Robust Multi-User Analog Beamforming in mmWave MIMO Systems

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Abstract—**In this paper, we propose a robust analog-only beamforming scheme for the downlink multi-user systems, which not only suppresses the interference and enhances the beamforming gain, but also provides robustness against imperfect channel state information (CSI). We strike a balance between the average beamforming gain and the inter-user interference by formulating a multi-objective problem. A probabilistic objective of leakage interference power is formulated to alleviate the effects of the channel estimation and feedback quantization errors. To solve the problem, we first use the sum-weighted method to transform the multi-objective problem into a single-objective problem. Then, we use the semi-definite programing technique to make the constant-magnitude constraints of the analog beamforming tractable. Simulation results show that our proposed robust beamformer can provide up to 120% improvement in the sum-rate compared to the beam selection method.**

I. Introduction

Millimeter wave (mmWave) communication has been considered as a key technology for future wireless communication systems. However, mmWave carrier frequencies suffer from relatively severe propagation losses, which reduce service coverage and impair communication performance [1]. Thus, large antenna arrays are usually proposed to be implemented at both transmitters and receivers to provide sufficient beamforming gain to mitigate the severe propagation attenuation [2]. To further improve the system throughput, multi-user systems, where a base station (BS) simultaneously serves a number of mobile stations (MS), are often adopted. To cancel the interference among MS, some form of precoding is usually applied at the BS. For conventional multi-user systems, precoding is commonly done at the baseband, where each antenna element has a radio frequency (RF) chain. This kind of precoding is called the fully digital beamforming schemes [3]. However, in the multi-user mmWave multiple-input and multiple-output (MIMO) systems, the fully digital beamforming schemes are not practical for large antenna arrays in mmWave systems due to the high complexity and the large power consumption.

To address the difficulty of the limited number of RF chains in multi-user mmWave MIMO systems, two approaches have been proposed. One is the hybrid multi-user beamforming, in which the beamformer is constructed by the concatenation of a low-dimensional baseband (digital) beamformer and an RF (analog) beamformer [4]–[6]. This method can achieve a performance close to a conventional digital beamformer [4], [5]. However, a two-stage feedback for both the RF beamforming and the baseband beamforming is needed, which requires a tremendous overhead for large antenna arrays. This may become a limitation for mmWave MIMO systems and should be avoided if possible.

The other approach is the analog-only multi-user beamforming, where the beamforming processing is only performed with RF analog components. In analog beamforming [7]–[9], both transmitter and receiver are equipped with an analog beamforming codebook, e.g. phase shifts. In [10]–[12], an analog beam selection method for mmWave multi-user systems was proposed. The BS chooses the best beamforming vector, which maximizes the beamforming gain, from the codebook. This method performs well for line-of-sight (LOS) channels. However, considering a multi-user system using non-LOS (NLOS) channel models, the performance of the beam selection method will be degraded due to the interference of different paths and different users. Besides, the beam selection method needs a training stage to find the best beam, whose overhead scales linearly with the number of users.

Channel information is also critical for mmWave MIMO systems. Imperfect channel state information (CSI) will lead to severe performance degradation. Some papers, such as [13], [14] and [15], analyzed the performance of the imperfect CSI and proposed communication schemes for imperfect CSI in traditional MIMO systems. However, to the best of our knowledge, there is no robust communication design for mmWave MIMO systems in the literature.

In this paper, we propose a robust design for the analog beamforming, which not only suppresses the interference and enhances the beamforming gain, but also provides robustness against imperfect CSI. To reduce the feedback overhead, we only use the angle of departures/angle of arrivals (AoD/AoA) of the channel instead of the full channel information. Then, We assume there exist estimated errors in the AoD/AoA and simplify the error model into an additive error model by using Taylor expansion method. Based on the statistical properties of the errors, a probabilistic objective similar to [16]–[18] is formulated. We maximize the average beamforming gain while keeping the probability of small leakage power as large as possible (i.e., we formulate a multi-objective problem (MOP) to maximize the average array gain and the probability of small leakage power at the same time). The probabilistic objective is transformed into a deterministic one by applying Markov's inequality. We then use the sum-weighted method to transform the MOP to a single-objective problem (SOP) and introduce the semidefinite programming (SDP) technique to deal with the constant-magnitude constraints for the analog beamforming. By using modern convex optimization algorithms, we efficiently solve the optimization problem.

