Persistification of Robotic Tasks using Control Barrier Functions*

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Abstract— **In this paper we consider the problem of rendering robotic tasks persistent by ensuring that the robots' energy levels are never depleted, which means that the tasks can be executed over long time horizons. This process is referred to as the *persistification* of the task. In particular, the state of each robot is augmented with its battery level so that the desired persistent behavior can be encoded as the forward invariance of a set such that the robots never deplete their batteries. Control barrier functions are employed to synthesize controllers that ensure that this set is forward invariant and, therefore, that the robotic task is persistent. As an application, this paper considers the persistification of a robotic sensor coverage task in which a group of robots has to cover an area of interest. The successful persistification of the coverage task is shown in simulation and on a team of mobile robots.**

I. Introduction

Many robotic tasks occur over long timescales during which the robots are deployed in an environment of interest to perform a desired task. These tasks include environmental monitoring [1], [2], exploration [3], [4], and sensor coverage [5], [6]. As the duration of a task increases, beyond the time a robot can operate on a single charge of a battery, energy consumption determines the successful execution of the task. Furthermore when a long duration task is performed by autonomous robots, the control algorithms need to strike a balance between executing the task and energy consumption.

Robotic task persistence has been addressed in a number of particular contexts [7]–[14], which will be discussed in Section II, by explicitly incorporating persistency into the controller design. In contrast, this paper presents a framework that can be used to render a generic robotic task persistent, allowing persistent behavior to be applied to a variety of tasks. The efficacy of the framework is shown by the persistification of a sensor coverage task. Sensor coverage is aimed at sensing environments that in application can evolve over long time horizons, making it an ideal candidate for persistification.

The proposed persistification strategy entails that the robots follow a given control input as closely as possible, while being constrained by the condition that sufficient energy is always available for them to return to a charging station and recharge their batteries. This constraint corresponds to ensuring that the set of positions and energy levels from which the robots can never get stranded with a depleted battery is rendered forward invariant. The forward invariance of this set ensures that, if the robots start with enough energy and sufficiently close to a charging station, their energy will never go below a minimum value. Forward invariance of sets is ensured by leveraging the recently developed Control Barrier Functions (CBFs) [15]–[18]. The CBFs are used to encode constraints for the robots that are enforced at each point in time by means of an optimization-based controller, which minimizes the difference between the inputs applied to the robots and a nominal input subject to the CBF-based constraint.

The rest of the paper is organized as follows: Section II discusses past work related to persistent behaviors, Section III presents the high level strategy to make a task persistent, which is then implemented to ensure persistent coverage in Section IV. Section V shows simulation and experimental results for the implementation of the persistent coverage behavior on a team of differential-drive mobile robots.

II. Related Work

In the literature, the concept of robotic task persistency has been interpreted in different ways: [7] considers a sensing task, where persistence is achieved if a changing environment does not change too much without being sensed by a robot. In [8] the robots behave persistently if they periodically revisit a discrete set of sites that have to be monitored. The work presented in this paper focuses on persistence with regard to the energy level of the robots, i.e. the robots performing a task never run out of energy.

In this paper, the persistification strategy is applied to the coverage control task which results in an energy-aware implementation of coverage control. Sensor coverage is a canonical example of a task that has to take place over long time horizons and as such energy-aware coverage control has been studied in [9]–[11]. In [9], the energy level of the robot is incorporated directly into a coverage cost that is to be minimized to cover a certain area; gradient descent is used to generate a motion control law for the robots. In [10], sensing robots switch, in a continuous fashion and depending on the current energy level, between the goal point defined by the coverage task and the docking station locations. The persistification strategy presented in this paper generalizes this approach and allows its application to different kinds of tasks. In [11], a battery aware coverage strategy is developed that takes into account how to return to a charging station in such a way that the area covered while traveling is maximized.

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Persistent robotic tasks have also been considered in application areas other than sensor coverage. In [12] a heuristic algorithm for the vehicle routing problem is proposed, which takes into account detours that pass through refueling stations. Limited energy reserves and finite recharge times are used as additional constraints in [19] for a path planning strategy for optimal deployment of multi-robot teams.

