

# Realization of a Vibration Diode using a Temporally Modulated Metabeam

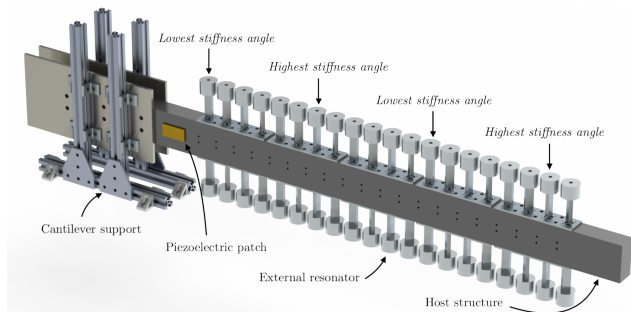
**Mohammad A. Attarzadeh<sup>1</sup>, Jesse Callanan<sup>1</sup>, Mostafa Nouh<sup>1</sup>**

<sup>1</sup>*Department of Mechanical & Aerospace Engineering,  
University at Buffalo (SUNY), Buffalo NY 14260, USA  
mattarza@buffalo.edu, jesseccal@buffalo.edu, mnouh@buffalo.edu*

**Abstract:** This work entails the design and physical realization of a vibration diode which exploits the inherent geometric-dependence of stiffness in non-axisymmetric cross sections. The diode relies on a phase shift between the geometric orientations of an array of resonators attached to a host beam to prescribe a spatial modulation of the metamaterial stiffness, accompanied with a uniform rotation induced via small motor action to effectively onset a spatiotemporal stiffness profile.

Motivated by their unique wave manipulation capabilities, periodic structures exhibiting tunable band gaps and directional effects have culminated in a spurt of research efforts. Among these are the attempts to realize structures analogous to electronic diodes, switches and logic functions which have long eluded the mechanical domain. In theory, a vibrating structure which comprises a time-dependent elastic field is no longer bound by reciprocal constraints [1]. As such, materials with properties that vary simultaneously in space and time have been recently utilized to investigate wave amplification and non-reciprocal propagation in mechanical systems. This problem has been investigated in the context of one- and two-dimensional structures, most commonly using a plane wave expansion (PWE) approach [2, 3]. Similar traits have also been reported in acoustic metamaterials and systems with internally resonating components [4, 5]. Locally resonant elastic metabeams (EMs) typically comprise a continuous flexural beam which hosts an array of mechanical resonators, and a non-reciprocal behavior is onset following a temporal modulation of the elastic properties of the host beam, the resonators, or both. Inducing a traveling-wave-like variation in an elastic medium has shown promising theoretical results capitalizing on band tilting in dispersion diagrams to create unidirectional band gaps. A physical realization of the concept, however, has been shown to be significantly challenging. Recent attempts include the use of piezoelectric shunt and switch circuits as well as an external magnetic field to impose the desired modulation pattern. While both are valiant efforts, the need for hard-wired setups and complex experimental configurations in such methods pose legitimate concerns pertaining to practicality as well as ease of build-up and operation.

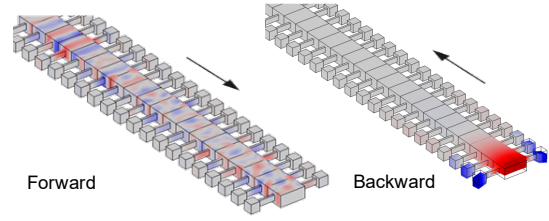
In this effort, we introduce a magnet-free design of a sub-wavelength vibration diode that incorporates a space-time variation of the resonators stiffness properties via changes in their geometric configuration. The device, which is shown in Figure 1, consists of a host beam attached to a set of configurable cantilever resonators. A phase shift between the geometric orientations of the resonators gives a prescribed spatial elastic profile. The entire array of resonators are then rotated synchronously using a set of miniature electric motors to introduce the required temporal variation. Each of the resonators consists of a heavy tip mass and a lightweight arm, and is attached to both sides of the host beam. The resonator stiffness  $k$  is equal to the arm's lateral stiffness. As a result, for a prismatic beam under Euler-Bernoulli assumptions, the lateral stiffness of the arm is a function of the



**Figure 1** A vibration diode which consists of a host beam attached to a set of configurable cantilever resonators. A phase shift between the geometric orientations of the resonators gives a prescribed spatial elastic profile which are then rotated by a set of miniature motors to introduce the temporal variation.

As a result, for a prismatic beam under Euler-Bernoulli assumptions, the lateral stiffness of the arm is a function of the

second area moment of the arm's cross section  $I_a$ . Consequently, for a non-axisymmetric cross section, we are able to harmonically alter the second area moment of the resonator's arm by rotating it at a chosen velocity. For circular cross sections, for example,  $I_{ax}$  remains unchanged with a rotation about the  $z$ -axis. However, for a non-axisymmetric cross section, such as an ellipse, this will no longer be the case. Let  $x'$  and  $z'$  be the principal axes for the arm's cross sectional area and  $\theta$  be the clockwise angular orientation of  $x$  with respect to  $x'$ . Following the coordinate rotation principle, we get  $I_{ax} = I_o + I_1 \cos(2\theta)$  where,  $I_o = (I_{x'} + I_{y'})/2$  and  $I_1 = (I_{x'} - I_{y'})/2$ . The previous implies that different orientations of the arm about the  $z$ -axis will result in different values of  $I_{ax}$  if  $I_{x'} \neq I_{y'}$ . Furthermore, a rotation of the arm about the  $z$ -axis is bound to generate a harmonically changing  $I_{ax}$  with a period  $\pi$ . A numerical study of the diode's dynamic response confirms that the imposed rotations induce a notable shift in band gap frequency ranges for forward and backward traveling waves, thereby creating the diode-like effect. In here, we present an overview of the diode's build-up, operation and performance.



**Figure 2** Unidirectional band gaps in a non-reciprocal elastic metabeam with rotating resonators

## References

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