A Study of Gravitational Wave Memory and Its Detectability With LIGO Using Bayesian Inference

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1 Abstract

Gravitational waves are produced by accelerating masses, but in most cases they are too weak to detect. In 2015 LIGO announced its first gravitational wave detection which was produced by the merging of two black holes 1.3 billions years ago. The detectable component of gravitational waves, known as the oscillatory waveform, is predicted to have a smaller, lower frequency counterpart called the memory: a permanent warping of spacetime. In addition to memorys small amplitude compared to the oscillatory waveform, low frequency noise sources on earth make it difficult for ground based detectors to reach the SNR (signal to noise ratio) needed to detect this component. While memory is likely not currently detectable due to LIGO limitations, it is of interest to characterize future detector sensitivities to know where and when to look for this phenomenon. Here we implement Bayesian parameter estimation to calculate the likelihood of a simulated set of LIGO data with a template, both of which include memory. Next we explore binary systems of varying masses and distances along with the noise curves of various observatories in order to establish the SNR needed to detect gravitational wave memory. Our final goal is to find a ballpark SNR value for when memory will be detectable.

2 Introduction: LIGO, Gravitational Waves, and Memory

In 1916 Albert Einstein proposed a theory that unifies gravity, spacetime, and energy which he called the general theory of relativity. In his theory he predicted that massive, accelerating objects would emit waves that physically distort spacetime; these objects were given the name gravitational waves. In hopes of directly testing for the existence of gravitational waves a detector called LIGO (Laser Interferometer Gravitational Wave Observatory) was built in 1995. Twenty years later in 2015 the first detection of a gravitational wave was announced: a merger of two black holes that occurred approximately 1.3 billion years ago [1]. This binary black hole merger process is an example of a compact binary coalescence (CBC). CBCs can consist of binary neutron stars, binary black holes, or black hole-neutron star binaries; all of these systems have the capability to produce gravitational waves that are detectable by ground-based interferometers. A binary black hole system will undergo three main phases. The first phase of this process is the inspiral which consists of the black holes orbiting one another with a shrinking orbit as energy is gradually lost through gravitational wave emission. Next is the merger where the black holes combine to make one black hole; gravitational wave emission peaks at this time. Finally the ringdown stage is where the resulting black hole oscillates between a spheroid, and an elongated spheroid through gravitational wave emission. Since this first occurrence, there have been a handful of gravitational waves detected by LIGO [2]. With each detection providing new information, there is constantly a push to analyze the data in hopes of further understanding the sources that produce the waves. Along with gaining information concerning some of the universe's most extreme events, LIGO observations provide unique tests of general relativity in the strong-field, highly dynamical regime. While we have begun to probe general relativity with these detections, there are other predictions that we have yet to test. Here we begin to characterize the detectability of one of these predictions, gravitational wave memory, through the study of binary black hole mergers.

3 Motivation for Memory Detection

General relativity predicts that gravitational waves will have an oscillatory component as well as a memory component (Figure 1). These oscillatory and memory components are polarized in the plus and cross orientations. These polarizations are similar to that of light except they are related by a 45° rotation compared to the 90° rotation for EM (electromagnetic) radiation. These polarizations also have oscillatory and non-oscillatory components. For a binary inspiral there is a non-oscillatory component to the + polarization which makes the amplitude of the gravitational wave end with a non-zero value [3]. This non-zero amplitude represents the gravitational wave memory, a weak stretching that permanently alters spacetime, which is displayed in Figure 1 [4].

