

# Blameworthiness in Multi-Agent Settings

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## Abstract

We provide a formal definition of blameworthiness in settings where multiple agents can collaborate to avoid a negative outcome. We first provide a method for ascribing blameworthiness to groups relative to an epistemic state (a distribution over causal models that describe how the outcome might arise). We then show how we can go from an ascription of blameworthiness for groups to an ascription of blameworthiness for individuals using a standard notion from cooperative game theory, the *Shapley value*. We believe that getting a good notion of blameworthiness in a group setting will be critical for designing autonomous agents that behave in a moral manner.

## 1 Introduction

As we move towards an era where autonomous systems are ubiquitous, being able to reason formally about moral responsibility will become more and more critical. Such reasoning will be necessary not only for legal ascription of responsibility, but also in order to design systems that behave in a moral manner in the first place. Unfortunately, though, pinning down the many notions related to moral responsibility has been notoriously difficult to do, even informally. In this work, we lay foundations on which these problems can be solved.

Halpern and Kleiman-Weiner (2018) (HK from now on) made important headway on this work by providing a definition of blameworthiness based on a causal framework. An epistemic state, that is, a distribution over causal models, can be used to capture an agent’s beliefs about the effects that actions may have. Given an epistemic state, they then provide a definition of blameworthiness for an outcome by taking into account both whether the agent believed they could affect the likelihood of the outcome and the cost they believed would be necessary to do so.

While their definition seems compelling in single-agent settings, as HK themselves observe, the definition does not capture blameworthiness in multi-agent settings where, if the group could coordinate their actions, they could easily bring about a different outcome (see Section 3.2 for more discussion of this issue). Being able to analyze blameworthiness in multi-agent scenarios seems critical in practice; if

an accident is brought about due to the actions of multiple agents, we would like to understand to what extent each one is blameworthy.

We tackle that problem here by defining a notion of group blameworthiness in multi-agent scenarios. Our notion can be viewed as a generalization of the single-agent notion of blameworthiness defined by HK. However, as we shall see, subtleties arise when considering groups. Once we have a way of ascribing blameworthiness to a group, we show that a standard notion from cooperative game theory, the *Shapley value* (Shapley 1953), can be used to apportion blame to individual members of the group, and is in fact the only way to do so that satisfies a number of desirable properties.

The rest of this paper is organized as follows. In Section 2 we review the basic causal framework used. Section 3 constitutes the core of the work; in it, we review the Halpern and Kleiman-Weiner definition of blameworthiness in the single-agent setting, define group blameworthiness, show how to apportion it to individual agents, and demonstrate how the definition plays out in an illustrative example. In Section 4 we review related work and in Section 5 we conclude.

## 2 Causal Models

At the heart of our definitions is the causal model framework of Halpern and Pearl (2005). We briefly review the relevant material here.

A causal situation is characterized by a set of variables and their values. A set of *structural equations* describes the effects that the different variables have on each other. Among the variables we distinguish between *exogenous variables* (those variables whose values are determined by factors not modeled) and *endogenous variables* (those variables whose values are determined by factors in the model).

A causal model  $M = (\mathcal{S}, \mathcal{F})$  consists of a signature  $\mathcal{S}$  and a set  $\mathcal{F}$  of structural equations. A signature  $\mathcal{S} = (\mathcal{U}, \mathcal{V}, \mathcal{R})$  is in turn composed of a (finite but nonempty) set  $\mathcal{U}$  of exogenous variables, a (finite but nonempty) set  $\mathcal{V}$  of endogenous variables, and a range function  $\mathcal{R}$  that maps each variable in  $\mathcal{U} \cup \mathcal{V}$  to a set of values it can take on.  $\mathcal{F}$  associates with each endogenous variable  $X \in \mathcal{V}$  a function denoted  $F_X$  such that  $F_X : (\times_{U \in \mathcal{U}} \mathcal{R}(U)) \times (\times_{Y \in \mathcal{V} - \{X\}} \mathcal{R}(Y)) \rightarrow \mathcal{R}(X)$ ; that is,  $F_X$  determines the value of  $X$ , given the values of all the other variables in  $\mathcal{U} \cup \mathcal{V}$ .

We assume that there is a special subset  $\mathcal{A}$  of the endogenous variables known as the *action variables*. Since we consider scenarios with multiple agents, we need to be able to identify which agent each action is associated with. Thus, given a set  $G$  of agents, we augment the signature of the causal model to be  $S = (\mathcal{U}, \mathcal{V}, \mathcal{R}, \mathcal{G})$ , where  $\mathcal{G} : \mathcal{A} \rightarrow G$  associates an agent with each action variable in  $\mathcal{A}$ . The range of a variable  $A \in \mathcal{A}$  is simply the set of actions available to the agent  $\mathcal{G}(A)$ . So, for instance, if there are five members of a committee and each of them can vote yes or no then there will be action variables  $A_1$  through  $A_5$  with  $\mathcal{G}(A_i) = i$  and  $\mathcal{R}(A_i) = \{\text{yes}, \text{no}\}$  for all  $i$ . We restrict here to scenarios where there is one action per agent; in future work, we hope to consider blameworthiness in planning scenarios, where agents may take multiple actions sequentially.

