Scheduling Post-disaster Repairs in Electricity Distribution Networks with Uncertain Repair Times

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ABSTRACT

Natural disasters, such as hurricanes, large wind and ice storms, typically require the repair of a large number of components in electricity distribution networks. Since power cannot be restored before the completion of repairs, optimally scheduling the available crews to minimize the cumulative duration of the customer interruptions reduces the harm done to the affected community. We have previously proposed approximation algorithms to schedule post-disaster repairs in electricity distribution networks with complete damage information [1]. In this paper, we extend our previous work to the case with incomplete damage information. We model this problem as scheduling a set of jobs with stochastic processing times on parallel identical machines in order to minimize the total weighted energization time. A linear programming (LP) based list scheduling policy is proposed and then analyzed in terms of theoretical performance.

1. INTRODUCTION

Natural disasters have caused major damage to the electricity distribution networks and deprived homes and businesses of electricity for prolonged periods. The resulting damages may also have secondary economic and environmental impact due to unpreparedness for such severe events. Examples include Hurricane Harvey in August 2017 [2, 3], the Christchurch Earthquake in February 2011 [4] and the June 2012 Mid-Atlantic and Midwest Derecho [5]. The recent Hurrican Harvey affected 2.02 million customers and over 6200 distribution poles were downed or damaged [2]. Physical damage to grid components must be repaired before power can be restored [6, 7]. From an operational perspective, approaches to scheduling the available repair crews to minimize the cumulative weighted customer downtime and reduce the harm done to the affected communities have been proposed in [1, 8, 9]. In particular, given the complete damage information and knowledge of deterministic repair times, our previous work [1] modeled the problem as a parallel machine scheduling problem with *outtree soft precedence constraints* in order to minimize the total weighted energization time, or equivalently, $P|outtree\ soft\ prec|\sum w_j E_j$, following Graham's notation [10]. This problem was proven to be strongly $\mathcal{NP}$-hard and 2 constant-factor polynomial-time approximation algorithms was proposed in [1].

In our prior work [1], all job repair times were assumed to be known with certainty prior to decision-making. Since this is hardly realistic in practice, in this paper, we relax that assumption and model the job repair times as random variables. We refer to the former scenario as *deterministic scheduling* and the latter scenario as *stochastic scheduling*.

1.1 Review of Deterministic Scheduling

We now briefly review the operational problem of scheduling post-disaster repairs in distribution networks with multiple repair crews when the repair times and other parameters (such as the number of repair crews) are known with uncertainty. A detailed discussion can be found in [1].

We model a distribution network as a tree graph G (alternately, a radial topology) with a set of nodes N and a set of edges L. Without any loss of generality (w.l.o.g), we assume that there is only one source node and the rest are sink nodes which connect to load centers. An importance factor, w_n, is assigned to every node. This parameter depends on multiple factors, including but not limited to, the amount of load connected to node n, the type of load served, and interdependency with other critical infrastructures. An edge in G represents a distribution feeder or some other connecting component. Network connectivity is ensured using a simple network flow model. While our work considers the availability of multiple repair crews, we assume that only one repair crew is assigned to repair a damaged line (component). Let p_l denote the repair time for line l. Obviously, nodes downstream of l can be energized only after line l is repaired. Based on conversations with an industry expert, we make the assumption that crew travel times in a typical distribution network are small compared to actual repair times and can be ignored as a first order approximation.

We construct two simplified directed radial graphs to model the effect that the topology of the distribution network has on scheduling. The first graph, G', is called the *damaged*

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component graph. All nodes in G that are connected by intact edges are contracted into a supernode in G'. The set of edges in G' is the set of damaged lines in G, L^D. The directions to these edges follow trivially from the network topology. The damage component graph G' would allow us to assume that, without loss of generality, all lines are damaged, i.e., $L^D = L$. The second graph, P, is called a *soft precedence constraint graph*, and is constructed as follows. The nodes in P are the damaged lines in G and an edge exists between two nodes in this graph if they share the same node in G'. Such a graph enables us to consider the hierarchical relationships between damaged lines, which we define as *soft precedence constraints* [1]. See Fig. 2 for examples of P and G'.

