Playful Mathematics Learning: Beyond Early Childhood and Sugar-Coating

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Abstract: In this symposium, we ask what mathematical engagement looks like in the context of play, focusing on contexts designed to support mathematical thinking through open-ended activity, and looking at ages that are traditionally overlooked in studies of play. We heed Dewey's admonition to look beyond sugar-coating: we do not seek to claim that "play is a good method for supporting traditional mathematics learning," but rather, to explore the ways that mathematics is engaged through play, and how such different engagement with mathematics might transform students' relationships with the domain. Taken as a collective, the papers in this symposium address this issue by looking across technical and physical contexts, using different lenses on play, and studying different experiences of play. Despite these contrasts, we jointly ask questions about what mathematics thinking and learning could look like if we changed the rules of the game.

Keywords: play, mathematics education, informal learning

Everything is made play, amusement. This means over-stimulation; it means dissipation of energy. Will is never called into action. The reliance is upon external attractions and amusements. Everything is sugar-coated for the child, and he soon learns to turn away from everything that is not surrounded with diverting circumstances.

John Dewey, Interest and Effort in Education, *pp. 4-5*

There is widespread agreement about the importance of play in supporting learning. Early childhood education has a long tradition of play-based curriculum, with significant evidence demonstrating how children substantively engage the world through play. Some play scholars demonstrate the ubiquity of play as a developmental strategy, extending this lens into the animal kingdom, and developmental theorists often articulate how different kinds of play develop over childhood (Piaget, 1932/1965; Vygotsky, 1967). Studies of children as they move into formal schooling consider the role that play takes in supporting development of social-emotional skills, in providing exercise, and generally allowing for periods of focus in the classroom. However, little research investigates the role of play in learning mathematics in older children.

Indeed, once children enter school, discussions of learning almost never reference play, and work investigating the design of learning environments has seldom explored whether or how play might be involved. And yet, much of what is considered to be sophisticated disciplinary engagement involves many of the same features as play, including exercising personal agency to set and reach goals, exploration, imagination, and joy. Thus, it seems likely that there is quite a bit of play involved in learning in the later years, and the field could benefit from better understanding how and when play is involved in disciplinary engagement.

In the mathematics education community, the context for this symposium, there is a significant history for studying students' mathematical problem solving (Schoenfeld, 1985), and how their engagement with tasks might extend to their relationships to the discipline (Boaler & Greeno, 2000). Most of this work has taken place in the context of formal schooling, which is often sufficiently constrained so students have few opportunities to engage in the kind of open exploration of ideas foundational to mathematical problem solving. In settings less pressed by time and uniformity, mathematical problem solving can involve visualization and imagination, exploration and play, and investigating *what if* questions—all elements of engagement that are very much like play (Sutton-Smith, 1997; Gray, 2015). However, little is known about students' engagement in such spaces, what is playful or enjoyable, and what mathematical ideas students think about when playing therein.

Thus, in this symposium, we ask what mathematical engagement looks like in the context of play, focusing on contexts designed to support mathematical thinking through open-ended activity, and looking at ages that are traditionally ignored in studies of play (that is, children who are in elementary and middle school). We heed Dewey's admonition to look beyond sugar-coating: we do not seek to claim that "play is a good method for supporting traditional mathematics learning," but rather, to explore the ways that mathematics is engaged through play, and how such different engagement with mathematics might transform students' relationships with the domain.

Specifically, Paper 1 considers how a teacher's' use of a multi-touch app called *TouchCounts* and a collection of web-based Dynamic Geometry sketchpads help create propitious conditions for mathematical play in K-7 students. In exploring the relations between mathematics and play, they consider what children were aiming to do, how those aims emerged, and how the aesthetic features of their activity shaped their play. Paper 2 investigates middle school students' engagement in a large-scale number line activity called *Secret Pattern*, where students performed a walking pattern along a giant number line on the floor for others to guess. They consider how this activity became a kind of socio-dramatic play as students explored the tension between wanting to develop and perform a pattern difficult for the other team to guess while also producing a good performance, and how this tension supported the flexible and creative recruitment of existing mathematical understandings in the service of meeting the challenge. Paper 3 explores how children's self-chosen and self-directed play with mathematically structured objects creates opportunities for spontaneous mathematical sense-making. Specifically, they reveal that as children persisted in their attempts to repair trouble, they often became increasingly systematic in their efforts. This systematicity involved careful exploration of objects' properties and relations between them, which, by design, were also mathematical properties. Finally, Paper 4 explores what play can mean in the context of an educational videogame, and asks what students might be playing at. The paper explores how a gamer initially set aside the game goal according to the game narrative and took a playful, explorative stance toward the game, and in doing so, he began to uncover and question the underlying mathematical structure of the game world.