In causal models, we can reason about interventions. Specifically, we have formulas of the form  $[A \leftarrow a]\varphi$ , which can be read “if action  $a$  were performed, then the outcome would be  $\varphi$ ”, where an *outcome* is a Boolean combination of primitive events of the form  $X = x$ . We give semantics to such formulas in a *causal setting*  $(M, \vec{u})$  consisting of a causal model  $M$  and a *context*  $\vec{u}$ , an assignment of values to all the exogenous variables. We do not need the details of the semantics in what follows; they can be found in (Halpern 2016; Halpern and Pearl 2005).

### 3 Blameworthiness

With this background, we now turn to the question of how blameworthy an agent  $ag$  is for an outcome  $\varphi$ . We begin by reviewing the HK definition, and then propose a way of dealing with settings that allow coordinated group actions. One caveat: as noted by HK, words like “blame” have a wide variety of nuanced meanings in natural language. While we think that the notion that we are trying to capture (which is essentially the same as the notion that HK tried to capture) is useful, it corresponds at best to only one way that the word “blame” is used by people.

#### 3.1 Blameworthiness in a single-agent setting

HK identified two factors that play a role in determining blameworthiness:  $ag$ ’s beliefs about his ability to affect  $\varphi$  and  $ag$ ’s beliefs about the cost necessary to affect  $\varphi$ . Here we present a slightly simplified version of their formalization of these notions.

An agent  $ag$  has an *epistemic state*  $\mathcal{E} = (Pr, \mathcal{K})$  relative to which his or her blameworthiness is determined.  $\mathcal{K}$  is the set of all causal settings that  $ag$  considers possible, and  $Pr$  is a probability on  $\mathcal{K}$ .<sup>1</sup> Given  $\mathcal{E} = (Pr, \mathcal{K})$ , two actions  $a$  and  $a'$ , and an outcome  $\varphi$ , we can define  $\delta_{a,a',\varphi}^{\mathcal{E}}$  to be how much more likely  $\varphi$  was to occur if the agent performed action  $a$  than if he performed  $a'$ . Let  $\llbracket \psi \rrbracket_{\mathcal{K}}$  denote the set of all settings in  $\mathcal{K}$  where  $\psi$  is true, so that  $Pr(\llbracket [A = a]\varphi \rrbracket_{\mathcal{K}})$  is the probability that  $ag$  ascribes to outcome  $\varphi$  occurring given that action  $a$  is taken. Then  $\delta_{a,a',\varphi}^{\mathcal{E}} = \max(0, Pr(\llbracket [A = a]\varphi \rrbracket_{\mathcal{K}}) - Pr(\llbracket [A = a']\varphi \rrbracket_{\mathcal{K}}))$ .

<sup>1</sup>In HK’s original formalization, an epistemic state also contained a utility function. This is not necessary for our purposes, so to simplify matters we leave it out.

Thus,  $\delta_{a,a',\varphi}^{\mathcal{E}}$  is 0 if performing action  $a'$  is at least as likely to result in outcome  $\varphi$  as performing action  $a$ .

Intuitively, this  $\delta_{a,a',\varphi}^{\mathcal{E}}$  term ought to play a significant role in how we define blameworthiness: if  $ag$  does not believe that he can have any effect on outcome  $\varphi$ , then we can hardly blame him for its occurrence. At the same time, though, this does not seem to tell the whole story. For if  $ag$  believed he could change the outcome but only by giving up his life, we would not blame him for  $\varphi$ ’s occurrence. Thus there seems to be a second factor at play, the expected cost  $c(a)$  that  $ag$  ascribes to each action  $a$ . Intuitively, the cost measures such factors as the cognitive effort, the time required to perform the action, the emotional cost of the action, and the potential negative consequences of performing the action (like death). (HK provide further discussion and intuition for cost.)

Noting that the balance between these two terms seems to be situation-dependent, HK propose that a parameter  $N > \max_{a'} c(a')$  be used to weight the cost term in a given scenario. The degree of blameworthiness of  $ag$  for  $\varphi$  relative to action  $a'$ , given that  $ag$  took action  $a$ , is then defined to be  $db_N^c(a, a', \mathcal{E}, \varphi) = \delta_{a,a',\varphi}^{\mathcal{E}} \frac{N - \max(c(a') - c(a), 0)}{N}$ . The degree of blameworthiness of  $ag$  for  $\varphi$  given that  $ag$  took action  $a$  is then  $db_N^c(a, \mathcal{E}, \varphi) = \max_{a'} db_N^c(a, a', \mathcal{E}, \varphi)$ .

As pointed out by HK, blameworthiness judgments are not always made relative to the beliefs of the agent. It may be more appropriate to consider the beliefs that we believe that the agent *ought* to have had. Consider, for example, a drunk driver who gets into an accident; in his inebriated state, he may have believed that it was perfectly safe to drive, but we still consider him blameworthy because we do not consider that belief acceptable. The definition just takes an epistemic state as input, without worrying about whose epistemic state it is.

#### 3.2 Blameworthiness of groups

As HK already note, this definition of blameworthiness seems to provide unsatisfactory results in settings where multiple agents are involved. Consider for instance the well-known *Tragedy of the Commons* (Hardin 1968).

**Example 3.1.** 100 fishermen live by a lake. If at least 10 of them overfish this year then the entire fish population of the lake will die out and there will be nothing left to fish in coming years. Each fisherman, however, believes that it is very likely that at least 10 other fishermen will overfish. So given that all the fish will die out no matter his particular action, each fisherman decides to maximize his utility that year and so overfishes. By the end of the year the entire fish population has died out.

