattern of results does not simply indicate that the traditional way of coding of children's number word understanding is incorrect. An adult-like understanding of $N + 1$ clearly necessitates an understanding that the number just provided for $N + 1$ should not also be used for $N + 2$. Thus, simply ignoring the amount that children provide for incorrect answers would leave out important information about how children are interpreting number words. Moreover, the traditional way of coding the give-*N* task has evidence of validity based on its correlation with other measures of cardinality understanding (e.g., Lee & Sarnecka, 2011; Le Corre et al., 2006; Wynn, 1990, 1992) and its ability to predict future mathematics understanding (e.g., Geary et al., 2017). Nonetheless, children who reliably gave $N + 1$ when asked for $N + 1$ did reach a higher knower level at posttest compared to their fellow *N*-knowers who did not reliably give $N + 1$ when asked for $N + 1$. This finding suggests that children's ability to give $N + 1$ when asked for $N + 1$ may be the first step toward progressing to the next knower level.

The lack of partial $N + 1$ knowledge shown for the one-knowers who provided two for "two" and two for "three" may be due to children initially treating number words greater than one as "plural" (e.g., Carey, 2009). That is, early in the learning process when children have learned "one" they are differentiating singular and plural, and simply treat all higher number words as "more than one." However, once children have developed an exact understanding of "one" and "two" this distinction is no longer used, and higher number words can begin to be interpreted not as singular or plural but in reference to specific set sizes.

These results have important implications for how children's knower levels on the give-*N* task are conceptualized. Typically, researchers have treated number word development as something that occurs in an all-or-none, stage-like fashion. However, these results add to recent evidence suggesting that word learning proceeds in a noisier fashion (e.g., Apfelbaum & McMurray, 2017; Barner & Bachrach, 2010; Yurovsky et al., 2014). The results also align with the idea that children's knowledge can exist in transitional or graded states (e.g., Alibali & Goldin-Meadow, 1993; Munakata et al., 1997). Similar to the research in gesture-speech mismatch, entertaining multiple hypotheses or strategies (thinking $N + 1$ applies to $N + 2$) can signal readiness to learn (Alibali & Goldin-Meadow, 1993). The partial knowledge of $N + 1$ may also reflect the strengthening process of children's early numerical representations, showing how early knowledge shapes later behaviors. Although these results suggest number word development is less straightforward than traditional ways of coding the give-*N* task capture, it is unclear how best to measure such development.

The current study suggests that partial knowledge predicts children's growth in performance on the give-*N* task over time, but it remains to be seen just how this partial knowledge manifests itself in a given time point. In other words, outside of providing $N + 1$ when asked for $N + 1$ on the give-*N* task, how else can this partial knowledge be seen, if at all? Other common measures of children's understanding of cardinality require less action on the child's part to respond. For example, on the point-to-*X* task (e.g., Wynn, 1992), similar to give-*N*, children are told the target set size for the trial, but in the point-to-*X* task they simply need to point to one of two possible sides. Such pointing behavior may not require as strong of a representation to display an understanding of a given number as the give-*N* task does, possibly creating another measure that may be sensitive to children's partial number knowledge. Another popular measure, the what's on this card? (WOC; Le Corre et al., 2006) task may not provide the same sensitivity. In the WOC task, children are shown a card and have to come up with the cardinal label on their own, instead of being given the cardinal label and having to identify or create the relevant set. Without the given label in the environment to activate the relevant representation it's possible that children with partial knowledge of $N + 1$ would perform similarly to children with no such knowledge. Future research will be needed to determine whether the partial-*N* knowledge observed in the present study extends to performance on other measures of an understanding of *cardinality* as well as other measures of early mathematics understanding.

It is also important to consider the possibility that the knowledge observed in the present study is more than partial knowledge. Perhaps the traditional way of coding the give-*N* task is simply too strict and not reflective of children's true knowledge. We do not expect that to be the case, given the past evidence of the give-*N* task's validity (mentioned above). However, if this were the case, then partial-$N+1$ knowers should simply be coded as $N + 1$ knowers and they should perform similarly to other $N + 1$ knowers both in terms of their pre-to-post growth in understanding cardinality, and in terms of their performance on other measures of counting and cardinality. Future research can begin to test this possibility by comparing these two groups.

Overall, the current study suggests that children know more about number words than is currently captured by traditional ways of coding the give-*N* task. These findings are consistent with other recent research suggesting that the give-*N* task, on its own, may not provide a complete picture of children's knowledge of the cardinality principle (Baroody, Lai, & Mix, 2017). Further research is needed to determine how exactly different strengths of number knowledge may influence children's development, and how better to design measures to capture individual differences in the development of children's understanding of cardinality. In the meantime, results suggest that researchers may benefit from including multiple measures of children's understanding of cardinality to provide converging evidence of where children are in their development.

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