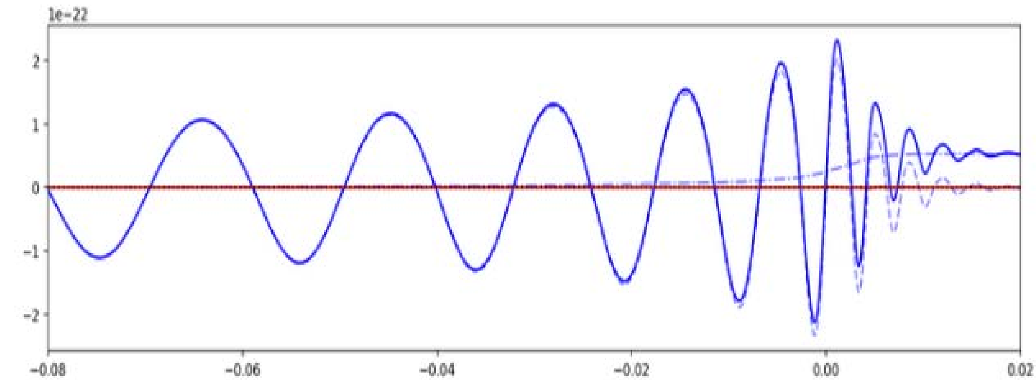

Figure 1: Waveform showing oscillatory binary black hole merger with memory (solid blue line) and without memory (dashed blue line). Made with package GWMemory from *Gravitational-wave memory: waveforms and phenomenology* [5].

Linear memory, discovered in the 1970s, arises from near-zero-frequency changes in the time derivatives of the source's multipole moments. Multipole moments are a combination of the mass moment, the extent to which an object resists rotational acceleration about a particular axis, and the mass-current moment which corresponds to the star's spin angular momentum (the star's moment of inertia about its spin axis multiplied by its spin angular frequency ω) [6]. Linear memory also appears in systems that experience kicks such as a rogue black hole, or systems that eject particles such as neutrinos from supernovae [3]. Non-linear memory, also known as Christodoulou memory [3][7], grows slowly and is a non-oscillatory contribution to to the gravitational wave's amplitude. It originates from gravitational waves that are sourced by the previously emitted waves. All gravitational waves carry a component of nonlinear memory which means it should be included in LIGO waveform models [7].

Since linear and nonlinear memory depend on the form of general relativity field equations, a set of ten coupled non-linear differential equations that describe gravity as a result of spacetime being curved by mass and energy, it is possible that different forms of memory could be uncovered if general relativity were to be modified [8]. Since memory is difficult for LIGO to detect, it has mostly been disregarded by scientists studying gravitational waves. However, the memory scales linearly with the black hole's mass which means there will likely be a detectable contribution to the calculated waveform amplitude of the resulting gravitational waves [9]. This memory effect is computed to be non-negligible as it enters the waveform at approximately the same order as the quadrupole. From this one can conclude that the memory effect should not be impossible to detect with the proper equipment and analysis techniques [3]. Now that LIGO has published seven CBC (compact binary coalescence) events, there is more data to explore and a greater potential to detect memory.

4 Background - Gravitational Wave Memory

While there are numerous methods one can utilize to begin to understand gravitational radiation, one helpful analogy is electromagnetic radiation. As electric charges move they create electromagnetic waves that propagate outward from their source at the speed of light. The waves carry energy and their energy flux falls off as $\frac{1}{r^2}$ where r is the distance away from the source, while the amplitude falls off as $\frac{1}{r}$. They can be detected by the forces they apply to electrons, or by the amount of energy the source loses from the wave propagation. In a similar fashion, gravitational waves arise when moving masses send out waves that are the fluctuating curvature of spacetime. The amplitude of the waves also falls off as $\frac{1}{r}$ over long distances and they can be detected either by the gravitational strain they apply to groups of massive objects in free fall, or the amount of energy that is lost by the source. While there are strong similarities between gravitational and electromagnetic radiation, the differences become apparent when the strength of the two forces are compared. Due to the weakness of gravity, only very powerful astrophysical interactions are capable of producing gravitational waves that are detectable on earth. Some of these interactions include mergers of neutron stars, black holes, or a combination of both [10].