A different way of addressing the persistification of robotic tasks is the deployment of multiple types of robots, as in [20], where a mixed integer quadratic program is designed to coordinate the interaction of "task robots", which execute tasks, and "delivery robots", which provide the task robots with required energy resources. Similarly, [13] considers a team of aerial robots which is to be refueled by a group of ground mobile docking stations. The trajectories of the docking stations are planned based on the aerial robots' trajectories in order to guarantee recharge without suspending the operation of the aerial robots.

While the existing persistification strategies are designed to address robotic tasks with specific robots and environments, in this paper we present a general persistification strategy that can be applied to a large variety of robotic tasks.

III. Persitification of Robotic Tasks

A. System Model

The robotic task which will undergo persistification is assumed to be performed by a collection of N mobile robots where robot i has a state $x_i \in \mathbb{R}^n$ and each robot is modeled by a control affine nonlinear model:

$$\dot{x}_i = f(x_i) + g(x_i)u_i,$$

where $u_i \in U \subseteq \mathbb{R}^m$ is the input to robot i. The functions f and g are assumed to be locally Lipschitz continuous.[1] As the robotic task is to be made persistent with respect to energy consumption, the model of the robots has to describe the dynamics of the robots' battery as well. Therefore, the augmented state of each robot is defined as $\chi_i = \begin{bmatrix} x_i, & E_i \end{bmatrix}^{\mathrm{T}}$, where $E_i \in \mathbb{R}_+$ is the robot energy level. Correspondingly, the robot dynamics are augmented with the discharging dynamics of the battery, leading to the new dynamic model:

$$\dot{\chi}_i = F(\chi_i) + G(\chi_i)u_i, \tag{1}$$

where $F(\chi_i) = \begin{bmatrix} f(x_i), & \hat{f}(\chi_i) \end{bmatrix}^{\mathrm{T}}$ and $G(\chi_i) = \begin{bmatrix} g(x_i), & \hat{g}(\chi_i) \end{bmatrix}^{\mathrm{T}}$. The functions $\hat{f}$ and $\hat{g}$ are also assumed to be locally Lipschitz continuous [21].

The final element required for the persistification of robotic tasks is the ability for the robots to charge. For this we map the robots' state to a position that can be related to the position of charging stations. Let $\pi : x_i \in \mathbb{R}^n \mapsto p_i \in \mathbb{R}^d$ be the function that maps the robot state to the robot position, where $d = 2$ for planar robots and $d = 3$ for aerial robots. Let the closed set $P_{i,\text{charge}} \subset \mathbb{R}^d$ denote the *charging station*, i. e., the set of all positions p_i where robot i recharges its battery, and let $\partial P_{i,\text{charge}}$ be the boundary of $P_{i,\text{charge}}$. Note that no assumptions are made about the charging modality: it may correspond to swapping the battery, as in [14], or actually resting at a charging station until the battery is charged. With these preliminaries in place, the high level persistification strategy can be explained.

[1] Although the case of homogeneous robots is considered here, the resulting persistification framework can be extended to the heterogeneous case, in which robots can have different dynamic models.

B. Persitification Strategy

The goal of the persistification strategy is to allow the robots to execute a task while ensuring that their energy never goes below a minimum threshold, allowing the task to be perpetually executed. The task is assumed to be performed if robot i executes the Lipschitz continuous controller $u_{i,\text{nom}}$. The desired energy level is maintained by enforcing constraints formulated by control barrier functions (CBFs). As will be shown in Section III-C, the designed CBF describes a set in the robots' state space in which the robots are able to reach their charging station when they are in need of charging. In order to ensure the persistification of a task, this set has to be rendered forward invariant.

