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Phononic Band Gap Effects in Finite Serially Pivoted Pendulum Chains

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Abstract: This work shows evidence of an inherent attenuation mechanism in periodic chains of serially pivoted pendulums, reminiscent of band gaps in PnCs. The main contribution lies in the extraction of a "pseudo" unit cell of an equivalent phononic lattice as opposed to a traditional self-repeating cell. Such cell is directly obtained from the derived equations of motion of a finite chain with a terminal payload. The presented framework is specifically valuable to load-carrying and delivery applications such as gantry cranes, robotic arms and space tethers.

Over the past few decades, the use of elastic periodic structures and lattices to address challenges in vibroacoustic mitigation and control has become prominent[1]. One of the hallmark features of such periodic systems is the emergence of frequency band gaps as has been shown and physically realized in Phononic Crystals (PnCs). Periodic structures have been studied in the context of discrete spring-mass systems, continuous bars, beams, membranes, flexural plates, and more recently pendulum-like systems. Periodic arrangements of pendulums have been shown to give rise to a variety of intriguing phenomena pertaining to nonlinear oscillators. These have included solitons[2], breathers[3] and energy transmission in band gaps regions[4], among others. The notion of periodically arranged pendulum arrays has been also used to induce helical edge states in mechanical topological insulators. The latter is achieved by carefully coupling two one-dimensional pendulum arrays in a prescribed manner[5].

The work presented here focuses on a specific configuration of dynamically coupled pendulums arranged in a finite chain. The end of each pendulum acts as a pivot for the following one and a tip mass (payload) is attached to the unconstrained end of the chain. The dynamics of such systems frequently appear as a benchmark problem in active vibration control of overhead cranes and multi-mode systems[6]. To this end, however, the analysis of such systems in the context of elastic wave propagation in periodic media remains lacking. In doing so, we highlight the potential of such systems to provide self-reliant vibration isolation owing to their periodic architecture. We start by analyzing the dynamics of a periodic chain of lumped pendulums with an end payload (tip mass) as portrayed in Figure 1. The effects of the lumped masses and the length of the linkages on the chain's dynamical behavior are detailed. We then investigate the feasibility of creating frequency band gaps similar to those associated with conventional PnCs. Despite the apparent periodic pattern, the determination of a traditional unit cell which corresponds to an infinitely long chain is not trivial. Such a cell cannot be defined as two successive masses due to the fact that the dynamics of each mass is coupled with the entire chain as dictated by its equations of motion. Consequently, we depart from the conventional approach of defining a unit cell (i.e. based on an infinite chain) and resort to a radically different approach where we consider the closed-form dynamics of the finite chain of pendulums as a starting point and ultimately, converge on a "pseudo" unit cell for wave propagation purposes which accurately depicts the desired wave dispersion behavior.

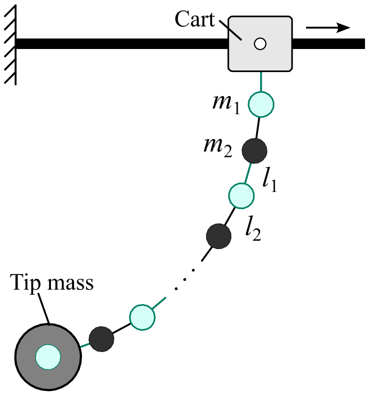

Figure 1 Schematic diagram of a serially pivoted periodic pendulum chain. The chain carries a tip mass and is suspended from a cart which moves along a horizontal rail.

The study shows that the finite pendulum chain behaves similar to a conventional PnC lattice at large values of the tip mass. It is also shown that a periodic variation in the masses or links of the pendulum chain onsets a frequency band gap which is numerically verified in both the frequency and time domains. But more interestingly, we show that a periodic variation in the masses of the pendulum chain mimics the effect of a stiffness variation in a conventional PnC; from the standpoint of the derived mathematics. A periodic change in the pendulum linkages, on the other hand, is shown to be analogous to a PnC with: (1) A periodic elastic foundation and (2) A periodic variation of masses (See Figure 2). Both cases, however, act identically in terms of the equivalent dispersion relations since the hypothetical elastic foundation comprises a negative stiffness which nullifies its own effect.

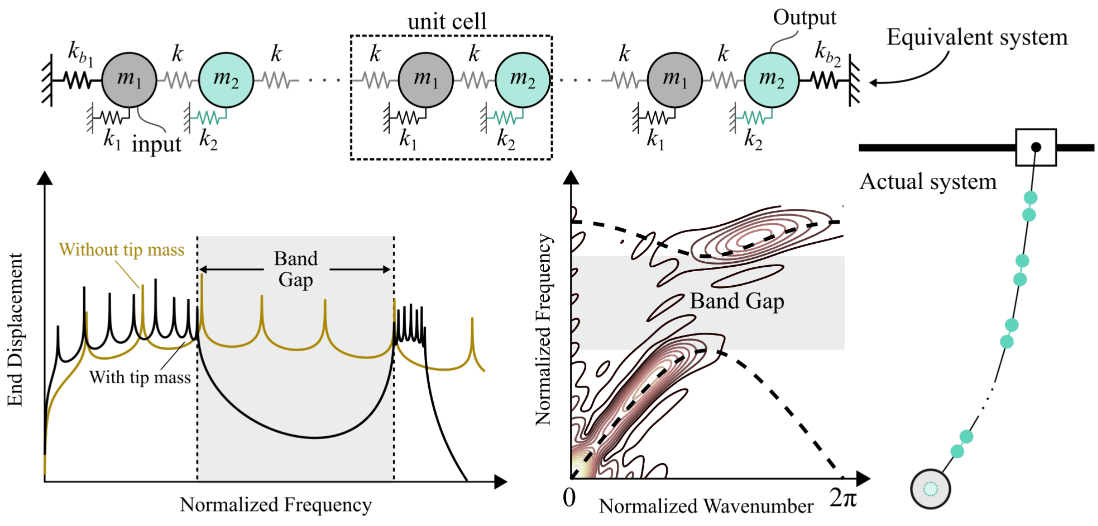

Figure 2 An illustrative schematic of a serially pivoted periodic pendulum chain (with a variation in linkage length) and the equivalent PnC. Frequency response of the end angular displacement with and without an end payload are shown (bottom left), as well as dispersion contours obtained from a spatiotemporal analysis of a wide band excitation (bottom right).

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