An additional factor that differs between electromagnetism and gravitation is gravitational waves have a large nonlinearity [11]. This nonlinearity is intriguing to study because it will help us further understand the fundamental nature of gravitational waves. Gravitational waves are sourced by energy and mass which allows the particles that carry gravity, called gravitons, which carry energy, to source gravitational waves and emit more gravitons. These gravitons interact or couple with one another which gives rise to the residual warping of spacetime; this is what we refer to as gravitational wave memory. By better understanding this nonlinear memory we may also reach a stronger comprehension of other objects in our universe such as black holes. While black holes are produced by collapsed massive stars, the theoretical point at its center, the singularity, is thought to have infinite spacetime curvature. For this reason black holes can be considered a physical representation of memory. However, since quantum mechanics is not currently able to account for this and we do not have a clear picture of strong field quantum gravity, there are certainly flaws in the idea of infinite spacetime curvature. Studying nonlinear memory will allow us to test whether this firm prediction of general relativity holds up to our quantitative predictions. While the nonlinear component of gravitational radiation is very important, it becomes easier to understand the background physics when only the linear portion is considered at first. In this case, the relevant Einstein field equation reduces to a form similar to that of one of Maxwells equations. After taking the time derivatives of the source's multipole moments the resulting equation for the gravitational wave strain far from the source becomes

$$h(r,t) = \frac{2G}{c^4 r}\frac{d^2 I_{ij}}{dt^2}(t - r/c) \tag{1}$$

where I_{ij} represents the mass quadrupole moment of the source [12]. This is known as the quadrupole formula of general relativity which is used to calculate energy loss. The equation for this quadrupole moment gives rise to an indirect way to detect gravitational waves, that is, by considering a system where motion is measured very accurately. An example of this is a binary neutron star system with one pulsar and one neutron star where the rate general relativity predicts the system will lose energy can be calculated. The work of Hulse-Taylor [13] concluded that the loss of energy will cause the orbit of the neutron star and the pulsar, and therefore the orbital period, to shrink. The change in orbital features can be tracked through the Doppler shift of the arrival time of the radio pulses. From the formula for gravitational wave energy loss one can then predict what the orbital period of the binary system will be at a particular moment in time. The measure of a decreasing orbital period and the energy lost through gravitational radiation were shown to match which means gravitational waves were indirectly detected [13].

5 Limitations of Memory Detection

While we have good reason to believe gravitational wave memory exists, it is the detection process that has prevented us from obtaining substantial results. In understanding why gravitational wave memory has not yet been directly detected, it is helpful to first examine the details that make finding it difficult. The first reason is due to the extremely small size of the memory effect. The size of memory at its peak value is roughly $\frac{1}{5}$ the size of the maximum value of the oscillatory waveform making it significantly more difficult to detect.

Another reason why the detection of gravitational wave memory is difficult with LIGO is due to the presence of low-frequency detector noise. While some noise sources are relatively well understood, quantum and instrument noise are difficult to suppress. Quantum noise arises from the radiation pressure fluctuations causing random motion of the interferometer mirrors [14]. Instrument noise is a concern as it can overwhelm or mimic the gravitational wave strain pattern that is being looked for. The instrument noise is smallest around a few hundred Hertz, but increases sharply at low and high frequencies. Throughout the LIGO frequency band there are narrow spikes due to vibrating fibers that suspend the mirrors and test masses in the interferometers [15]. In summary, the memory effect is dominant at low frequencies but the noise that limits LIGOs sensitivity to gravitational wave strain is orders of magnitudes larger at frequencies below ten to twenty Hertz. This low-frequency noise makes it significantly more difficult to detect the low-frequency strain signal from gravitational wave memory.