Under the definition of blameworthiness discussed, each fisherman will have blameworthiness close to 0, as  $\delta_{a,a',\varphi}^{\mathcal{E}}$  will be close to 0: each fisherman deemed the probability of the fish population dying to be close to 1 independent of his or her own action. And it seems unreasonable to say their beliefs were unacceptable, as in fact what each fisherman predicted is exactly what occurred. But it seems problematic for each fisherman to have a degree of blameworthiness that is almost 0 when the fishermen as a group are clearly to blame for this outcome.  $\square$

How blameworthy are the fishermen for the outcome? We claim that in order to assign blame to the group, we need to assess the cost to the fishermen of coordinating their actions, just as was done with actions in the case of assigning individual blameworthiness. If in fact it was impossible or extremely difficult for the fisherman to coordinate (perhaps they had no means of communication, or all spoke different languages), then they each should have very little blameworthiness. On the other hand, if coordination would have been relatively straightforward, then the group should be viewed as quite blameworthy.

Computing the costs and expected effects of different ways that the group could coordinate seems, in general, quite difficult. To do this, we would need to have a model of what kinds of actions the group could perform in order to bring about coordination. Perhaps some members of the group could convince politicians to pass laws that put caps on how large the catch could be; perhaps they could arrange for sensors that would be able monitor how much each individual fisherman caught. Note that in this discussion we are not considering whether fisherman would want to undertake these actions; only whether there are feasible actions that might lead to coordination, and how much they would cost. For example, the fishermen might choose not to lobby politicians to pass laws because doing so would be quite expensive, but if it was possible for them to do so, we still consider it an action the group could have taken. Unfortunately, while we may understand how concrete actions might affect the likelihood of the fish population dying out, completely describing a set of rich causal models that capture the possible dynamics that can lead to collaboration may be prohibitively difficult, if not impossible.

We instead consider a simpler way to capture difficulty of coordination in group settings by abstracting away from these details. We directly associate a cost with various distributions over causal settings (i.e., epistemic states) that we view as the possible outcomes of attempts to coordinate. Intuitively, these are the distributions that could have been induced by feasible collective actions given beliefs about the probability of different causal settings in the richer models. The cost of a distribution represents the expected cost of performing whatever collective actions were needed in the richer models to bring about that distribution, as well as the expected costs of whatever actions will be taken in the simpler model. For example, suppose that the richer causal model for the tragedy of the commons example allowed for installing sensors, and after installing sensors we believe each fisherman  $i$  will have an independent probability  $p_i$  of overfishing. This leads to a distribution over the settings of the simple causal models that were implicit in the description of Example 3.1, where there is presumably an exogenous variable  $U_i$  that determines how much fishing fisherman  $i$  does. This exogenous variable is endogenous in the richer model, and is affected by the installation of sensors. In any case, the cost of that distribution on causal settings in the simpler model is the cost of performing whatever collective actions are required in the richer model to arrange for the installation of sensors, plus the expected collective costs of all the fishing the fishermen do. Note that there may not be

feasible collective actions (i.e., ones with finite cost) in the richer model that lead to none of the fishermen overfishing.

It is also worth noting that modeling the effects of an attempt at coordination in the richer models as a distribution over causal settings in the simpler models allows us to capture other effects that the coordination process may have. For instance, consider a scenario where one of the fishermen believes that taking everyone out on a boat ride on the lake would be a particularly effective way to get the fishermen to feel social responsibility to not overfish. Such a boat ride may also effect the pollution levels in the lake, which in turn may also play a role in determining whether the fish population will die out. By viewing the coordination process as inducing a new distribution over causal settings, we can at the same time capture both effects that the boat ride is expected to have on how people behave and on pollution levels.

With this background, we can now give analogues to the HK definitions. We first give an analogue to the definition of  $\delta_{a,a',\varphi}^{\mathcal{E}}$ . As discussed above, rather than comparing two actions that an individual can perform, we are comparing two epistemic states (intuitively, ones brought about by different collective actions in the richer model). Let  $\mathcal{E}_i = (Pr_i, \mathcal{K}_i)$ ,  $i = 1, 2$ , be two epistemic states. Then, given an outcome  $\varphi$ , we can define the extent to which  $\varphi$  was more likely given  $\mathcal{E}_1$  than  $\mathcal{E}_2$  as

$$\delta_{\mathcal{E}_1, \mathcal{E}_2, \varphi} = \max(0, \Pr_1(\llbracket \varphi \rrbracket_{\mathcal{K}_1}) - \Pr_2(\llbracket \varphi \rrbracket_{\mathcal{K}_2})).$$

Just as in the HK definition, we are comparing the likelihood of outcome  $\varphi$  in two scenarios. For HK, the two scenarios were determined by the agent performing two different actions; here, they are determined by two different epistemic states that we can think of as arising from two coordination actions in the richer models combined with uncertainty regarding the richer causal setting.

We now want to get an analogue of the degree of blameworthiness function  $db$  for a group. Again, this will depend on two parameters, a cost function  $c$  and a parameter  $N$  that determines the relative weight we ascribe to the cost and the difference  $\delta$  defined above. However, now  $c$  has different arguments. One of its arguments is, as suggested above, an epistemic state. The second is a subset of agents. Let  $Ag = \{ag_1, \dots, ag_M\}$  be the set of all agents and consider a subset  $Ag' \subseteq Ag$ . We think of  $c(Ag', \mathcal{E})$  as the expected cost of the coordination actions in the richer game required for the agents in  $Ag'$  to bring about epistemic state  $\mathcal{E}$ , plus the expected total costs of the actions of the agents in the simpler models given that epistemic state (see below).<sup>2</sup> The key point here is that we may not require coordination among all the agents to bring about a particular epistemic state; it may only require a subset. Moreover different subsets of agents may have different costs for obtaining the same outcome; the cost function is intended to capture that.