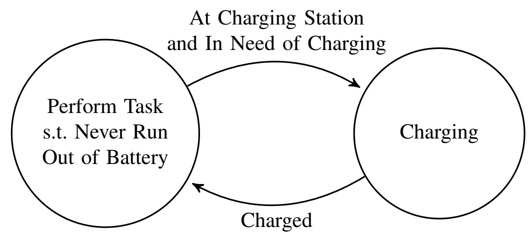


Fig. 1: Persistification framework: the robots switch between the execution of the desired task and the charging behavior

As soon as the robots reach their charging stations, they stop executing the task and start performing a charging operation. Once the robots are charged, they will return to following $u_{i,\text{nom}},\ i = 1 \dots N$ as closely as possible while being constrained by a CBF. This is achieved by executing the controller from the following Quadratic Program (QP):

$$\begin{aligned} u_i^* = \operatorname*{argmin}_u & \left\| u - u_{i,\text{nom}} \right\|^2 \\ \text{s.t. } & c_{i,\text{CBF}}\left(\chi_i\right) \geq 0. \end{aligned} \tag{2}$$

The constraint $c_{i,\text{CBF}}\left(\chi_i\right) \geq 0$ specifies a set of admissible inputs u_i that let robot i satisfy the energy constraints. Its definition will be discussed in the next section. The described behavior is summarized in Fig 1. Note that the charging behavior initiates when the robot i is both in need of charging and close enough to the charging station. This ensures that if a robot enters the charging station while pursuing the nominal task, it does not switch to the charging behavior if its energy level is high enough.

C. Ensuring Persistent Operation

We now introduce the CBF theory necessary to provide the desired forward invariance properties, i.e., that the battery levels never go below a desired minimum energy, $E_{\min} \geq 0$, while the robots are away from their charging station. CBFs are used to explicitly encode this behavior as a constraint that is enforced on each of the robots executing the task.

Following [17], define the set $\mathcal{C} \subset \mathbb{R}^{n+1}$ as the set where the state χ_i should be confined. Furthermore, assume that this set can be expressed as the superlevel set of a barrier function h, i. e. $\mathcal{C} = \left\{\chi_i \in \mathbb{R}^{n+1} \;\middle|\; h(\chi_i) \geq 0\right\}$, where $h : \mathbb{R}^{n+1} \to \mathbb{R}$ is a continuously differentiable function.

Definition 1. *(from [17]) Given the control affine system* (1) *together with the set $\mathcal{C} \subset \mathbb{R}^{n+1}$, the function h is a Zeroing Control Barrier Function (ZCBF) defined over a set $\mathcal{D}$ with $\mathcal{C} \subseteq \mathcal{D} \subset \mathbb{R}^{n+1}$ if there exists an extended class $\mathscr{K}$ function, α, such that for all $\chi_i \in \mathcal{D}$*

$$\sup_{u \in U} \left[L_F h(\chi_i) + L_G h(\chi_i) u_i + \alpha(h(\chi_i))\right] \geq 0,$$

where $L_F h(\chi_i)$ is the Lie derivative of $h(\chi_i)$ along $F(\chi_i)$ and $L_G h(\chi_i)$ is the Lie derivative of $h(\chi_i)$ along $G(\chi_i)$.

The following corollary gives us the conditions to ensure the forward invariance of a set by means of ZCBFs:

Corollary 1. *[17] Given a set $\mathcal{C} \subset \mathbb{R}^{n+1}$, if h is a Zeroing Control Barrier Function on $\mathcal{D}$, then any Lipschitz continuous controller $u_i(\chi_i) : \mathcal{D} \to U$ such that $L_F h(\chi_i) + L_G h(\chi_i) u_i(\chi_i) + \alpha(h(\chi_i)) \geq 0$ will render the set $\mathcal{C}$ forward invariant.*

We propose the candidate ZCBF $h_i(\chi_i) = E_i - E_{\text{min}} - \rho_i\left(\pi(x_i)\right)$ to allow for the persistification of the robotic task. The function $\rho_i\left(\pi(x_i)\right) : \mathbb{R}^d \to \mathbb{R}$, represents the energy required to reach the i-th charging station from the position $\pi(x_i)$. As such, the function $\rho_i\left(\pi(x_i)\right)$ is assumed to satisfy the following:

- $\rho_i\left(\pi(x_i)\right) \geq 0$,
- $\rho_i\left(\pi(x_i)\right) = 0$ only holds when $\pi(x_i) \in \partial P_{i,\text{charge}}$,
- $\frac{\partial \rho_i}{\partial \pi} \frac{\partial \pi}{\partial x_i} g(x_i) \neq \hat{g}(\chi_i)$.[2]

The following lemma considers the effect of the proposed ZCBF on the robot's battery life.