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II. System model

A. System model

We consider a multi-user system including a BS with N_t antennas serving K single-antenna users. The BS is equipped with K RF chains to enable the multi-user transmission. Only analog beamforming is used for each user. The BS generates the analog beamforming vector for User i based on the estimated multi-path angles of the channels. We denote s_i as the transmitted symbol intended for User i with $E[\|s_i\|^2]=1$ and $\boldsymbol{w}_i \in \mathbb{C}^{N_t\times 1}$ as the beamforming vector for s_i. The channel between User i and the BS is denoted by $\boldsymbol{h}_i^H \in \mathbb{C}^{1\times N_t}$. The operator H represents the Hermitian transpose. The received signal at User i can be expressed as

$$y_i = \boldsymbol{h}_i^H \boldsymbol{w}_i s_i + \sum_{k=1,k\neq i}^{K} \boldsymbol{h}_i^H \boldsymbol{w}_k s_k + n_i, \tag{1}$$

where n_i is the additive Gaussian noise with zero mean and σ^2 variance. The second term is the co-channel interference (CCI) caused by other users. How to cancel the CCI is a big issue in multi-user systems. Many papers such as [3] and [19] proposed a zero-forcing scheme to cancel the CCI, which enforces the beamforming vectors lie in the null space of the interference channels. However, this method is not optimal for SINR and has great performance degradation for imperfect CSI. To remedy these issues, we propose a robust analog beamforming scheme, which not only suppresses the interference and enhances the beamforming gain, but also provides robustness against imperfect CSI.

In the next section, we will introduce the mmWave channel model. Based on the mmWave channel model, we will formulate our robust analog multi-user beamforming problem for mmWave systems.

B. Channel model

MmWave channels are expected to have limited scattering characteristic [20], which means the assumption of a rich scattering environment becomes invalid. This is called sparsity in the literature and leads to the unreliability of traditional channel models. To characterize the limited scattering feature, we adopt the clustered mmWave channel model in [21] and [22] with L_i scatterers for the channel of User i. Each scatterer is assumed to contribute a single propagation path between the BS and the user. For our single-antenna user system, the channel is modeled as a vector described by

$$\boldsymbol{h}_i^H = \sqrt{\frac{N_t}{L}} \sum_{l=1}^{L} (a_l^i)^* \boldsymbol{\alpha}_t(\theta_l^i)^H, \tag{2}$$

where $\boldsymbol{\alpha}_t(\theta_l^i)$ is the antenna array response vector of the BS for path l with departure angle θ_l^i. Parameter $(a_l^i)^*$ is the complex path gain of path l modeled by a complex Gaussian distribution such as $\mathcal{CN}(0,1)$. While the algorithms and results in this paper can be applied to arbitrary antenna arrays, we use uniform linear arrays (ULAs) in the simulations for simplicity. The array response vectors take the following form

$$\boldsymbol{\alpha}_t(\theta_l^i) = \frac{1}{\sqrt{N_t}}[1, e^{j\frac{2\pi}{\lambda}d\sin(\theta_l^i)}, ..., e^{j(N_t-1)\frac{2\pi}{\lambda}d\sin(\theta_l^i)}], \tag{3}$$

where λ is the signal wavelength, and d is the distance between antenna elements. The departure angle θ_l^i is assumed to have a uniform distribution over $[0,2\pi]$.

To simplify the expression of the channels, we denote

$$\boldsymbol{h}_i^H = \tilde{\boldsymbol{h}}_i^H \mathbf{A}_i^H, \tag{4}$$

where $\mathbf{A}_i = [\boldsymbol{\alpha}_t(\theta_1^i), \boldsymbol{\alpha}_t(\theta_2^i), ..., \boldsymbol{\alpha}_t(\theta_L^i)] \in \mathbb{C}^{N_t\times L_i}$ contains all the array response vectors from the BS to User i and $\tilde{\boldsymbol{h}}_i = [a_1^i, a_2^i, ..., a_L^i]^T \in \mathbb{C}^{L_i\times 1}$ contains the complex gain of all the paths from the BS to User i.

We call $\mathbf{A}_i$ the AoD matrix of User i. In fact, to estimate the mmWave channels, we need to estimate the AoD matrix and the complex gains. Reference [23] proposed a CAPON-based method to estimate the AoD/AoA for MIMO radar systems. The method in [23], which relies on several time samples, can apply to MISO systems, like the ones in our paper, as well.