Taken as a collective, the papers in this symposium address this issue by looking across technical and physical contexts, using different lenses on play, and studying different experiences of play. Despite these contrasts, we jointly ask questions about what mathematics thinking and learning could look like if we changed the rules of the game.

Designing intellectual playgrounds for mathematics learning

Nathalie Sinclair and Victoria Guyevskey

Drawing on the work of historian Johan Huizinga, Featherstone (2000) argues for the central importance of what she calls *intellectual play* in mathematics learning. She draws many parallels between the characteristic features of play and mathematical activity as it is described by mathematicians. These characteristic features include: stepping outside of ordinary or "real" life; being orderly and, in consequence, beautiful; being governed by rules; creating social groupings; and, being voluntary. Featherstone does not argue that play and mathematics are identical, nor that teachers should include play periods as part of their mathematical lessons; instead she is interested in "the ways in which 'play' might expose children to aspects of the discipline that may not ordinarily be visible to them" (p. 16).

In this paper, we explore some of the features of environments in which Featherstone's intellectual play can emerge. We do so in the context of elementary school mathematics, because we are particularly interested in the way that play might free children "from the dictatorship of concrete objects," as Vygotsky (1967, p. 19) writes, and enable them to "develop the capacity to behave in accordance with meaning." This is all the more pertinent in the primary school grades, where the emphasis on the concrete — through manipulatives and real-life situations — may in fact delay children's mathematical development.

In addition to these characteristics, Featherstone underscores the fact that play is not an aimless, spontaneous activity, but it instead, always has an aim: "The player wants something to 'go', to 'come off; he wants to 'succeed' by his own expertise" (Huizinga, 1955, pp. 10-11). This aim is rooted in aesthetics: "Play has a tendency to be beautiful. It may be that this aesthetic factor is identical with the impulse to create orderly form, which animates play in all its aspects" (p. 10). Play thus typically involves aspects of aesthetic experience, such as tension and poise, balance and contrast, solution and resolution, repetition and variation.

Following Papert (1980), well-designed and open-ended computer-based environments are effective at providing mathematical microworlds that operate outside of ordinary life. Such microworlds also provide the kind of "mathland" that Papert described as being like the immersive environment in which language learning occurs—where many words may not be fully understood, but in which fluency can develop. We conjecture that they might be especially effective at promoting mathematical play. In order to investigate this conjecture, we analyse several different instances of intellectual play, in the sense of Featherstone, in which K-7 children are engaged in two open-ended computer-based environments. Our goal is to examine how these environments, including the teacher, help create propitious conditions for play. The first environment is a free, multi-touch App called *TouchCounts*, in which children use their fingers and gestures to count, add and subtract, and which provides auditory, haptic, visual and symbolic feedback. For example, when one finger taps the screen, the word "one" is said aloud, a disc appears and the symbol "1" appears on the disc. The environment is open-ended in that there are no prescribed tasks and no evaluative feedback provided. Teachers may propose some tasks, as may children. The multi-touch feature encourages children to work together on the screen, as multiple fingers can be used simultaneously. Prior research on the use of *TouchCounts* with young children has focused on the ordinal aspects of number that the app facilitates, as well as on the way that the gestures for making numbers, adding and subtracting change the way that children think about number. Coles and Sinclair (2017) have argued that children's tendency to create large numbers (beyond those prescribed by grade-level curricula) provides them with a sense of the structure of numbers, which facilitates their understanding of place value—not as a cardinal concept, but as a temporal and symbolic one. The second environment is an online Dynamic Geometry platform that features a collection of Web Sketchpads, where children manipulate various geometric shapes and create geometric designs within interactive sketches. Visual and dynamic component allows learners to play and explore, test conjectures, discover patterns, think creatively, and otherwise get engaged. Students are excited to interact with the software, which offers many opportunities for collaboration and discussion.