Lemma 1. *If $\chi_i \in \mathcal{C} = \left\{\chi_i \in \mathbb{R}^{n+1} \;\middle|\; h_i(\chi_i) \geq 0\right\}$, where $h_i(\chi_i)$ is given by the ZCBF:*

$$h_i(\chi_i) = E_i - E_{\mathit{min}} - \rho_i\left(\pi(x_i)\right), \tag{3}$$

then $E_i \geq E_{\mathit{min}}$, and, moreover, $E_i = E_{\mathit{min}}$ can only hold if $\pi(x_i) \in \partial P_{i,\mathit{charge}}$.

Proof. By the definition of the set $\mathcal{C}$, consider that $h_i(\chi_i) \geq 0$. Then, by the definition of $h_i(\chi_i)$, one has $E_i \geq E_{\text{min}} + \rho_i\left(\pi(x_i)\right) \geq E_{\text{min}}$, as $\rho_i\left(\pi(x_i)\right) \geq 0$. Further, the condition that $\rho_i\left(\pi(x_i)\right) = 0$ only holds when $\pi(x_i) \in \partial P_{i,\text{charge}}$ implies that $E_i = E_{\text{min}}$ can only hold when $\pi(x_i) \in \partial P_{i,\text{charge}}$. □

As the proposed ZCBF ensures the desired energy behavior of the robot, it must now be shown that it satisfies Definition 1 and therefore can be used to ensure forward invariance.

[2]If the ZCBF has relative degree greater than one, a similar condition involving higher-order derivatives of $\rho(x_i)$ can be derived.

Theorem 1. *The function $h_i(\chi_i) = E_i - E_{\mathit{min}} - \rho_i\left(\pi(x_i)\right)$ is a ZCBF when $U = \mathbb{R}^m$.*

Proof. With $U = \mathbb{R}^m$, if $L_G h_i(\chi_i) \neq 0$ then there exists a $u_i \in U$ that will ensure that $L_F h_i(\chi_i) + L_G h_i(\chi_i) u_i + \alpha(h_i(\chi_i)) \geq 0$ [22]. Given that:

$$\begin{aligned} L_G h_i(\chi_i) &= \frac{\partial h_i}{\partial \chi_i} G(\chi_i) = \frac{\partial h_i}{\partial E_i} \hat{g}(\chi_i) + \frac{\partial h_i}{\partial x_i} g(x_i) \\ &= \hat{g}(\chi_i) - \frac{\partial \rho_i}{\partial \pi} \frac{\partial \pi}{\partial x_i} g(x_i) \neq 0, \end{aligned}$$

where the inequality holds by the definition of ρ_i, the proposed ZCBF is valid. □

Now that the proposed ZCBF has been shown to be valid, it can be applied to the persistification of the desired task. In order to apply the ZCBF to the persistification of a robotic task, the optimization-based controller (2) is modified by introducing the expression of the constraint $c_{i,\text{CBF}}(\chi_i) \geq 0$, leading to the following quadratic program (QP):

$$\begin{aligned} u_i^* = &\operatorname*{argmin}_u \|u - u_{i,\text{nom}}\|^2 \\ &\text{s.t. } L_F h_i(\chi_i) + L_G h_i(\chi_i) u + \alpha(h_i(\chi_i)) \geq 0, \end{aligned} \tag{4}$$

which produces an input that is as close as possible to the nominal input, $u_{i,\text{nom}}$, while ensuring that the residual energy of each robot is always above the minimum threshold E_{min}. Fig. 2 shows the described strategy implemented in the framework introduced in Fig. 1.