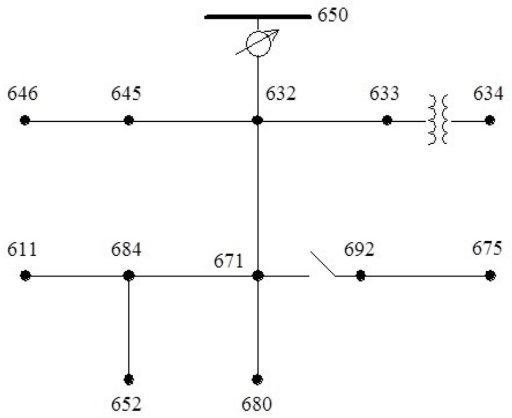
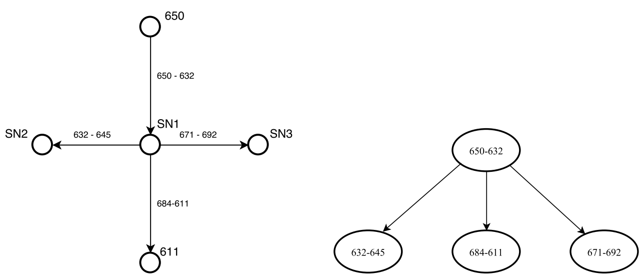


Figure 1: IEEE 13 Node Test Feeder



(a) G' graph (b) P graph

Figure 2: (a) The damaged component graph, G', obtained from Fig. 1, assuming that the damaged edges are $650-632$, $632-645$, $684-611$ and $671-692$. (b) The corresponding soft precedence graph, P.

Two different time vectors are of interest: (i) a vector of completion times of line repairs, denoted by C_l's, and (ii) a vector of energization times of nodes, denoted by E_n's. Due to the radial topology, it is possible to define energization times on the lines. Given a directed edge $l \in G'$, let $h(l)$ and $t(l)$ denote its head and tail nodes, i.e., $l = h(l) \to t(l)$. Then, $E_l = E_{t(l)}$, where E_l is the energization time of line l and $E_{t(l)}$ is the energization time of the node $t(l)$ in G. Analogously, the weight of node $t(l)$, $w_{t(l)}$, can be interpreted as a weight on the line l, w_l. The *soft precedence constraint* $i \prec_S j$, necessitated by network flow, therefore implies that line j cannot be energized unless line i is energized, or equivalently, $E_j \geq E_i$. Line j can be energized immediately after all its predecessors, including itself, are repaired, i.e., $E_j = \max_{i \preceq_S j} C_i$. The set of soft precedence constraints of cardinality $|L|-1$ embodied in the precedence graph is also denoted by P.

In the deterministic setting, the goal is to come up with an m-crew schedule by which the damaged lines should be repaired such that the total weighted energization time, or analogously, the *aggregate harm*, $\sum w_j E_j$, is minimized. See [1, 11] for a discussion on how the minimization of aggregate harm objective relates to the infrastructure resilience metric.

1.2 Stochastic Scheduling

In deterministic scheduling, all problem data is known beforehand and a schedule assigns a job to a machine at a specified time. In a general scheduling framework considering uncertainties, however, we may not have complete or exact information about the job processing times, the number of machines (repair crews) or even the set of jobs to be scheduled (incomplete knowledge of the damage statuses), prior to decision-making. In this paper, we allow the job repair times to be uncertain, while still assuming complete and exact knowledge of the number of repair teams and the set of jobs to be scheduled. Albeit a slight abuse of notation, we have used P to denote the soft precedence constraint graph/set and $\mathcal{P}$ to denote the random vector of repair times. Accordingly, the processing time of each job j is modeled as a non-negative random variable, $\mathcal{P}_j$, and its realization, p_j, is known with certainty only after completion of the job. We assume that $\mathcal{P}_j$'s are pairwise independent. Since the repair times are random, so are the resulting start times, completion times and energization times, which we denote by $\mathcal{S}_j$, $\mathcal{C}_j$, and $\mathcal{F}_j$ respectively[1]. In contrast to the deterministic case, we seek to minimize the *expected aggregate harm*, $\mathbb{E}\left[\sum w_j \mathcal{F}_j\right]$.