In the paper, we consider three different episodes involving two or more children and first show how they can be identified as examples of intellectual play. We then analyse these episodes to better understand the children's aims, how those aims emerged, and how aesthetic features of their activity shaped their play. We also study how children's play enabled them to develop relational meanings in the absence of any concrete objects or real-life settings. In doing so, we identify features of the environment that gave rise to intellectual play in young children including, most significantly: (1) the *symbolically structured* nature of the environments, where the symbols are mathematical in nature and the structure enables relational meanings to emerge (Coles & Sinclair, in press); and (2) the participative revoicing of the teacher in the children's process of goal setting.

Guess my secret pattern: Imaginative explorations through operating as points on a walking scale number line

Jasmine Y. Ma and Sarah C. Radke

This paper investigates fourteen middle schoolers' engagements in a large-scale number line activity called *Secret Pattern*, in which students performed a walking pattern along a giant number line on the floor for others to guess. The somewhat competitive nature of the activity, the open-ended possibilities for completing the task, as well as the imaginative requirements of operating as a point along a giant number line supported a playful atmosphere for mathematical reasoning. We present findings here that describe how these students' mathematical engagements shaped and were shaped by their play.

Secret Pattern took place on the first day of an elective mini-course called *Math Battle!* that ran for three hours a day for five days during the school's mid-winter "minimester," designed for students to explore innovative extensions of the school's regular curriculum. *Math Battle!* was advertised, in part, as an opportunity to design a giant game of Human Battleship, in the context of which students would explore the number line and the coordinate system using their whole bodies. It took place in an empty gymnasium-sized space in the school building. On the first day the students engaged in activities on what we call a Walking Scale Number Line, a long orange number line made of painter's tape placed on the floor (Figure 1). The number line was partitioned by unnumbered hash marks made of short pieces of blue tape placed at equal intervals (the length of the second author's shoe). Every other hash mark was slightly shorter than its neighbors. *Secret Pattern* took place after students had each chosen a "home position" on a hash mark, placing an index card labelled with his name there (all students in the mini-course were boys). The group then explored the number line, negotiating physical logistics of moving along the line as a group and developing shared language for discussing their location on the line and relative to each other.

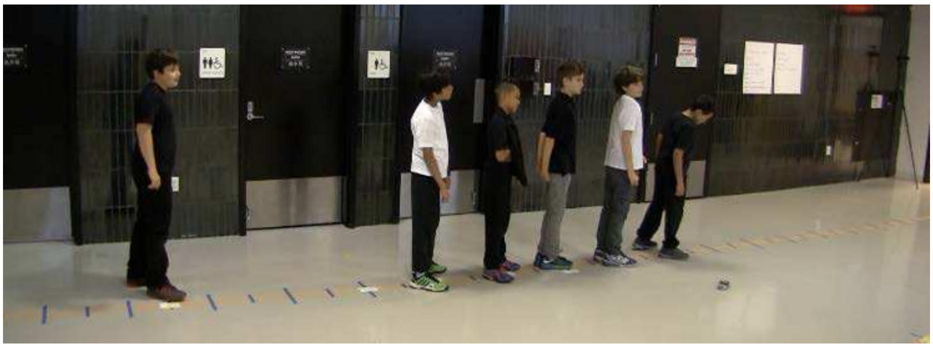

Figure 1. *Math Battle!* students standing on their home positions on the walking scale number line.

Our analysis is framed by contemporary theories in the learning sciences that foreground whole bodies and movement as constitutive of processes of meaning-making (e.g., Hall & Stevens, 2016; Marin, 2013; Taylor, 2017). From this point of view, multimodal and multisensorial aspects of mathematics activity are not epiphenomenal to reasoning, but instead must be taken into account in an analysis of knowledge and learning. Walking and attendant embodied sensations are forms of place-making, of coming to know and simultaneously producing place (Ingold, 2007). Our analysis focuses on walks on the thin strip of tape that we called a number line, and how students infused it with mathematical meanings and developed corresponding representational practices (Hall, 1996) in repeated walks along its surface.