In this paper, to further reduce the feedback overhead, we assume the BS only knows the AoD of the channels (i.e., the BS only knows $\mathbf{A}_i$).

To cancel the CCI, we need to minimize the effect from User i to other users, which is called the leakage interference. We define the leakage interference matrix of User i as

$$\tilde{\mathbf{I}}_i = [\mathbf{A}_1, ..., \mathbf{A}_{i-1}, \mathbf{A}_{i+1}, ..., \mathbf{A}_K]^H, \tag{5}$$

where $\tilde{\mathbf{I}}_i \in \mathbb{C}^{\sum_{k=1,k\neq i}^{K} L_k \times N_t}$ is a matrix that contains the AoD matrices from the BS to all users except User i. In the following section, we will minimize the leakage power of User i based on this leakage interference matrix.

III. Robust Beamforming

To design the robust beamforming scheme, we first need to model the estimation errors. In the clustered mmWave channel model, the errors cannot be simply modeled as the additive estimation errors, since the estimated angle errors appear in the index of the exponential function in the array response vectors. Therefore, we need to simplify the error model before designing a robust beamforming scheme.

A. Error model

We assume that for the angle θ_l^i of the l^{th} path, there exists an angle estimation/quantization error $\Delta\theta_l^i$ with mean 0 and variance τ_l^i. A Gaussian distribution $\mathcal{N}(0,\tau_l^i)$ is a reasonable assumption, although we only use the first and second order statistics and do not need the distribution. Then, the array response vector with error $\Delta\theta_l^i$ can be expressed as

$$\boldsymbol{\alpha}(\theta_l^i + \Delta\theta_l^i) = \frac{1}{\sqrt{N_t}}[1, e^{j\frac{2\pi}{\lambda}d sin(\theta_l^i+\Delta\theta_l^i)}, ..., e^{j(N_t-1)\frac{2\pi}{\lambda}d sin(\theta_l^i+\Delta\theta_l^i)}]^T. \tag{6}$$

To extract the error out of the exponential function in (6), we expand the exponential function using the first-order Taylor expansion. To simplify the expression, we denote $\frac{2\pi}{\lambda}d$ as κ. Each element in Eq. (6) can be expanded as

$$e^{jn\kappa\sin(\theta_l^i+\Delta\theta_l^i)} \approx e^{jn\kappa\sin(\theta_l^i)} + jn\kappa\cos(\theta_l^i)\Delta\theta_l^i e^{jn\kappa\sin(\theta_l^i)}. \tag{7}$$

We denote $e_l^{i,n}$ as $jn\kappa\cos(\theta_l^i)\Delta\theta_l^i e^{jn\kappa(\sin(\theta_l^i)}$, which represents the error for the n^{th} element in the response vector of the l^{th} path of User i. Then, we define the error vector

$\boldsymbol{e}_l^i \triangleq \frac{1}{\sqrt{N_t}}[e_l^{i,0}, e_l^{i,1}, ..., e_l^{i,N_t-1}]^T$ as the error for the l^{th} path of User i, we now simplify the errors in the AoD into an additive random error as

$$\tilde{\boldsymbol{\alpha}}(\theta_l^i) = \boldsymbol{\alpha}(\theta_l^i + \Delta\theta_l^i) \approx \boldsymbol{\alpha}(\theta_l^i) + \boldsymbol{e}_l^i. \tag{8}$$

Based on the mean and the variance of $\Delta\theta_l^i$, vector $\boldsymbol{e}_l^i$ has zero mean and the covariance matrix $\mathbf{C}_l^i$ can be calculated as

$$\mathbf{C}_l^i = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & (\kappa\cos(\theta_l^i)\tau_l^i)^2 & \dots & (N_t-1)\kappa^2 cos^2(\theta_l^i)(\tau_l^i)^2 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & (N_t-1)\kappa^2 cos^2(\theta_l^i)(\tau_l^i)^2 & \dots & (N_t-1)^2\kappa^2 cos^2(\theta_l^i)(\tau_l^i)^2 \end{bmatrix}. \tag{9}$$

Note that the first row and the first column of $\mathbf{C}_l^i$ are all zeros. This is because the first element of the array response vector (6) is always 1, which is independent of error. In other words, the first element of Eq. (6) is deterministic and this leads to the zeros in the first row and the first column of $\mathbf{C}_l^i$.