The cost function is meant to take into account not only the costs of bringing about  $\mathcal{E}$ , but the expected cost of performing action  $A_i$  for  $i \in Ag'$  in the simpler causal models.

<sup>2</sup>If there is more than one way for the agents in  $Ag'$  to bring about  $\mathcal{E}$ , we can think of  $c(Ag', \mathcal{E})$  as being the cost of the cheapest way to do so.

This cost may vary from one causal model to another (e.g., it may be more costly to overfish if the probability of getting caught is higher, and this may depend on the causal model). Given an epistemic state, we can compute the expected costs of performing  $A_i$ .

Given a cost function  $c$  and “balance parameter”  $N$ , define the degree of blameworthiness for outcome  $\varphi$  of group  $Ag'$  and epistemic state  $\mathcal{E}_1 = (\text{Pr}_1, \mathcal{K}_1)$  relative to epistemic state  $\mathcal{E}_2 = (\text{Pr}_2, \mathcal{K}_2)$  such that  $c(Ag', \mathcal{E}_2)$  is finite as

$$gb_N^c(Ag', \mathcal{E}_1, \mathcal{E}_2, \varphi) = \delta_{\mathcal{E}_1, \mathcal{E}_2, \varphi} \frac{N - \max(c(Ag', \mathcal{E}_2) - c(Ag', \mathcal{E}_1), 0)}{N}.$$

Although we replace actions  $a_1$  and  $a_2$  in the definition of  $db$  by epistemic states  $\mathcal{E}_1$  and  $\mathcal{E}_2$ , and use a cost function with different arguments, the intuition for both the group degree of blameworthiness function  $gb$  and the individual degree of blameworthiness function  $db$  defined by HK are very much the same.

Just as for individual degree of blameworthiness, we can define the group blameworthiness of group  $Ag'$  for outcome  $\varphi$  given epistemic state  $\mathcal{E}_1$  as the max over all possible choices of  $\mathcal{E}_2$ .

$$gb_N^c(Ag', \mathcal{E}_1, \varphi) = \max_{\{\mathcal{E}_2: c(Ag', \mathcal{E}_2) \text{ is finite}\}} gb_N^c(Ag', \mathcal{E}_1, \mathcal{E}_2, \varphi).$$

Note that the degree of group blameworthiness of the empty group or any other group that cannot coordinate any alternative actions will be 0; the only epistemic state  $\mathcal{E}_2$  such that  $c(Ag', \mathcal{E}_2)$  is finite will be  $\mathcal{E}_1$  itself, and  $\delta_{\mathcal{E}_1, \mathcal{E}_1, \varphi}$  is 0.

One thing worth mentioning is that we require a monotonicity property for group blame: if  $Ag'' \subseteq Ag' \subseteq Ag$  then  $gb_N^c(Ag'', \mathcal{E}, \varphi) \leq gb_N^c(Ag', \mathcal{E}, \varphi)$ . The reason for this is that if group  $Ag''$  could coordinate in a particular way for a particular cost then that subset of group  $Ag'$  could do exactly the same thing. Essentially, when considering the possibility of a group coordinating, we must really consider the possibility of coordination of any subset of that group.

Up to now, we have not said anything about *whose* epistemic state we should use for the epistemic states  $\mathcal{E}_1$  and  $\mathcal{E}_2$  in the definitions above. In the case of individual blameworthiness, the typical assumption is that they represent the epistemic state of the agent whose blameworthiness is being considered, although as HK already observed, it may at times be reasonable to assume that it is the epistemic state that society thinks that agent should have. Here we are talking about group blameworthiness, so it is less clear whose epistemic state should be used. It is certainly not clear what a “group epistemic state” should be. It still makes sense to think about “society’s epistemic state”; that is, society’s view of what a reasonable agent’s beliefs should be. We can also take the epistemic state to be the subjective beliefs of one of the agents. Indeed, we will often consider the epistemic state of an agent in the group. We could also view the cost function as subjective—again, it could be society’s cost function or the cost function from the perspective of a particular agent. The definition is agnostic as to where the epistemic state and cost function are coming from, but to apply the definition we need to be explicit.

It is now worth returning briefly to Example 3.1, to see how this definition plays out there. Given an epistemic state  $\mathcal{E}_i = (\text{Pr}_i, \mathcal{K}_i)$  and cost function  $c_i$  representing the beliefs of agent  $ag_i$ , first consider a scenario where it would be essentially impossible for the fishermen to coordinate (e.g., no two fishermen speak the same language). If  $ag_i$  believed this, then the cost of coordinating any possible alternative distribution would likely be very high, so the term  $\frac{N - \max(c_i(Ag', \mathcal{E}_2) - c_i(Ag', \mathcal{E}_1), 0)}{N}$  would be close to 0. Because this is true for all epistemic states  $\mathcal{E}_2$ , maximizing over  $\mathcal{E}_2$  would still give that  $gb_N^{c_i}(Ag', \mathcal{E}_i, \varphi)$  is close to 0. On the other hand, suppose that  $ag_i$  believed that there was some possible coordination of group  $Ag'$  that was not tremendously expensive and that could lead to an epistemic state  $\mathcal{E}_2$  relative to which the probability of the fish population dying was lower (e.g., imposing a fine on anyone who overfished). In this case, to the extent that  $\mathcal{E}_2$  was believed to be effective and low cost, the group  $Ag'$  of fishermen would in fact be quite blameworthy. Note that  $Ag'$  might not consist of all the fisherman; it is possible that a subset of fishermen is powerful enough to impose fines. In general, different subgroups will have different degrees of blameworthiness.