6 Approach: Potential Methods of Detection

Even though there are numerous challenges to overcome before a clear memory signal is obtained, there are still strategies that can be implemented to begin the search. One approach to compute Christodoulou memory is to first calculate the oscillatory waveform associated with a detected event, $h_{osc}(t)$, from general relativity solutions obtained through numerical relativity models [16]. Then one can compute $h_{mem}(t)$ from $h_{osc}(t)$ by utilizing the equations

$$\delta h_{jk}^{TT}(T_R, \Omega) = \frac{4G}{Rc^4} \int_{-\infty}^{T_R} dt \int_{S^2} d\Omega' \frac{dE}{dtd\Omega} \left[\frac{n_j n_k}{1 - n^l N_t} \right]^{TT} \tag{2}$$

$$\frac{dE}{dtd\Omega} = \frac{R^2 c^3}{16\pi G} \left| h'(t, \Omega) \right|^2 \tag{3}$$

where Δh_{jk}^{TT} represents the memory component of the waveform, $h' \equiv \frac{dh}{dt}$, and h represents the gravitational wave strain [5].

After computing the h_{mem} component one can create a one-parameter model of the signal in the data: $h(t) = h_{osc}(t) + \lambda h_{mem}(t)$. Here, λ represents a parameter that equals 1 if the general relativistic predictions are correct, and if the memory component is present. If the memory component is either not present or detectable then λ will be equal to zero. Then one can implement Bayesian parameter estimation to compare the model to data that contains a real signal [17]. Bayesian parameter estimation is achieved by implementing Bayes' theorem which is given by

$$P(\,\lambda|\,d) = \frac{P(\,d|\,\lambda)P(\lambda)}{\int P(\,d|\,\lambda)P(\lambda)} \tag{4}$$

P(λ) is the prior which is the prediction of the range of values the parameter λ will likely take [18]. For our purposes we set the prior from -10 to 10. $P(\,d|\,\lambda)$ is the probability of the data given the prior that was just set and the denominator is the normalization.

By Implementing Bayes' theorem we obtain a posterior probability distribution function (PDF) which allows us to graph the posterior against λ. This tells us the likelihood of the λ parameter taking on a certain value. As predicted, the graph should peak at one and spread across a range of λ values that will depend on the arguments placed into the GWMemory code [18]. The width of the PDF corresponds to how accurately the parameter λ can be measured; the narrower the peak, the more exact the λ measurement. This process can then be repeated for different distances and masses which will alter the SNR, and therefore how accurately λ can be measured.

Another potential method of detection is to integrate along the signal to $t = \frac{1}{f_{opt}}$ where f_{opt} (optimal frequency) is the frequency at which the detector is the most sensitive to ordinary gravitational wave bursts. If the length of the burst with memory (BWM) is smaller than $\frac{1}{f_{opt}}$, the detectors sensitivity to BWM is practically equivalent to that of bursts without memory that are one cycle long and whose frequency is f_{opt}. A benefit of this method is it has the potential to be implemented for any type of detector and signal used for the study [19].

Additionally, there is the method of stacking events. Combining information from the mergers could over time, as more compact binary coalescence events are detected, boost the detectability of memory enough to obtain a clearer picture. Lasky has predicted that 35 to 90 black hole mergers similar in mass and distance as G150194 may be enough for LIGO to detect memory [4]. Also since LIGO is going through advancements until 2021, which will make it more sensitive, there is potential that fewer mergers than predicted will be needed to detect memory [20].

7 Specific Approach to Memory Detection with LIGO

As a prerequisite to attempting the stacking method mentioned above, we have been working on a general data analysis technique with an oscillatory LIGO waveform in preparation to work with waveforms with memory. The first steps involve creating simple waveforms with black hole masses of an arbitrary size. This preliminary step also involves creating a PSD (Power Spectral Density) with real LIGO data. A power spectral density is a measure of the strain-equivalent noise in a detector. The ASD (Amplitude Spectral Density) will also be used in our analysis and is simply the square root of the PSD.