Since we have simplified the AoD error of each path for each user into an additive error, we can further model the errors for the whole AoD matrix as an additive error. Denoting the presumed AoD matrix of User i as $\mathbf{A}_i^p$, the AoD matrix of User i with errors can be modeled as

$$\mathbf{A}_i = \mathbf{A}_i^p + \mathbf{E}_i, \tag{10}$$

where $\mathbf{E}_i = [\boldsymbol{e}_1^i, \boldsymbol{e}_2^i, ..., \boldsymbol{e}_L^i] \in \mathbb{C}^{N_t \times L}$ is a matrix that contains all the error vectors for User i. We assume the errors of different paths and users are independent. Therefore, the covariance matrix of $\mathbf{E}_i$ is

$$\mathbf{C}_i = \sum_{l=1}^{L_i} \mathbf{C}_l^i. \tag{11}$$

The imperfect leakage interference matrix of User i could also be modeled in the same way as the imperfect AoD matrix. We denote the presumed leakage interference matrix of User i as $\tilde{\mathbf{I}}_i^p = [\mathbf{A}_1^p, ..., \mathbf{A}_{i-1}^p, \mathbf{A}_{i+1}^p, ..., \mathbf{A}_K^p]^T$. The imperfect leakage interference matrix of User i with errors can be modeled as

$$\tilde{\mathbf{I}}_i = \tilde{\mathbf{I}}_i^p + \tilde{\mathbf{E}}_i, \tag{12}$$

where $\tilde{\mathbf{E}}_i = [\mathbf{E}_1, ..., \mathbf{E}_{i-1}, \mathbf{E}_{i+1}, ..., \mathbf{E}_K]^T \in \mathbb{C}^{\sum_{k\neq i}^K L_k \times N_t}$ is a matrix that contains all the error matrices for all the users except User i. We assume the errors of different users are independent. Therefore, the covariance matrix of $\tilde{\mathbf{E}}_i$ is

$$\tilde{\mathbf{C}}_i = \sum_{k\neq i}^{K} \mathbf{C}_k. \tag{13}$$

Now, we have simplified both the errors in the AoD matrix and the leakage interference matrix into the additive error. Based on this error model, we will propose a robust beamforming scheme to confront the uncertainty in the channel information.

B. Robust beamforming

The leakage interference matrix is random due to the uncertainty of errors. To deal with this problem, we use a probabilistic approach to restrict the leakage interference (i.e., we maximize the outage probability). The small interference leakage probability can be expressed as

$$P_{leakage} = Pr\{\boldsymbol{w}_i^H \tilde{\mathbf{I}}_i^H \tilde{\mathbf{I}}_i \boldsymbol{w}_i \leq \gamma_i\}, \tag{14}$$

where γ_i denotes a pre-specified leakage power level. Besides the leakage power, we also want to maximize the average beamforming gain of User i, which is defined as

$$BG_{avg} = E[\boldsymbol{w}_i^H \mathbf{A}_i \mathbf{A}_i^H \boldsymbol{w}_i]. \tag{15}$$

Based on (14) and (15), a multi-objective optimization problem is constructed as

$$\begin{aligned} &\boldsymbol{w}_i^{opt} = \text{argmax } \{E[\boldsymbol{w}_i^H \mathbf{A}_i \mathbf{A}_i^H \boldsymbol{w}_i], Pr\{\boldsymbol{w}_i^H \tilde{\mathbf{I}}_i^H \tilde{\mathbf{I}}_i \boldsymbol{w}_i \leq \gamma_i\}\} \\ &s.t. \ \boldsymbol{w}_i \in \mathcal{W}, \end{aligned} \tag{16}$$

where $\mathcal{W}$ is the set of all constant-magnitude vectors with each element having a magnitude of $1/\sqrt{N_t}$. Problem (16) is an MOP with a constant-magnitude constraint and a probabilistic objective function. We first use Markov's inequality to transform the probabilistic objective into the expectation objective. Then, we use the sum-weighted method and the SDP to deal with the multi-objective and constant-magnitude constraint, respectively. Based on the Markov's inequality, the probabilistic objective can be simplified as