It is important to observe that the probabilistic nature of the repair times fundamentally alters the associated scheduling problem. Whereas the deterministic setting yields an actual repair schedule, the stochastic setting leads to a *scheduling policy*. Loosely speaking, a scheduling policy makes decisions at certain times, say t, and a decision is made at time t based on an *a priori* knowledge of the input data and jobs which have already been completed by time t. In particular, a scheduling policy should be *non-anticipatory*; i.e., no assumption can be made regarding the jobs which may have started before t but not completed by t. For a complete discussion of the differences between deterministic and stochastic scheduling and rigorous definitions of scheduling policy, see [12–16]. A scheduling policy is called an α-approximation if its expected performance is within a factor of α of the optimal expected performance.

1.3 Our Approach

We focus on a static list scheduling policy, where a set of jobs is obtained ahead of time that does not change during the scheduling process. The policy simply assigns jobs from the ordered list to whichever crew happens to be idle at any point in time. We construct the list based on an LP-relaxation model, built upon the valid inequalities proposed in [12]. This LP-based list scheduling policy only requires the knowledge of the means and variances of the repair jobs.

[1]The switch in notation from E_j to F_j to represent the energization times is due to our extensive use of the expectation operator $\mathbb{E}$.

We also provide a theoretical performance bound for the proposed policy.

2. LP-BASED LIST SCHEDULING POLICY

As mentioned above, our scheduling policy is derived from an LP-relaxation model based on completion and energization time vectors. A set of inequalities was shown to be the convex hull of completion time vectors for general single machine scheduling problems and a slightly modified set of inequalities is valid for general parallel machine scheduling [17]. These inequalities have proven to be very useful in developing approximation algorithms for single and parallel identical machine scheduling problems with precedence constraints and release time, in order to minimize the total weighted completion time [18, 19]. In a deterministic framework, we have previously shown [1] how a set of valid inequalities could help in developing an approximation algorithm for the multi-crew repair scheduling problem with soft precedence constraints. In the stochastic setting, with the knowledge of mean and variance for the repair times, a set of valid inequalities is extended for stochastic parallel machine scheduling:

Theorem 1. *[12] Let Π denote any policy for general stochastic parallel machine scheduling and m the number of machines (repair crews). The following inequalities (1) are valid for the corresponding vector of expected completion times, $\mathbb{E}[\mathcal{C}^{\Pi}]$.*

$$\sum_{j\in A}\mathbb{E}[\mathcal{P}_j]\,\mathbb{E}[\mathcal{C}_j^{\Pi}] \ge \frac{1}{2m}\left(\sum_{j\in A}\mathbb{E}[\mathcal{P}_j]\right)^2+\frac{1}{2}\sum_{j\in A}\mathbb{E}[\mathcal{P}_j]^2 -\frac{m-1}{2m}\sum_{j\in A}\mathrm{Var}[\mathcal{P}_j] \quad \forall A\subset L \tag{1}$$

The proof of this theorem in [16] requires the processing times to be pairwise independent and the scheduling policy to be non-anticipatory. With an additional assumption on the squared coefficients of variation for all processing time distributions, eqn. (1) can be refined as follows:

Corollary 1.1. *[12] If $\mathrm{Var}[\mathcal{P}_j]/\mathbb{E}[\mathcal{P}_j]^2\le\Delta$, then inequalities (2) are valid for the corresponding vector of expected completion times $E[\mathcal{C}^{\Pi}]$.*

$$\sum_{j\in A}\mathbb{E}[\mathcal{P}_j]\,\mathbb{E}[\mathcal{C}_j^{\Pi}] \ge f(A) \quad \forall A\subset L, \tag{2}$$

where the set function $f: 2^L\to\mathbb{R}$ is defined as:

$$f(A) := \frac{1}{2m}\left(\sum_{j\in A}\mathbb{E}[\mathcal{P}_j]\right)^2+\frac{m-\Delta(m-1)}{2m}\sum_{j\in A}\mathbb{E}[\mathcal{P}_j]^2 \tag{3}$$

Observe that under any scheduling policy, the following inequalities:

$$\mathbb{E}[\mathcal{F}_j^{\Pi}] \ge \mathbb{E}[\mathcal{F}_i^{\Pi}],\ \forall (i,j)\in P \tag{4}$$

$$\mathbb{E}[\mathcal{F}_j^{\Pi}] \ge \mathbb{E}[\mathcal{C}_j^{\Pi}],\ \forall j\in L \tag{5}$$

$$\mathbb{E}[\mathcal{C}_j^{\Pi}] \ge \mathbb{E}[\mathcal{P}_j],\ \forall j\in L \tag{6}$$

are valid, since they are true for every realization of the repair times. For each realization, eqn. (4) establishes the soft precedence constraints (recall that P is the soft precedence constraint graph), eqn. (5) requires that a damaged line cannot be energized before completion of its repair, and eqn. (6) ensures that the completion time should be at least equal to the repair time. Combined with Corollary 1.1, an LP relaxation, similar to those in [1, 12], can be written as follows:

$$\begin{aligned}
\underset{F,C}{\text{minimize}}\quad & \sum_{j\in L} w_jF_j && \text{(7a)}\\
\text{subject to}\quad & F_j \ge C_j,\ j\in L && \text{(7b)}\\
& C_j \ge \mathbb{E}[\mathcal{P}_j],\ j\in L && \text{(7c)}\\
& F_j \ge F_i,\ (i,j)\in P && \text{(7d)}\\
& \sum_{j\in A}\mathbb{E}[\mathcal{P}_j]C_j \ge f(A)\quad \forall A\subset L, && \text{(7e)}
\end{aligned}$$

where the decision variable F_j represents the *expected energization time* for line j and C_j represents its *expected completion time*. Eqn. (7c) can be applied to eqns. (7b) and (7e), resulting in the following simplified model:

$$\begin{aligned}
\underset{F}{\text{minimize}}\quad & \sum_{j\in L} w_jF_j && \text{(8a)}\\
\text{subject to}\quad & F_j \ge \mathbb{E}[\mathcal{P}_j],\ j\in L && \text{(8b)}\\
& F_j \ge F_i,\ (i,j)\in P && \text{(8c)}\\
& \sum_{j\in A}\mathbb{E}[\mathcal{P}_j]\,F_j \ge f(A)\quad \forall A\subset L && \text{(8d)}
\end{aligned}$$

Although there are an exponential number of constraints in (8), the separation problem for these inequalities can be solved in polynomial time using the ellipsoid method [17].

Let F^{LP} denote any feasible solution to the constraint set (8b) - (8d). We consider a static list obtained by sorting the damaged lines in a non-decreasing order of F_j^{LP}'s. Ties are broken according to the soft precedence constraints or arbitrarily if there is none. Assume w.l.o.g that:

$$F_1^{LP}\le F_2^{LP}\le\cdots\le F_{|L|}^{LP}, \tag{9}$$

This implies that the list is $(1,\cdots,|L|)$. Since the scheduling policy simply assigns the job at the top of the list to some idle crew, the damaged lines will be assigned in the order $1,2,\cdots,|L|$.