### 3.3 Apportioning group blameworthiness among agents

Now that we have defined group blameworthiness, the question naturally arises: how should group blameworthiness be apportioned among the members of the group? In this subsection, we suggest three axioms that we believe apportionment of blame should satisfy. It turns out that these axioms are natural analogues of axioms that have been used to characterize the Shapley value. Shapley (1953) introduced the Shapley value as an approach to distributing benefits to individual agents in scenarios where agents might coordinate to obtain greater total benefits than they could individually. The Shapley value has since also been interpreted as way of appropriately distributing costs for shared resources (see e.g. (Roth and Verrecchia 1979)). It is thus not surprising that it can be applied in our setting as a way of apportioning group blame.

Given a cost function  $c$  and balance parameter  $N$ , let  $db_N^{c, \mathcal{E}}(j, \varphi)$  be the degree of blameworthiness ascribed to  $ag_j$  for outcome  $\varphi$  relative to epistemic state  $\mathcal{E}$ . Consider the following three axioms for  $db_N^{c, \mathcal{E}}(j, \varphi)$ :

**Efficiency.** All of the blame assigned to the full group of agents must be apportioned to the agents in the group:

$$\sum_j db_N^{c, \mathcal{E}}(j, \varphi) = gb_N^c(Ag, \mathcal{E}, \varphi).$$

This axiom essentially encapsulates what we are trying to do: apportion the total group blame among individuals. Note that we do not want an analogue of this for subgroups  $Ag'$  of  $Ag$ . For example, if a small subgroup  $Ag'$  of fishermen cannot coordinate so as to affect the outcome, they would have quite a low degree of blameworthiness, although the group consisting of all the fishermen might have degree of blameworthiness 1. Thus, we do not

necessarily want the sum of the degrees of blameworthiness of the individual fishermen in  $Ag'$  to be the group blameworthiness of  $Ag'$ .

**Symmetry.** The names of agents should not affect their blameworthiness, so if we simply rename them then the blameworthiness ascribed to them should remain the same. Formally, let  $\pi$  be a permutation of  $\{1, \dots, M\}$ . Given a model  $M = ((\mathcal{U}, \mathcal{V}, \mathcal{R}, \mathcal{G}), \mathcal{F})$ , let  $\pi \circ M = ((\mathcal{U}, \mathcal{V}, \mathcal{R}, \mathcal{G}'), \mathcal{F})$ , where  $\mathcal{G}'(A) = \pi(\mathcal{G}(A))$  for all action variables  $A \in \mathcal{A}$ . Given a set  $\mathcal{K}$  of causal settings, define  $\pi \circ \mathcal{K} = \{(\pi \circ M, \vec{u}) : (M, \vec{u}) \in \mathcal{K}\}$ . That is to say, for any action that is assigned to agent  $i$  in any model, we now instead assign it to agent  $\pi(i)$ . Given a distribution  $\text{Pr}$  over causal settings  $(M, \vec{u})$ , define  $(\pi \circ \text{Pr})((\pi \circ M, \vec{u})) = \text{Pr}((M, \vec{u}))$ ; if a setting had a particular probability then we want the corresponding setting with the actions renamed according to  $\pi$  to have the same probability. Finally, given an epistemic state  $\mathcal{E} = (\text{Pr}, \mathcal{K})$ , let  $\pi \circ \mathcal{E} = (\pi \circ \text{Pr}, \pi \circ \mathcal{K})$ . The *symmetry* axiom requires that

$$db_N^{c, \mathcal{E}}(i, \varphi) = db_N^{\pi \circ c, \pi \circ \mathcal{E}}(\pi(i), \varphi),$$

where  $(\pi \circ c)(Ag', \mathcal{E}) = c(\{b' : \pi(b') \in Ag'\}, \pi^{-1} \circ \mathcal{E})$  (i.e., costs in the new models correspond to costs pre-renaming, which we get by taking the  $\pi$ -preimage).

**Strong Monotonicity.** If agent  $ag_j$  contributes more to the group blameworthiness of all groups in one scenario than another, then  $ag_j$  also ought to have a greater degree of (personal) blameworthiness in the first scenario. Formally, define the *marginal contribution* of  $ag_j$  to the degree of blameworthiness of group  $Ag'$  as

$$mb_N^{c, \mathcal{E}}(j, Ag', \varphi) = \begin{cases} gb_N^c(Ag', \mathcal{E}, \varphi) - gb_N^c(Ag' \setminus ag_j, \mathcal{E}, \varphi) & \text{if } ag_j \in Ag' \\ gb_N^c(Ag' \cup ag_j, \mathcal{E}, \varphi) - gb_N^c(Ag', \mathcal{E}, \varphi) & \text{if } ag_j \notin Ag'. \end{cases}$$