$$\begin{aligned} Pr\{\boldsymbol{w}_i^H \tilde{\mathbf{I}}_i^H \tilde{\mathbf{I}}_i \boldsymbol{w}_i \leq \gamma_i\} &= Pr\{\boldsymbol{w}_i^H (\tilde{\mathbf{I}}_i^p + \tilde{\mathbf{E}}_i)^H (\tilde{\mathbf{I}}_i^p + \tilde{\mathbf{E}}_i) \boldsymbol{w}_i \leq \gamma_i\} && \text{(17a)} \\ &\geq 1 - \frac{E[\boldsymbol{w}_i^H (\tilde{\mathbf{I}}_i^p + \tilde{\mathbf{E}}_i)^H (\tilde{\mathbf{I}}_i^p + \tilde{\mathbf{E}}_i) \boldsymbol{w}_i]}{\gamma_i} && \text{(17b)} \\ &= 1 - \frac{trace(((\tilde{\mathbf{I}}_i^p)^H \tilde{\mathbf{I}}_i^p + \tilde{\mathbf{C}}_i)\mathbf{W})}{\gamma_i}. && \text{(17c)} \end{aligned}$$

Matrix $\mathbf{W} = \boldsymbol{w}_i \boldsymbol{w}_i^H$ is a symmetric semi-definite matrix with rank 1.

The average beamforming gain for User i is an expectation over the instant beamforming gain, which is not easy to deal with. To make the problem tractable, we perform some algebraic transformation and convert it into a deterministic and convex function of $\mathbf{W}$ as below.

$$\begin{aligned} E[\boldsymbol{w}_i^H \mathbf{A}_i \mathbf{A}_i^H \boldsymbol{w}_i] &= E[\boldsymbol{w}_i^H (\mathbf{A}_i^p + \mathbf{E}_i)(\mathbf{A}_i^p + \mathbf{E}_i)^H \boldsymbol{w}_i] && \text{(18a)} \\ &= trace((\mathbf{A}_i^p (\mathbf{A}_i^p)^H + \mathbf{C}_i)\mathbf{W}). && \text{(18b)} \end{aligned}$$

The introduction of matrix $\mathbf{W}$ will transform the non-convex constraints on $\boldsymbol{w}_i$ into

$$\mathbf{W}_{ii} = \frac{1}{N_t}, \ \forall i = 1, ..., N_t, \tag{19}$$

where $\mathbf{W}_{ii}$ represents the i^{th} diagonal element in $\mathbf{W}$. These constraints are convex constraints and are easy to deal with.

Based on the above three simplifications, using the sum-weighted method, we can reformulate Problem (16) into an SDP with rank-1 constraint. To deal with the rank-1 constraint, we introduce the semidefinite programming relaxation (SDR) [24] by dropping the rank constraint. Therefore, an upper bound can be achieved by solving the following Problem (20)

$$\begin{aligned}&\mathbf{W}^{opt} = \text{argmax}\{\lambda_1 trace((\mathbf{A}_i^p(\mathbf{A}_i^p)^H + \mathbf{C}_i)\mathbf{W}) + \\ &\lambda_2\left(1 - trace(((\tilde{\mathbf{I}}_i^p)^H\tilde{\mathbf{I}}_i^p + \tilde{\mathbf{C}}_i)\mathbf{W})/\gamma_i\right)\} \\ &s.t.\ \mathbf{W}_{ii} = \frac{1}{N_t},\ \forall i = 1,...,N_t; \\ &\quad \mathbf{W} \succeq 0.\end{aligned} \tag{20}$$

In Problem (20), parameter λ_i represents the importance of the i^{th} component in the cost function and $\lambda_1 + \lambda_2 = 1$. Different values for $\lambda_i's$ will result in different solutions to the problem. We will evaluate the performance under different values of $\lambda_i's$ by simulation.

The optimal solution $\mathbf{W}^{opt}$ can be found by standard tools of mathematical programming [25]. Note that Problem (20) is the relaxed version of Problem (16), which means we cannot guarantee a rank 1 for $\mathbf{W}^{opt}$. When the rank of $\mathbf{W}^{opt}$ is larger than 1, we cannot recover $\boldsymbol{w}_i^{opt}$ from $\mathbf{W}^{opt}$ straightforwardly. In such cases, we will use the randomization technique in [26] to make an approximation.

IV. Simulation Results

In this section, we evaluate the performance of the robust beamforming method. Note that our objective in this paper is not to optimize the sum-rate due to the intractability of doing so. In fact, we strike a balance between maximizing the beamforming gain and minimizing the inter-user interference. Since λ_1 and λ_2 represent the importance of each term in the objective function of the MOP, we expect to find the best balance by searching over different values of $\lambda_i's$. Therefore, we pick the combination of λ_1 and λ_2 that achieves the highest sum-rate. We also compare our multi-user analog beamforming with the beam selection method and the traditional ZF beamforming method.