We will start with analyzing the basic properties of the list scheduling policy and the LP relaxation. Let $\mathcal{S}^{\Pi}$ and $\mathcal{C}^{\Pi}$ denote the random vectors of starting times and completion times respectively, corresponding to the LP-based list scheduling policy Π.

Consider any job j in the list scheduling policy. Since jobs 1 to $j-1$ are scheduled in order with no idle time in between, all machines are busy before j starts. Therefore, for any realization of the repair time vector p, the start time of job j satisfies:

$$S_j^{\Pi} \le \frac{1}{m}\sum_{i=1}^{j-1}p_i, \tag{10}$$

which implies:

$$\mathbb{E}[\mathcal{S}_j^{\Pi}] \le \frac{1}{m}\sum_{i=1}^{j-1}\mathbb{E}[\mathcal{P}_i] \tag{11}$$

Applying eqn. (8d) with the set A being $\{1,2,\cdots,j\}$ and eqn. (9), we have:

$$\left(\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i]\right)F_j^{LP} \geq \sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i]F_i^{LP}$$
$$\geq \frac{1}{2m}\left(\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i]\right)^2 + \cdots$$
$$\frac{m-\Delta(m-1)}{2m}\left(\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i]^2\right) \quad (12)$$

Dividing both sides by $\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i]$, we have:

$$F_j^{LP} \geq \frac{1}{2m}\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i] + \frac{m-\Delta(m-1)}{2m}\left(\frac{\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i]^2}{\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i]}\right) \quad (13)$$

We now consider two cases:

- **Case 1**: $\Delta \leq m/(m-1)$. Since the expression within parentheses on the r.h.s of eqn. (13) is non-negative, it follows that:

$$F_j^{LP} \geq \frac{1}{2m}\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i]. \quad (14)$$

- **Case 2**: $\Delta > m/(m-1)$. Since:

$$\frac{\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i]^2}{\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i]} \leq \max_{i=1,\cdots,j}\mathbb{E}[\mathcal{P}_i] \leq F_j^{LP}, \quad (15)$$

  it follows that:

$$F_j^{LP} \geq \frac{1}{2m}\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i] + \frac{m-\Delta(m-1)}{2m}\left(\frac{\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i]^2}{\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i]}\right)$$
$$\geq \frac{1}{2m}\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i] + \frac{m-\Delta(m-1)}{2m}F_j^{LP} \quad (16)$$

Combining eqns. (14) and (16), we have:

$$\frac{1}{m}\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i] \leq \left(1+\max\left\{1, \frac{m-1}{m}\Delta\right\}\right)F_j^{LP} \quad (17)$$

Observe that the energization time for job j is the point-wise maximum of the completion times of all jobs in the ordered list $(1, 2, \cdots, j)$. In order to establish an upper bound on the expected energization time, we will invoke the following lemma to bound the expectation of the maximum of a sequence of random variables from above.

Lemma 2. *[20] Let $X_1, X_2, \cdots, X_n$ be a sequence of (real valued) random variables, each with finite mean and variance. Then:*

$$\mathbb{E}[\max_{1\leq i\leq n} X_i] \leq \max_{1\leq i\leq n}\mathbb{E}[X_i] + \sqrt{\frac{n-1}{n}\sum_{j=1}^{n}\mathrm{Var}[X_i]} \quad (18)$$

Note that this bound is not necessarily tight when the means and variances of the X_i's are arbitrary. We are now ready to state the key result in this paper.

Proposition 3. *Let $\mathcal{F}^{\Pi}$ denote the random vector of energization times corresponding to the LP-based list scheduling policy Π. Then the following inequality holds:*

$$\mathbb{E}[\mathcal{F}_j^{\Pi}] \leq C(\Delta, m)\, F_j^{LP}, \ \forall j \in L \quad (19)$$

where

$$C(\Delta, m) = \left(2+\max\left\{1, \frac{m-1}{m}\Delta\right\}\right)$$
$$+\sqrt{\Delta m\left(1+\max\left\{1, \frac{m-1}{m}\Delta\right\}\right)} \quad (20)$$