Data collected included documents related to instructional design of workshop activities, detailed fieldnotes written after each workshop day, multiple streams of video and audio recordings, including stationary video cameras mounted on tripods, GoPro cameras worn on students' chests, and digital audio recorders placed on the floor. Analysis followed multimodal, microethnographic methods (Jewitt, Bezemer, & O'Halloran, 2016; Streeck & Mehus, 2005), following students' coordinated movements and talk as the site of mathematics sense-making, and attending deliberately to sociomaterial phenomena of relevance to participants in the data, and meanings produced in unfolding interaction.

Findings explore the tension between wanting to develop and perform a pattern difficult for the other team to guess while also producing a good performance. This tension supported the flexible and creative recruitment of existing mathematical understandings in the service of meeting the challenge. Enyedy and his colleagues (2012) described how students in socio-dramatic play "often spend more time articulating and negotiating the rules of a play situation than they spend actually in character 'playing' their parts…as a result, the rules that govern a situation become visible and explicit for children" (p. 353). If we take students operating as points on a number line to be a form of socio-dramatic play, then we see how negotiations between members of the group made explicit, challenged, and expanded their understandings of the mathematical relations that would govern their pattern and how it would be performed on the walking scale number line. These negotiations included suggestions for mathematical patterns proposed in talk (e.g., adding one each time or the Fibonnaci sequence), trial walks implementing the pattern on the number line, and revisions aimed at either making the pattern walkable given the constraints of the taped line or making the pattern more complicated. Analysis illuminates how, through imaginative exploration, students' moving bodies and the tape on the floor accumulated mathematical meaning over the course of negotiation.

The emergence of mathematical thinking through construction play

Lara Jasien, Ilana Horn, and Melissa Gresalfi

Mathematics education research emphasizes student sense-making as foundational to mathematics learning (Hiebert, Carpenter, Fennema, Fuson, Wearne, & Murray, 1997). However, we need tools and frameworks that support mathematical inquiry, particularly ones that engage students' personal agency with respect to decision-making and reflection in mathematical activity. Within the constraints of the U.S. school system, a focus on making progress on content can become so dominant that it overshadows a focus on exploration. This is particularly ironic, since play is a core activities of childhood, yet it is increasingly eliminated from U.S. schools. The reduction of play represents a loss not just to children's physical and emotional health, but to their intellectual health as well: play supports sophisticated forms of negotiation, reflection, experimentation, persistence, and learning — all disciplinary practices of mathematics. Indeed, Lev Vygotsky (1933/1978) proposed that when playing, children are able to act "...a head above themselves," (p.95) literally by engaging in a zone of proximal development because of their freedom from the constraints of the "real" world. Following this perspective, in this study, we look at how students' play with mathematical objects to consider specifically how play centered on building or designing — often called *construction play* (Forman, 2006) — supports formal concepts and reasoning.

Prior studies have shown that preschool children who engage in construction play in informal environments develop relationships to materials and ideas that seem to support later success in math (Stannard, Wolfgang, Jones, & Phelps, 2001). Similarly, older children who do well on construction tasks also tend to perform well in mathematics (Casey, Pezaris, & Bassi, 2012). However, little of this scholarship has explored how or why such playful activities might support mathematical thinking, and indeed have only rarely made claims about the mathematical concepts that *might* be engaged in children's play (Seo & Ginsburg, 2004), leaving us with little knowledge about children's actual mathematical sense-making in constructive play. In addition, few studies of children's play take place in environments with materials explicitly designed to direct children's attention to mathematical concepts. This is a large theoretical and methodological gap, as the mathematical generativity of children's play is linked to material constraints that lead children to experience trouble as they work to achieve their goals (Bergen, 2009; Forman, 2006; Papert, 1980). In overcoming the constraints of materials, children not only engage in creative problem solving but also often become increasingly systematic in their examination of the object's properties (Karmiloff-Smith & Inhelder, 1974). We argue that examining children's self-chosen and self-directed play with mathematically structured objects is likely to provide rich information about children's emerging mathematical concepts and sense-making.