Let  $mb_N^{c, \mathcal{E}}$  and  $mb_N^{c', \mathcal{E}}$  be the marginal contributions to the degree of blameworthiness for two different scenarios; let  $db_N^{c, \mathcal{E}}$  and  $db_N^{c', \mathcal{E}}$  be the associated degree of (personal) blameworthiness for the two scenarios. Then we require that if

$$mb_N^{c, \mathcal{E}}(j, Ag', \varphi) \geq mb_N^{c', \mathcal{E}}(j, Ag', \varphi) \text{ for all } Ag' \subseteq Ag$$

then

$$db_N^{c, \mathcal{E}}(j, \varphi) \geq db_N^{c', \mathcal{E}}(j, \varphi).$$

Young (1985) showed that the only distribution procedure that would satisfy Efficiency, Symmetry, and Strong Monotonicity is the *Shapley value*. The Shapley value has an elegant closed-form expression. It follows that the only way of assigning individual degree of blameworthiness, given a group blameworthiness function  $gb$  has the form:

$$db_N^{c, \mathcal{E}}(j, \varphi) = \sum_{\{Ag' \subseteq Ag : ag_j \in Ag'\}} \frac{(|Ag'| - 1)!(|Ag| - |Ag'|)!}{|Ag'|!} mb_N^{c, \mathcal{E}}(j, Ag', \varphi).$$

We thus have a technique for assigning a degree of blameworthiness for an outcome to individuals in group settings. However, this is relative to an epistemic state, a cost function, and a balance parameter. The question still remains

how these inputs should be chosen. As in HK, one approach when assigning a degree of blameworthiness to an individual would be to take that individual's epistemic state, cost function, and balance parameter. But society may decide that other choices are more reasonable.

Recall that in the last subsection we required that group blame always be monotonic in the group, as if  $Ag''$  could coordinate in some manner then they should also be able to do so as a subset of  $Ag'$ . It is not hard to see (and we show in the full paper) that this suffices to ensure that individual blameworthiness will always be non-negative.

Note that in the single-agent setting, where the only agent choosing an action is some particular  $ag_1$ , if we assume (as HK implicitly did) that an agent can completely decide his or her own actions without the decision process itself incurring costs beyond the costs of the action, then the definition above agrees with the HK definition. Consider  $ag_1$ 's blameworthiness relative to  $ag_1$ 's epistemic state  $\mathcal{E}_1 = (Pr_1, \mathcal{K}_1)$  and cost function  $c_1$ . Note that  $|Ag| = 1$  and the only set  $Ag'$  containing  $ag_1$  is  $\{ag_1\}$ . So there is only one term to sum over in  $db_N^{c_1, \mathcal{E}_1}(1, \varphi)$ , and in that term we have that  $\frac{(|Ag| - 1)!(|Ag| - |Ag'|)!}{|Ag'|!} =$

1. Thus,  $db_N^{c_1, \mathcal{E}_1}(1, \varphi) = mb_N^{c_1, \mathcal{E}_1}(1, \{ag_1\}, \varphi)$ . Because  $ag_1 \in \{ag_1\}$ ,  $mb_N^{c_1, \mathcal{E}_1}(1, \varphi) = gb_N^{c_1}(\{ag_1\}, \mathcal{E}_1, \varphi) - gb_N^{c_1}(\emptyset, \mathcal{E}_1, \varphi) = gb_N^{c_1}(\{ag_1\}, \mathcal{E}_1, \varphi)$ . But now, because we assumed that an agent can decide his or her own actions, the alternatives that group  $\{ag_1\}$  could have coordinated will be precisely the set of actions available to  $ag_1$  at the costs they would incur to  $ag_1$ , so this is the HK definition of blameworthiness.

### 3.4 An illustrative example

The following example illustrates some features of these definitions: Consider a scenario where a committee of 7 people,  $ag_1$  through  $ag_7$ , vote for whether or not to pass a bill. If at least 4 agents vote yes, then the bill will pass. Everyone agrees that it would be better for the bill to pass, but there are external reasons (such as opinions of constituents) that might result in agents benefiting from voting no as long as the bill is passed. The committee votes and agents  $ag_1$  through  $ag_5$  all vote no, so the bill does not pass. How blameworthy is each agent for this outcome?

We now consider the degree of blameworthiness of some of the agents and show how the degree of blameworthiness varies as a function of the agents' beliefs:

- $ag_1$  :  $ag_1$  believed that each of the 6 other agents started with a 60% chance of voting yes. For any coalition of  $n$  agents,  $ag_1$  also believed that for a cost of  $n \times 100$  each agent's probability of voting yes (including that of agents not in the coalition) could be increased by  $n \times 5\%$  by applying social pressure. In addition, if  $ag_1$  herself was in the coalition, then for an additional cost of 2000 she would have switched her vote to yes. Given these beliefs, the degree of blameworthiness for the entire group is  $\approx 0.390$ , while  $ag_1$ 's degree of blameworthiness is  $\approx 0.073$ .
- $ag_2$  :  $ag_2$  held essentially the same beliefs as  $ag_1$ , except that the additional cost necessary for her to change

her vote for coalitions that she was in was 500 instead of 2000. Given these beliefs, the degree of blameworthiness of the entire group is  $\approx 0.390$ , while  $ag_2$ 's degree of blameworthiness is  $\approx 0.120$ .

$ag_2$  holds essentially the same beliefs as  $ag_1$ , but her cost of changing her own vote to yes is lower than  $ag_1$ 's. The degree of blameworthiness of the entire group is the same with respect to both  $ag_1$ 's and  $ag_2$ 's cost functions. In the epistemic state that both agents share, with  $ag_i$ 's cost function (for  $i = 1, 2$ ), the action that maximizes degree of blameworthiness is the action where social pressure is applied by all, but  $ag_i$  does not change her view. Thus, the cost of  $ag_i$  changing her view does not play a role in determining the degree of group blameworthiness. However, the blameworthiness of  $ag_2$  (according to  $ag_2$ 's cost function) is in fact greater than that of  $ag_1$  (according to  $ag_1$ 's cost function), as for some smaller groups the action that maximizes blame consists of  $ag_i$  changing her view, so the lower cost for  $ag_2$  to do so will end up making her more blameworthy. This is what we would expect; since it is less costly for  $ag_2$  to vote yes, she intuitively ought to be more blameworthy for not doing so.