Proof. Again, consider any job j from the list. As explained in Section 1.1, for each realization of the repair time vector, we have:

$$F_j = \max_{i \preceq_S j} C_i \quad (21)$$

Recall that the list scheduling policy assigns the damaged lines in the order of 1 through $|L|$ and that ties are broken according to the soft precedence constraints or arbitrarily if there is none. Therefore, for all $\{i : i \preceq_S j\} \subset \{1, \cdots, j\}$:

$$F_j \leq \max_{1\leq i\leq j} C_i := C_{\max}^{1:j}, \quad (22)$$

where the expression $\max_{1\leq i\leq j} C_i$ can be interpreted as the makespan for the job set $\{1, \cdots, j\}$, denoted by $C_{\max}^{1:j}$. Since j is the last job to start in this set, it follows that:

$$C_{\max}^{1:j} \leq S_j + \max_{1\leq i\leq j} p_i \quad (23)$$

Since the above arguments are valid for all realizations, it also holds in an expected sense:

$$\mathbb{E}[\mathcal{F}_j^{\Pi}] \leq \mathbb{E}[\mathcal{S}_j^{\Pi}] + \mathbb{E}[\max_{1\leq i\leq j}\mathcal{P}_i] \quad (24)$$
$$\leq \frac{1}{m}\sum_{i=1}^{j-1}\mathbb{E}[\mathcal{P}_i] + \max_{1\leq i\leq j}\mathbb{E}[\mathcal{P}_i] + \sqrt{\frac{j-1}{j}\sum_{i=1}^{j}\mathrm{Var}[\mathcal{P}_i]}, \quad (25)$$

where the last inequality follows from eqn. (11) and Lemma 2. We now bound all three terms in eqn. (25). Applying eqns. (9) and (17):

$$\frac{1}{m}\sum_{i=1}^{j-1}\mathbb{E}[\mathcal{P}_i] \leq \frac{1}{m}\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i] \leq \left(1+\max\left\{1, \frac{m-1}{m}\Delta\right\}\right)F_j^{LP} \quad (26)$$

Applying eqns. (8b) and (9):

$$\max_{1\leq i\leq j}\mathbb{E}[\mathcal{P}_i] \leq \max_{1\leq i\leq j} F_i^{LP} \leq F_j^{LP} \quad (27)$$

And finally:

$$\sum_{i=1}^{j}\mathrm{Var}[\mathcal{P}_i] \leq \Delta\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i]^2 \quad (28)$$
$$\leq \Delta\Big(\sum_{i=1}^{j}\mathbb{E}[\mathcal{P}_i]\Big)F_j^{LP} \quad (29)$$
$$\leq \Delta m\left(1+\max\left\{1, \frac{m-1}{m}\Delta\right\}\right)\left(F_j^{LP}\right)^2, \quad (30)$$

where eqn. (28) follows from the mean-variance assumption in Corollary 1.1, eqn. (29) follows from eqn. (15), and eqn. (30) follows from eqn. (17). The fact that $\frac{j-1}{j} \leq 1$ completes the proof.

Theorem 4. *The static list scheduling policy using LP relaxations is a $C(\Delta, m)$-approximation.*

PROOF. Proposition 3 and the fact that linear program (8) is a relaxation of the stochastic scheduling problem concludes the proof.

3. CONCLUSION AND FUTURE WORK

To cope with the uncertainties associated with estimation of repair times of the damaged components in the event of a disaster, we have proposed a static list scheduling policy which has a proven performance guarantee of $\mathcal{O}(\sqrt{m})$. Compared to the $\mathcal{O}(1)$ bound for its deterministic counterpart, we note that this deterioration is primarily due to an upper bound on the expected energization times. The bound, however, is not tight and has room for improvement, especially with more knowledge of the distributions. Another line of work is associated with extending our solution approach in the deterministic case, the so called 'conversion algorithm', to the stochastic setting.

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