We then begin to analyze the memory waveform by computing the oscillatory waveform associated with a detected event (in this study we chose O1 data) from general relativity solutions obtained with numerical relativity models [16]. We then calculate the memory waveform by using the package GWMemory [18] which uses a surrogate model: a combination of commonly used waveforms including those from numerical relativity. After obtaining the memory waveform we form a model of the signal in the data with one parameter λ: $h(t) = h_{osc}(t) + \lambda h_{mem}(t)$. From here we use Bayes' theorem (Equation 4) to calculate the posterior and we graph the results. The graphs obtained by this method are discussed in the subsequent section.

8 Posterior Probability Distribution Graph Results

Figure 2 shows the posterior, $p(\lambda | d)$, graphed against λ using the O1 GW15914 PSD for the noise weighted inner product [21]. For this graph we fix the total mass to be 60 solar masses, but allow the distance from earth to the CBC, and therefore the SNR, to vary. This confirms the prediction that the farther away the signal, the harder the memory component will be to detect as the peak will then cover a wider range of λ values. The results also match our original prediction as the graph is distributed across a range of λ values but peaks at one. This peak communicates that the most likely value for λ is one, however these results are not significant without performing further calculations.

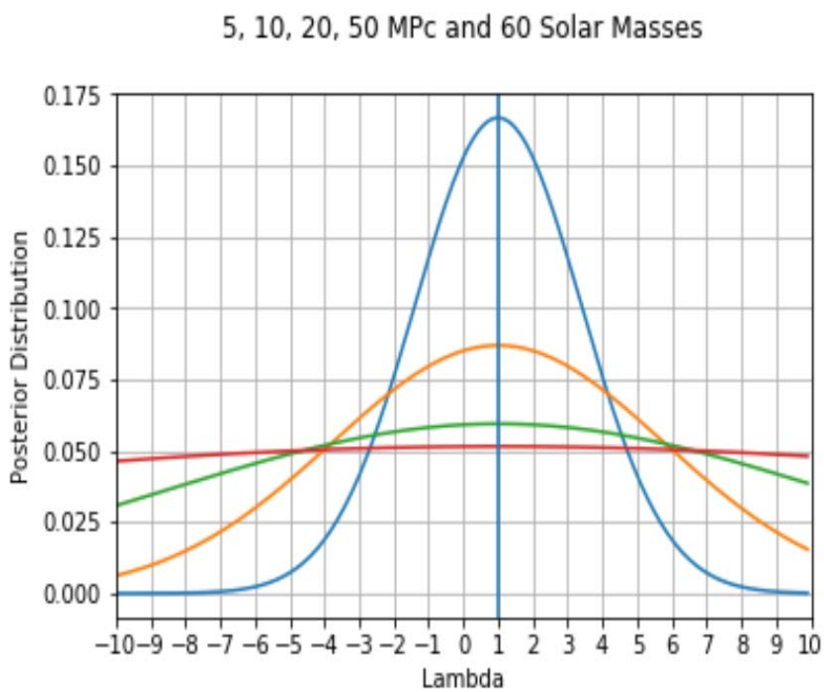

Figure 2: Graph of Likelihood versus lambda for a binary black hole system with a total of 60 solar masses at distances of 5, 10, 20, 50 megaparsecs.

In order to determine if our results were significant we took the 90% percent confidence interval of the posterior. Figure 3 shows the 90% confidence interval of a 60 solar mass merger at 0.9 megaparsecs away. Here the confidence interval is 0.1 to 1.9 which does not include zero. This means our results are statistically significant as the memory parameter being equal to zero is not probable. While this confidence interval does not include zero, there are many distance and mass combinations that will include zero which emphasizes the importance of this test when studying if memory could be detectable with given distance and mass values.