- $ag_3$  and  $ag_4$ :  $ag_3$  held essentially the same beliefs as  $ag_1$  except that she believed that social pressure would be less effective. In particular, she believed that a coalition of  $n$  agents applying social pressure for a cost of  $n \times 100$  would result in an increase of only  $n \times 3\%$  in each agent's probability of voting yes. With these beliefs and cost function, the degree of blameworthiness for the entire group is  $\approx 0.317$ , and  $ag_3$ 's degree of blameworthiness is  $\approx 0.079$ .

$ag_4$  held essentially the same beliefs as  $ag_1$  except that she believed that it would cost  $n \times 150$  to get the social pressure applied by  $n$  agents to increase each agent's probability of voting yes by  $n \times 5\%$ . Given these beliefs, the degree of blameworthiness for the entire group is  $\approx 0.361$ , and  $ag_4$ 's degree of blameworthiness is  $\approx 0.068$ .

$ag_3$  and  $ag_4$  each share beliefs similar to  $ag_1$ 's, but they believe that social pressure will not be quite as effective, either because it won't have as much of an impact or because it will be more costly. As expected, in both of these cases the degree of blameworthiness of the whole group decreases, as there is not as much the group could have been expected to do to ensure that the bill passed. It is worth noting, however, that the blameworthiness of a particular agent may still go up, as it does here for  $ag_3$ . The reason for this is that, while the total group blame goes down, if the group does not have effective alternatives to ensure the desired outcome, then it may be even more important for that particular agent to take an action that can significantly affect the outcome. There are several factors that will affect whether (and to what extent) individual blameworthiness increases or decreases, such as the difference in cost, difference in expected effect, and the balance parameter  $N$ .

- $ag_5$ :  $ag_5$  shared the same beliefs as  $ag_1$  with regard to what actions can be taken and the costs of taking those actions, but was more doubtful as to whether committee

members would vote yes without action being taken. In particular,  $ag_5$  believed that each of the 6 other agents started with a 40% chance of voting yes. She still believed that a coalition of  $n$  agents could increase each agent's probability of voting yes by  $n \times 5\%$  for a price of  $n \times 100$ , and if she was in the coalition would have changed her vote to yes for an additional cost of 2000. With these beliefs and cost function, the degree of blameworthiness for the entire group is  $\approx 0.560$ , and  $ag_5$ 's degree of blameworthiness is  $\approx 0.125$ .

The only difference between  $ag_5$  and  $ag_1$  was that  $ag_5$  believed there was a higher probability of the bill failing in the first place. Relative to this belief,  $ag_5$  (as well as the total group) is deemed to be more blameworthy, as it is more critical that the group do something to ensure the bill have a higher chance of passing. To see how this plays out formally, consider the case where all 7 agents are involved in applying social pressure. Then the effect this would have if the base probability was 60% would be a  $\approx 0.453$  increase in the probability of the bill passing. If, on the other hand, the base probability was only 40%, then the social pressure would lead to a  $\approx 0.651$  increase in the probability of a positive outcome. It is worth noting that if the base probabilities of agents voting yes were too low, then the blameworthiness would decrease, as the probability of the social pressure being able to actually effect a change would be low.

- $ag_6$ : Finally,  $ag_6$  held exactly the same beliefs and used the same cost function as  $ag_1$ , but unlike  $ag_1$ , she voted yes. In this case, the degree of blameworthiness for the entire group is  $\approx 0.157$ , and  $ag_6$ 's degree of blameworthiness is  $\approx 0.022$ .

As we would expect,  $ag_6$  is deemed to be less blameworthy than  $ag_1$ . The total group blame is also lower relative to this epistemic state, as the probability of the bill failing to pass is lower (because there is one definitive yes vote) and so group action was less important.

## 4 Related Work

Not surprisingly, there has been a tremendous amount of work on notions of blameworthiness across a wide range of fields. In this section, we survey some of the literature most relevant to this work from computer science, philosophy, and law.

Our definitions of blameworthiness are based directly on those of HK. Chockler and Halpern (2004) also defined a notion of blame that is related to but somewhat different from blameworthiness; see (Halpern and Kleiman-Weiner 2018) for a discussion.

Our use of Shapley value in defining how to apportion group blame is similar to (and partly inspired by) the work of Datta et al. (2015). They define a measure of the influence that each feature has on the classification of a dataset. So, for instance, if one feature is gender, their measure is intended to give a sense of how much influence gender had on how the data was classified. They provide a set of desired axioms for influence and show that there is a unique measure that satisfies these axioms, which roughly corresponds

to the probability that changing that feature would change the classification. This seems to have natural relevance to our setting if we consider each feature to be the action of an agent and the classification to be the outcome. It is not sufficient, however, as it is not clear how factors such as the cost of an action (which is not relevant in the classification setting) should be incorporated. The Datta et al. approach also does not deal with the “group” aspects of group blame. It is in a sense closer to the work of HK than to ours. For example, in the Tragedy of the Commons, it would assign a low degree of blameworthiness to individual agents. While the group aspects are not relevant in the setting of classification influence, in our setting they are critical.