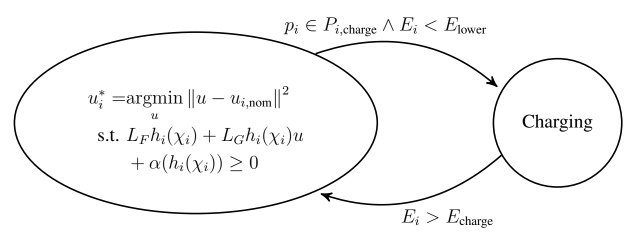


Fig. 2: Persistent task strategy: here E_{lower} is a threshold which captures the robots' need to be recharged

The QP (4) is solved by each robot leading to a decentralized task persistification. This means that, if the nominal input $u_{i,\text{nom}}$ does not require a centralized approach, solving for u_i^* can be done in a decentralized fashion.

To summarize, the persistification of a robotic task is realized as follows:

1) the robotic system model is augmented to take into account the robots' battery discharge dynamics
2) the set of positions and energy levels where the robots do not run out of battery is defined through the use of control barrier functions that allow the synthesis of constraints on the robots' inputs
3) a quadratic program is formulated to allow robots to perform the nominal task while satisfying the energy constraints.

IV. Persistent Coverage

In this section, the persistification strategy introduced in the previous sections is applied to a specific multi-robot task, that of sensor coverage. The persistent sensor coverage task will take place on the *Robotarium*, a remotely accessible swarm robotics research testbed [23], which is populated by differential-drive robots.

A. Robot dynamics

Each differential-drive robot is assumed to move in a planar environment with single integrator dynamics, i. e. $\dot{x}_i = u_i$, where $x_i \in \mathbb{R}^2$ is the position of robot i and $u_i \in \mathbb{R}^2$ is the velocity control input. Here the function $\pi(x_i)$ introduced in Section III-A is the identity function $\pi : x_i \mapsto x_i$, and its gradient becomes the 2×2-identity matrix I_2. Therefore, in the following, the symbols x_i, p_i and $\pi(x_i)$ will be used interchangeably.

The dynamic model used for the discharge of the robots' battery is the following:

$$\dot{E}_i = -K_{\text{d}}, \tag{5}$$

where $K_{\text{d}} > 0$ has been experimentally estimated such that the simulated dynamics represent the worst case scenario of the actual discharging behaviors. This way, the use of a linear approximation of the battery dynamics, introduced in (5), still allows the robots to reach their charging station before their stored energy goes below the minimum threshold E_{min} as prescribed by the constraint $c_{i,\text{CBF}}(\chi_i) \geq 0$.

The Robotarium has wireless charging stations and each robot i is assigned to a dedicated station located at $\hat{x}_i$. Therefore, $P_{i,\text{charge}}$ is defined as follows:

$$P_{i,\text{charge}} = \{\pi(x_i) \in \mathbb{R}^2 \mid \|x_i - \hat{x}_i\| \leq d_{\text{charge}}\}, \tag{6}$$

where d_{charge} is the maximum distance from the charging station at which the robots are able to charge.

B. Coverage Control

Let $D \subset \mathbb{R}^2$ be a closed and connected area of interest that has to be covered, and associate a density function $\phi : D \to \mathbb{R}_+$ with each point $q \in D$, which ranks how important the different points are through the value of $\phi(q)$. Given N planar mobile sensors located at $x_i \in D,\ i = 1, \ldots, N$, the following locational cost can be defined [5]:

$$C = \sum_{i=1}^{N} \int_{V_i(x)} \|x_i - q\|^2 \phi(q) dq, \tag{7}$$

where the measuring performance of robot i is assumed to decrease with the square of the distance $\|x_i - q\|$, and $V_i(x) = \{q \in D \mid \|x_i - q\| \leq \|x_j - q\|,\ \forall j \neq i\}$ is the Voronoi cell related to robot i. As shown in [5], given robots with single integrator dynamics, the locational cost C can be minimized using gradient descent employing the following closed-loop control law: $u_i = K(C_{V_i} - x_i)$, where C_{V_i} is the centroid of the Voronoi cell V_i and $K > 0$. By using Lloyd's algorithm [24], the robots will asymptotically converge to the Centroidal Voronoi Tessellation (CVT), the configuration in which each robot is in the centroid of its Voronoi cell, i. e., $x_i = C_{V_i},\ i = 1, \ldots, N$.