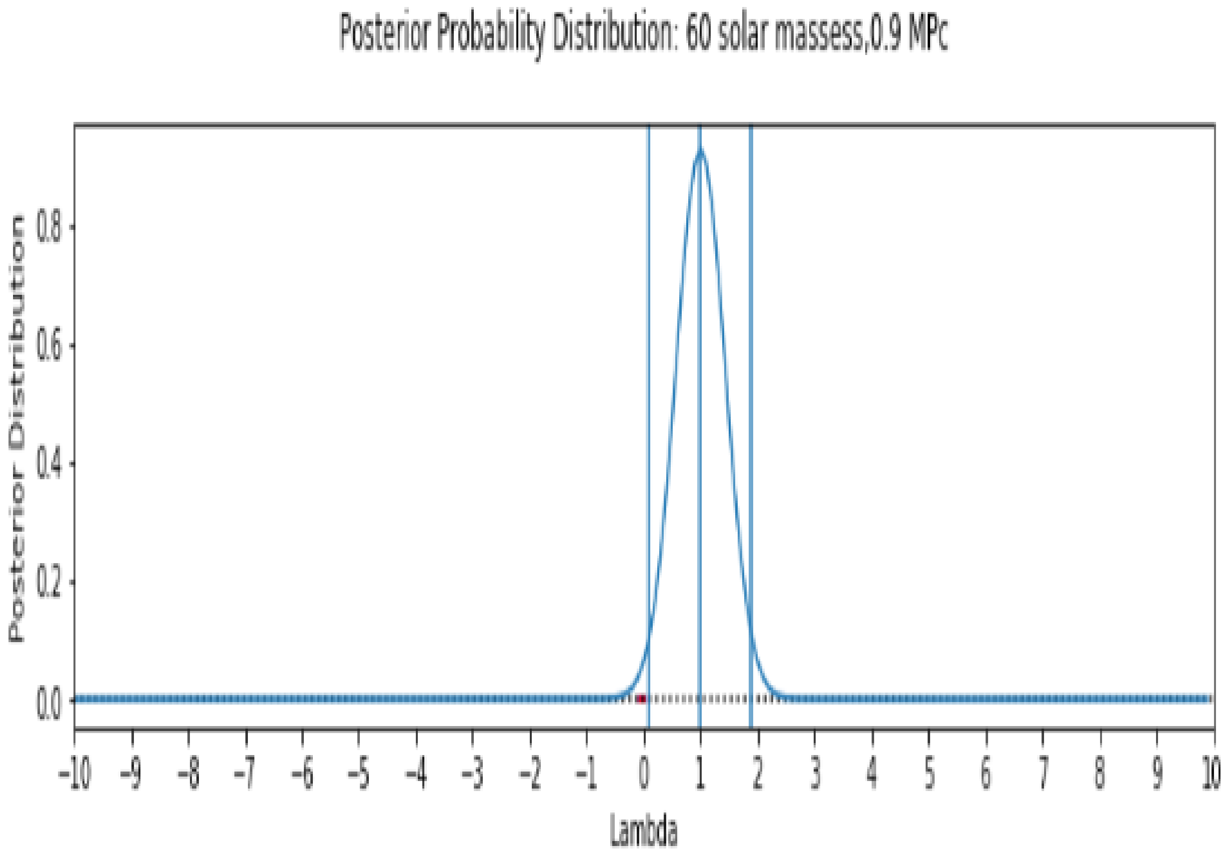

Figure 3: 90% confidence interval: 0.1 to 1.9 of a 60 solar mass merger 0.9 megaparsecs away.

The next step is to attempt to characterize the detectability of the memory component using PSD curves from current and future detectors. Our first attempt is by making use of the O1 GW150914 [22] noise curve graphed with the oscillatory waveform and memory component in the frequency domain as shown in Figure 4. Before analyzing this graph it is important to note that the memory component is never smaller than the oscillatory component; it is due to the limitations of the package [5] that the graph is presented in this manner. After viewing this graph we are able to better characterize the detectability of memory. In the O1 ASD graph the memory component always lies below the noise curve which means it will not be identified by the specified detector at the chosen distance (200 megaparsecs). However, in Figure 5 the memory component appears to just cross the Advanced LIGO ASD noise curve which implies that memory will be easier to detect with future, more sensitive detectors.