Ferey and Dehez (2016) applied the Shapley value to sequential-liability tort cases, cases where the amount of damage each agent’s action brings about depends on the actions of earlier agents. The court must decide how restitution of the damages should be divided among the agents in such cases. Ferey and Dehez used reasoning similar to ours to show that the Shapley value gives reasonable outcomes in this context. They also showed that the outcomes seems to align well with some prior case law and legal literature.

There has been much work in the philosophy literature on moral responsibility, including its nature and the conditions under which one ought to be held morally responsible. Particular attention has been paid to the relation of moral responsibility to such issues as free will and agency. Eshleman (2016) provides a good overview and further references.

There has also been significant discussion in the philosophical literature on issues of collective moral responsibility: can it ever really exist, under what conditions would it exist, can group moral responsibility be in turn divided among the member agents, and how ought it be divided if and when it can be? May and Hoffman (1992) provide an excellent collection of essays exploring some of the major ideas in this area. Cooper (1968) argues that collective moral responsibility is not always divisible among agents. He considers an analogy of a delicious stew made from various ingredients; we cannot say that any particular ingredient has a specific degree of impact on the overall flavor; rather, it is the precise way that the different flavors combined that led to such a delicious stew. Similarly, he argues, there may be instances where no particular agent can be ascribed blame for the mis-actions of the group, but rather it emerges from the collective as a whole. In these examples, it seems that Cooper would reject the Efficiency axiom.

Van de Poel et al. (2015) focus on what they call *the problem of many hands* (a term originally due to Thompson (1980)): that is, the problem of allocating responsibility to individual agents who are members of a group that is clearly responsible for an outcome. They formalize some of their ideas using a variant of the logic CEDL (*coalition epistemic dynamic logic*) (De Lima and Royakkers 2015). Unfortunately, CEDL cannot directly capture counterfactuals, nor can it express quantitative notions like probability. Thus, it cannot capture more quantitative tradeoffs between choices that arise when defining degree of blameworthiness.

Finally, it is worth mentioning some of the factors that come into play in legal notions of blameworthiness. Here

we focus on two in particular: *joint and several liability* and *normality*. In tort cases where defendants are jointly and severally liable, each defendant can be considered to be independently liable for the full extent of damages. Thus the injured party can recover the full amount of damages from any of the defendants; it is up to that defendant who ends up paying damages to then attempt to recover some of the payment from other guilty parties. Thus, if two parties are guilty for an outcome but one does not have the means to make restitution or is inaccessible, the other party must make full restitution. This may be viewed as suggesting that there are cases where the law deems each agent who is sufficiently responsible as being fully blameworthy for the outcome rather than just having a portion of the blameworthiness. However, a more reasonable interpretation is that the law takes into account considerations other than just degree of blameworthiness when imposing penalties. Nevertheless, considerations of blameworthiness are likely to come into play when the defendant who is compelled to pay attempts to recover some damages from the other defendants. When joint and several liability should be applied is a complicated matter in the legal literature (see, e.g., (Prosser 1936)).

Another notion at play in legal considerations of blameworthiness is the legal norm. The only considerations we have built into our definitions are expected affect on the outcome and the cost of actions. In the law, however, the extent to which an agent is judged to have deviated from the legal norm may play a role in judgments of blameworthiness for outcomes that were brought about by multiple individuals (American Law Institute 2000). In future work we hope to further explore formalizations of some of the notions at play in legal ascription of blameworthiness. Work done on combining notions of normality with causality (Halpern 2016; Halpern and Hitchcock 2015) may prove relevant in dealing with issues like legal norms.

## 5 Conclusion

We have provided a way to ascribe blameworthiness to groups of agents that generalizes the HK definition. We then showed how, given ascriptions of group blameworthiness, the Shapley value can be used to ascribe blameworthiness to individual agents. These two contributions are separable; if an alternative definition of group blameworthiness is used, the Shapley value could still be used to ascribe blameworthiness to individual agents.

In considering these issues carefully, one obvious question is whether we view our definitions as descriptive or prescriptive. The answer is “both”. We plan to do experiments to see if the perceived difficulty of coordination really does affect how people ascribe group blameworthiness, and to see whether an agent’s potential marginal contribution to an account affects his ascribed degree of blameworthiness. To the extent that we can view legal penalties as proxies for degree of blameworthiness, we can also examine the legal literature to see how these issues affected outcomes in legal cases (although, as we observed earlier, there is clearly more to how penalties are apportioned in legal cases than just blameworthiness). Whether or not our definitions exactly match how people seem to ascribe blameworthiness, we might still ask

whether these definitions might be useful as guides for ascribing blameworthiness in situations involving self-driving cars (or a combination of self-driving cars and humans).

Formalizing notions of moral responsibility will be critical for the eventual goal of designing autonomous agents that behave in a moral manner. We believe that blameworthiness as we have considered it in this work is one important component of moral responsibility, though not the whole story. In future work we hope to continue exploring how these notions can be formalized and applied to a wide variety of settings, especially legal settings; we hope that others will join us in considering these problems.

**Acknowledgments:** This work was supported in part by NSF grants IIS-1703846 and IIS-1718108, ARO grant W911NF-17-1-0592, and a grant from the Open Philanthropy project. We would like to thank Bruce Chapman for pointing out the work of Ferey and Dehez, and the anonymous reviewers of the paper for comments that provided some interesting food for thought.

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