C. CBFs For Persistent Coverage

In order to implement the persistification strategy on the coverage control task as introduced in Section III, a function $\rho_i(\pi(x_i)) = \rho_i(x_i)$ which satisfies the conditions of Theorem 1 must be found. Consider the following candidate function:

$$\rho_i(x_i) = \frac{K_{\text{d}}}{k} \log \frac{\|x_i - \hat{x}_i\|}{d_{\text{charge}}}, \tag{8}$$

which is the energy required to drive robot i from position x_i to a distance d_{charge} from the i-th charging station located at $\hat{x}_i$. This has been evaluated assuming a proportional control law with proportional gain $k > 0$ and a robot model given by

$$\dot{\chi}_i = \begin{bmatrix} 0_{1\times 2}, & -K_{\text{d}} \end{bmatrix}^{\text{T}} + \begin{bmatrix} I_2, & 0_{2\times 1} \end{bmatrix}^{\text{T}} u_i,$$

where $0_{n\times m} \in \mathbb{R}^{n\times m}$ is a $n\times m$-zero matrix and $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix.

Theorem 2. *Given $P_{i,charge}$ defined in* (6)*, the function $\rho_i(x_i)$ defined in* (8) *satisfies the conditions of Theorem 1 when $\|x_i - \hat{x}_i\| \geq d_{charge}$.*

Proof. To satisfy the conditions of Theorem 1, it has to be shown that $\rho_i(x_i) \geq 0$, $\rho_i(x_i) = 0$ only holds when $x_i \in \partial P_{i,\text{charge}}$, and $\frac{\partial \rho_i}{\partial \pi} \frac{\partial \pi}{\partial x_i} g(x_i) \neq \hat{g}(\chi_i)$.

- $\log \dfrac{\|x_i - \hat{x}_i\|}{d_{\text{charge}}} \geq 0 \quad \forall\ x_i : \|x_i - \hat{x}_i\| \geq d_{\text{charge}}$ and therefore $\rho_i(x_i) \geq 0$.
- When $x_i \in \partial P_{i,\text{charge}}$, one has $\|x_i - \hat{x}_i\| = d_{\text{charge}}$, then $\log \dfrac{\|x_i - \hat{x}_i\|}{d_{\text{charge}}} = \rho_i(x_i) = 0$. Otherwise, for all x_i such that $\|x_i - \hat{x}_i\| > d_{\text{charge}}$, it holds that $\log \dfrac{\|x_i - \hat{x}_i\|}{d_{\text{charge}}} > 0$ and $\rho_i(x_i) > 0$.
- As $\frac{\partial \pi}{\partial x_i} = I_2$, $g(x_i) = I_2$ and $\hat{g}(\chi_i) = \begin{bmatrix} 0 & 0 \end{bmatrix}$, it is necessary to show that $\frac{\partial \rho_i}{\partial x_i} \neq \begin{bmatrix} 0 & 0 \end{bmatrix}$. As
$$\frac{\partial \rho_i}{\partial x_i} = \frac{K_{\text{d}}}{k} \left(\frac{x_i - \hat{x}_i}{\|x_i - \hat{x}_i\|^2} \right)^T$$
and $\|x_i - \hat{x}_i\| \geq d_{\text{charge}}$, it holds that $\frac{\partial \rho_i}{\partial x_i} \neq \begin{bmatrix} 0 & 0 \end{bmatrix}$.