To better understand how mathematics is engaged in children's play, we examine school-aged children's activity in a mathematics playground at the Minnesota State Fair called Math On-A-Stick (MOAS). MOAS has nine exhibits, each with unique mathematically structured objects such as tiling pentagons, pattern machines, tessellating turtles, and 6x5 egg crates with colorful plastic eggs. Over 10 days, we collected video data using head-mounted GoPros, capturing children's perspectives of their play (n = 348). Children's average MOAS visits lasted 26 minutes, with median visit per exhibit at approximately four minutes. Out of the 348 participants, this study focuses on older children (7 – 12 years old, n = 277, visit length: m = 28 minutes), because this age group's mathematical play is understudied.

To understand how mathematics was engaged in children's play with the mathematical objects at MOAS, we used interaction analysis (Jordan & Henderson, 1995), a method that centered the children's perspectives. Specifically, we initially reduced our data to look for sustained engagement towards one goal for at least two minutes, as we hypothesized longer episodes of engagement were more likely to contain more complex forms of play. Then we inductively coded our video data to look for when children experienced trouble (e.g., when they stopped to reorganize, expressed frustration or confusion, etc.) and then worked to repair it, what we called *episodes of trouble-and-repair*. *Trouble* was often signaled by audible expressions of frustration or confusion, and *repair* was signaled by solicitations for help or multiple revision attempts. We conducted a close examination of how children's strategies and goals shifted as they engaged with and experienced push-back from the mathematical objects. As children persisted in their attempts to repair, they often became increasingly systematic in their efforts. This systematicity involved careful exploration of the objects' properties and relations between them. When this kind of systematicity and exploration arose in children's play as they persevered through trouble in order to meet their goals, we have come term this activity *mathematically generative play*.

Our analyses suggest that *Mathematically generative play* arises when children explicitly engage with mathematical properties of objects (although they do not necessarily name them with mathematical terms), which often involves children's use of disciplinary practices such as *attending to precision* and *making generalizations*. This is not only a feature of play, but also of the objects in the exhibit; throughout our analyses it appeared that the mathematical affordances of the objects makes it more likely that children attend to and engage with mathematical concepts. This research serves to deepen our understanding of school-age children's mathematical sense-making as we begin to look at children's spontaneous mathematical concepts and ways of thinking in contexts that center children's decision making and agency.

"Playing" the game: Exploring the underlying mathematical structure of an immersive game

Panchompoo Wisittanawat

One challenge in understanding play is that play describes both 1) a form of activity and 2) a stance or orientation toward an activity (Malaby, 2009). In the first sense, play describes a *form* of activity that is game-like either with implicit and emergent rules (e.g. children's pretend play) or formal and more stable rules (e.g. soccer, chess, videogames). In another sense, play describes a *mode* of cultural experience that occurs in many forms of activity. This means that an activity that looks like a play activity may not be playful (e.g. students play a mathematics game to complete a homework assignment). On the other hand, an activity that does not look like a play activity may in fact be playful (e.g. students solve a homework math problem with a playful stance). Conceptualizing play in this second sense, as a mode of orientation toward an activity, raises the question: when students appear to be playing a mathematics videogame in a classroom, are they playing, and what are they playing at?

This case analysis draws data from a larger research project that investigates an educational videogame designed to support mathematical problem solving. The game, called "Boone's Meadow," builds on a project-based curriculum that was developed as part of the "Adventures of Jasper Woodbury" project (Cognition and Technology Group at Vanderbilt, 1997). In this game, students play the role of a wildlife rescue assistant who transports injured animals from a nature reserve to a clinic. Players make rescue plans (i.e., deciding which plane and which route to fly and) and act out their plans (i.e., flying a plane to transport an animal). The conjecture that framed the design of the game was that greater engagement with the game narrative would lead to *consequential engagement* with mathematics (Gresalfi & Barnes, 2015), when students increasingly used mathematics to make and reflect on decisions in the game.