In the simulation, we consider a multi-user MIMO system consisting of one BS equipped with a large antenna array and K single-antenna users. The channels are realized using Eq. (2). Due to the limited scattering characteristic of the mmWave channels, the number of paths should be small. Here, we assume each channel has $L = 6$ paths. The large antenna array at the BS is assumed to have $N_t = 64$ antennas, which is the same antenna configuration in [27]. We assume the total number of users $K = 6$. The θ_l^i of each path is assumed to be uniformly distributed in $[0, 2\pi]$. The results are averaged over $20,000$ channel realizations. The variance of AWGN noise per user is assumed to be the same for all users, i.e. $\sigma_1^2 = ... = \sigma_K^2 = \sigma^2$.

Fig. 1 illustrates the sum-rate of our analog beamforming under different λ_1 and λ_2 values, respectively. We, in general, evaluate 21 combinations of λ_1 and λ_2. To be specific, λ_2 ranges from 0 to 1 with step-size 0.05 and $\lambda_1 = 1 - \lambda_2$. In Fig. 1, as λ_2 increases from 0 to 1, the sum-rate first increases and then decreases. We set $SNR = 25dB$. In fact, the results in Fig. 1 can apply to all range of SNR values, because the trends of the beamforming gain and the leakage interference versus λ_1 are not affected by the power of noise. We will set $\lambda_1 = 0.9$ and $\lambda_2 = 0.1$ for our follow-up simulations.

For the robust beamforming, we evaluate the sum-rate performance when the error variance $\tau_1^1 = \tau_2^1 = ... = \tau_{L_K}^K = \tau = 0.005$. The leakage power level is set to be $\gamma_i = 0.1, \forall i = 1,...,K$. We compare the performance of the proposed robust

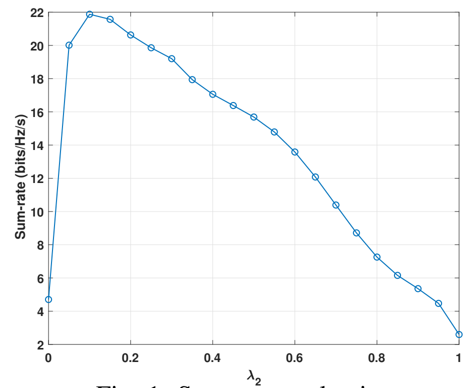

Fig. 1: Sum-rate evaluation

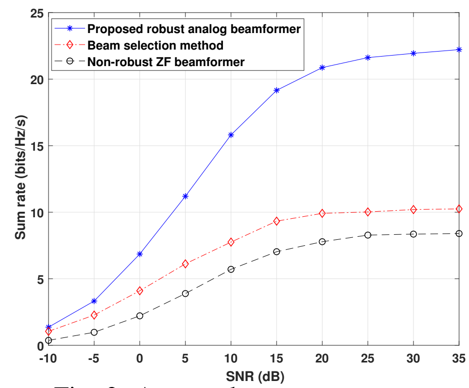

Fig. 2: Averaged sum-rate per user

analog beamfomer, the non-robust digital ZF beamformer, and the beam selection method. The beam selection methods in [10]–[12] use implicit channel information, for which we cannot apply the additive estimation error model. Therefore, to evaluate the robustness of the beam selection methods, we assume there exists an error in the beam alignment angle and this error has the same statistical characteristic as the error in AoDs.

Fig. 2 plots the averaged sum-rate per user of the three beamforming methods when SNR ranges from -10dB to 35dB with $\tau = 0.005$. The proposed robust analog beamformer outperforms both the beam selection method and non-robust ZF beamformer at every SNR. When SNR is 30dB, the proposed beamformer provides an improvement of 120% and 175% of the averaged sum-rate with respect to that of the beam selection method and beamformer, respectively.

V. Conclusion

In this paper, we proposed a robust analog beamforming scheme which not only strikes a balance between the beamforming gain and the inter-user interference, but also provides robustness against imperfect CSI. We formulated an MOP with probabilistic objectives to optimize the beamforming gain and the interference at the same time. The sum-weighted method was used to transform the MOP into an SOP and the SDP was adopted to make the constant magnitude constraints of analog beamforming tractable. The simulation results demonstrated the highest robustness of our beamforming scheme against channel errors.

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