□

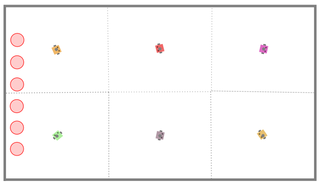
Now that the function $\rho_i(x_i)$ has been shown to satisfy the conditions of Theorem 1, a valid ZCBF based on (3) can be formulated. The ZCBF

$$h_i(\chi_i) = E_i - E_{\text{min}} - \frac{K_{\text{d}}}{k} \log \frac{\|x_i - \hat{x}_i\|}{d_{\text{charge}}} \tag{9}$$

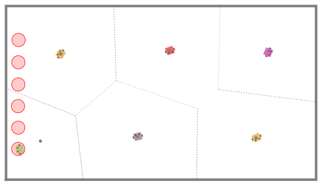
can be used to ensure that, at each point in time, robot i has sufficient stored energy to be able to reach the charging station at $\hat{x}_i$ before its energy E_i attains the value E_{min}. Once the robots are driven to the boundary of a charging station by the ZCBF, according to the strategy in Fig. 2, they will execute a simple docking maneuver realized by a traditional parking controller [25]. Once the battery is charged ($E_i \geq E_{\text{charge}} > E_{\text{min}}$), the robot resumes coverage control behavior.

V. Experiments

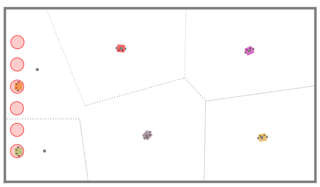
The persistent coverage task is implemented and tested both in simulation and on the Robotarium. First the persistification control strategy is implemented on the MATLAB simulator provided on the Robotarium website (www.robotarium.org). The Robotarium gives remote access to differential-drive robots that reside on an arena of 130×90 cm. This arena contains 9 wireless charging stations arranged along the left and right side of the testbed. In the simulation 6 robots are used to perform coverage of the testbed area and each robot is assigned to a designated charging station.



(a) All the robots follow the nominal inputs and do coverage control



(b) One robot is charging



(c) Two robots are charging

Fig. 3: Sequence of images from the Robotarium simulator. The colored squares represent the robots, the red circles depict the charging stations, whereas the thin lines show the boundaries of the Voronoi cells of the robots.

Fig. 3 shows a sequence of images taken from the Robotarium simulator. In Fig. 3a the coverage control task is executed by all the robots and $u_i = u_{i,\text{nom}}$ $i = 1, \ldots, 6$. Fig. 3b shows one of the robot that has been driven to the charging station by executing the QP (4) and is now charging. In Fig. 3c two robots are charging their batteries.

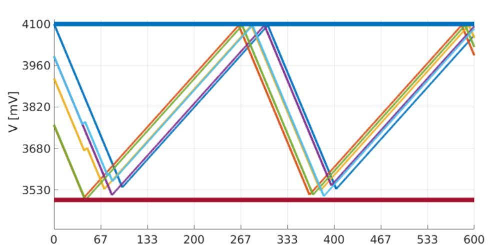
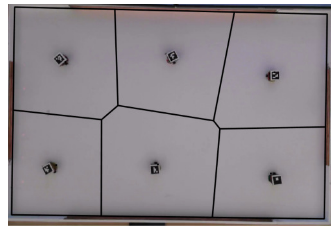


Fig. 4: Simulated battery voltage data of 6 robots collected during the simulation shown in Fig. 3. The red and blue horizontal lines depict the values of E_{min} and E_{charge}, respectively

The simulated battery values of the robots are shown in Fig. 4. As can be seen, every robot is able to reach the docking station and start charging before reaching the lower energy threshold E_{min}. The transition back to coverage control behavior is triggered as soon as the energy level goes above the threshold marked as E_{charge}.