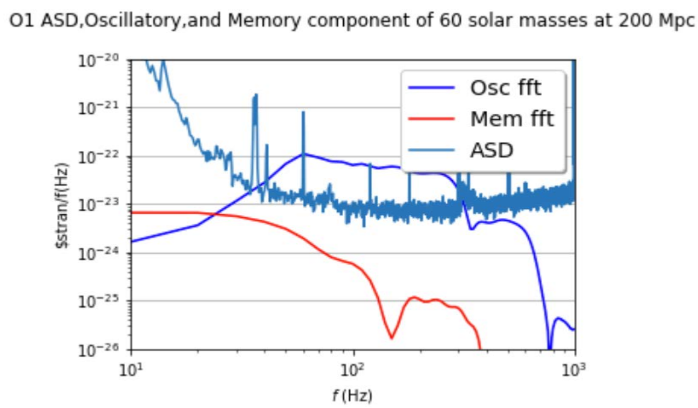

Figure 4: O1 ASD, oscillatory component, and memory component of 60 solar masses at 200 megaparsecs.

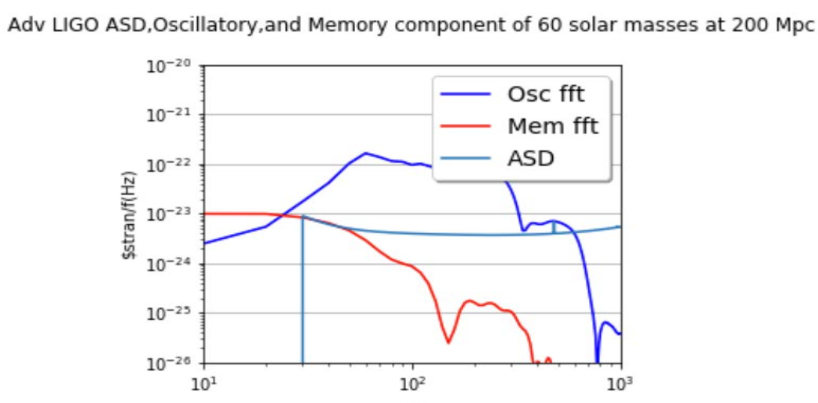

Figure 5: Advanced LIGO ASD, oscillatory component, and memory component of 60 solar masses at 200 megaparsecs.

9 Conclusion and Future Work

There is a continual effort to analyze gravitational wave signals in hopes of gaining a deeper understanding of their sources, and to explore the predictions of general relativity. While its effects have not yet been theoretically tested, detecting memory would be an important scientific feat that would allow us to gain both a deeper understanding of CBCs and confirm a firm prediction of general relativity. Nonlinear memory continues to be an intriguing research topic due to the interesting information it has revealed thus far, including the way it affects the waveform at a leading order equivalent to that of the quadrupole. One way to further develop our analysis will be to perform a Markov Chain Monte Carlo simulation that will allow us to sample the entire set of parameters of the gravitational wave signal opposed to just the memory component, mass, and distance. This process will allow us to look for degeneracies, or parameters that have a waveform signature similar to that of memory.

LIGO in combination with other ground-based laser detectors will potentially be able to detect memory after the discovery of dozens of extreme merger events. Additional gravitational wave detectors such as Virgo, KAGRA and LIGO-India will further increase the number of detections which will allow us to more accurately measure λ [9]. There is also potential that LISA [23] (to be launched in 2030) will be able to greatly enhance the detectability of gravitational wave memory due to its ability to measure much lower frequencies. While memory has not yet been observed, the capabilities of our technology is one of the many reasons why there is optimism surrounding the potential to directly detect gravitational wave memory.

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