This paper presents a case analysis that explores a unique form of play that emerged as Calvin, a student gamer (grade 6), played Boone's Meadow. This form of play, common in videogame communities, is unique in this context in the sense that this is the only observed case in our data corpus. While most of his peers played the game by adopting the goal according to the game narrative (i.e., rescuing an injured animal), Calvin temporarily set aside that goal and "played" the game to expose its mathematical structure. Calvin presented himself as a gamer, someone who played a lot of games and was knowledgeable about genre(s) of videogames. How Calvin played the game became of analytic interest because this is a case that did not fit our design conjecture. Prior analyses demonstrated that while Calvin and his partner were very engaged with the game narrative, they appeared as though they didn't engage with mathematics in the game, at least not in the ways intended by the designers. Calvin and his partner were not enthusiastic about conducting calculations that were required to make precise recommendations about saving the eagle, and did not use the calculations they did to inform their game choices. However, upon closer analysis, Calvin appeared to interact with the game in a deeply mathematical way. He took an explorative, playful stance toward the game, and intentionally "played" the game to expose its underlying mathematical structure.

Data include screencapture recordings of three class periods of game play. This analysis used methods of interaction analysis, attending to talk and game actions, and in particular, what Calvin noticed in the game and with whom he shared his noticing. Calvin and his partner sometimes addressed the computer ("Dear Boone's Meadow creators …") or directed his comments to researchers in the room. Comments directed to "designers" contained game jargon (e.g. "8-bit graphics" or "It'd be cool if it goes like a turn-based JRPG."), and they often included his evaluations of the game.

Analysis of Calvin's play revealed playful attunements to the activity that were different from the intended forms of engagement envisioned by the designers. First, Calvin engaged in a kind of *beta testing:* In gaming communities, the goal of beta testing is to creatively challenge the design in order to push the limit of the design, and it is an important mechanism for a co-creative game production (Wirman, 2012). The beta testing frame oriented Calvin to seek the limit of the game. Despite some objection from his partner, when he had a chance to maneuver a plane in the game, he tried different backflips and diverted the plane away from the destination to find the edge of the game world. He commented to one of the researchers, "Can you give us a little more room to fly the plane?" Calvin also noticeably *attended to visual details;* he first noticed many "texture issues" with the game. For example, "There are texture issues. This is when people went cheap on the game and they did not give Larry the proper texture, because they want to go cheap on him. … His thumb just went through his arm!" This suggests that Calvin's ways of seeing in the game included paying attention to very minute visual details. Finally, Calvin's play served to *expose the underlying mathematical structure.* His Calvin's careful attention to visual cues was also integrated in how he "played" the game to expose its underlying structure, which he seemed to assume to be mathematical. In running two of their eagle rescue operations, Calvin and his partner used two different planes, and they crashed because they ran out of gas both times. After the second plane crashed, Calvin expressed his surprise: "Our thing died at the exact same time as the one that had the most fuel efficiency." This comment suggests some intentionality in how Calvin varied his game actions, i.e., he was cognizant that what was varied between the two trials was fuel efficiency. Also implicit in this comment was an expectation that two planes with different fuel efficiency should not have crashed at the same location. This expectation entailed an understanding that some common relationship governed these two flight trials, which should not have yielded the same result given that an important factor in that relationship (i.e., fuel efficiency) was varied. In terms of a learning goal in this game, this form of reasoning demonstrated a sophisticated understanding of proportional relationships. Through his playful exploration, Calvin uncovered and (mildly) questioned the accuracy of these underlying relationships in the game.

At least in the early attempts, Calvin didn't play the game to save animals as most of his peers did. He took a playful stance toward playing the game to explore the underlying mathematical structure of the game world. As someone familiar with videogame media, he was highly attuned to visual cues, and the beta-testing frame possibly heightened this attunement that led to him noticing and questioning the mathematical structure of the game. After some playful explorations, Calvin started to take up the game mission according to the narrative, commenting to his partner, "Dude, we have to do this [again]. We've killed two eagles in the span of 5 minutes. … I feel so bad. I was starting to drive legit too."

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