(a) The robots perform coverage control

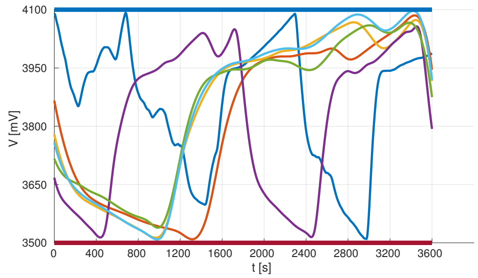
(b) One of the robots enters its corresponding set $P_{i,\text{charge}}$

(c) Two robots are charging

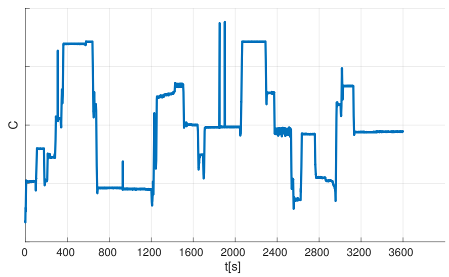
Fig. 5: Sequence of salient frames extracted from the video of the Robotarium experiment. The robots and the corresponding Voronoi cells are shown, the positions of the charging stations are similar to those in Fig. 3.

The code used for the simulation of the persistent coverage task has been submitted through the Robotarium web interface for execution on the Robotarium. Snapshots from the experiment are shown in Fig. 5. The robots are able to execute the desired behavior, covering the testbed area under the $u_{i,\text{nom}}$ input (Fig. 5a), and returning to the charging stations to keep their residual energy above E_{min} (Fig. 5b and Fig. 5c). Data collected from the experiment on the Robotarium are used to estimate the constant K_{d}, which is the maximum discharge rate that the robots experience over several discharging cycles. This value is required in the formulation of the CBFs (9). In Fig. 6, Fig. 6a shows the battery values of the robots on the Robotarium during the course of the persistent coverage experiment. Note that, as discussed in Section IV-A, the actual discharge behavior differs from the model (5) since it is only approximately linear. Nevertheless, the experiments confirm that the modeled discharge rate corresponds to the worst-case scenario and, therefore, the robots are able to reach their charging stations and start charging before their energy level go below the minimum value.

In order to evaluate the coverage performances during persistent operation, the locational cost C introduced in (7) is calculated. The definition of this cost assumes that the robot's performance in covering a point decreases proportionally to the square of the distance from that point. Fig. 6b shows the value of the cost C evaluated at each time instant during the experiment. The cost attains a minimum when the robots are discharging while performing coverage with no need to travel to the charging station. As one or more robots are charging the cost goes up, the robots being unable to adequately cover the testbed while sitting on the charging stations. The formulation of the persistification strategy as an optimization problem, as in (4), lets each robot execute a control input u_i^*



(a) Measured battery voltage data of 6 robots. The red and blue horizontal lines depict the values of E_{min} and E_{charge}, respectively



(b) Value of the locational cost (7): the cost is minimized when the majority of robots are away from a charging station

Fig. 6: Battery value and locational cost measured during the course of the Robotarium experiment shown in Fig. 5

that is as close as possible to the coverage control input, $u_{i,\text{nom}}$, while satisfying the energy constraints. In order to improve the performance, a set of "standby" robots could be employed to replace robots that are charging, as in [10].

The experiments show that the forward invariance property introduced in Subsection III-C allowed the robots to continuously operate during a one-hour experiment, which included at least one 10-minute recharging cycle for each robot.

With the implementation of the presented persistification strategy, one is able to achieve the best task performances as long as the energy constraints allow it, and to sacrifice the task only when there is no other way of keeping the energy level above a minimum desired threshold.

VI. Conclusions

In this paper we introduce a control strategy that renders robotic tasks persistent. *Persistification* is achieved by ensuring that the robots never run out of battery while executing the desired task. The definition of this objective by means of control barrier functions (CBFs) allows the formulation of the persistification strategy as an optimization problem.

The presented persistification strategy has been tested both in simulation and on a remotely accessible swarm robotics testbed. During the experiments, a group of robots performed persistent coverage of an environment, subject to the constraint that their energy level is confined above a minimum threshold. While the task considered in this paper is a persistent coverage task, it is important to note that the implementation of the framework presented in this paper is not task specific. Consequently, any robotic task can be made persistent via the application of the optimization-based control presented in this paper, allowing a wide range of behaviors to be performed over long periods of time.

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