Mathematics Problem-Posing in Informal Contexts

Mathematics instruction in the United States largely involves engaging students in correctly solving predetermined problems (Jacobs et al., 2006; Litke, 2015). This means that students have few opportunities to engage in a core element of problem solving—finding and articulating problems that are interesting to solve. Unsurprisingly, our understanding of the mathematical practice of problem-posing — a fundamental driver of disciplinary engagement (Einstein & Inhfeld, 1938; NCTM, 1991) — is underdeveloped (Singer, Ellerton, & Cai, 2013). In fact, neither teachers nor students gain much experience in problem-posing, as the people who pose the problems of school mathematics are often erased through the distant, authoritative voices of textbooks. This sterilization of problem-posing from our understanding mathematical practice is disingenuous to the essence of the discipline. In truth, "problem formulation and problem solution go hand in hand, each eliciting the other as the investigation progresses" (Davis, 1985, p. 23). In other words, mathematical problem-posing and problem-solving are reflexive practices.

In contrast, mathematics in everyday life is rich with examples of problem posing. In a study of family mathematics, Pea and Martin (2010) found that families reflexively articulate and resolve mathematical problems that arise in their daily routines as they deal with tensions between competing elements such as family values, needs, income (c.f. Lave, Murtaugh, de la Rocha, 1984).

Objectives and Setting

In this study, we seek to bridge the gaps between research on problem-posing in everyday life and in mathematics classrooms by examining student engagement in a mathematical playground called Math On-A-Stick (MOAS), located at the Minnesota State Fair. MOAS is a unique research setting because of the playful mathematical tools offered at its interactive exhibits (see Figure 1). This, along with children's voluntary participation and their freedom in how and how long to engage with the exhibits make it an ideal context for a study of how learners engage in mathematical problem-posing in informal contexts.

[INSERT FIGURE 1]

Theoretical Framework

In order to understand learners' problem-posing in this context, we examine the interaction between the mathematics embedded in the tools at MOAS with the mathematics that children might tacitly attend to in their play by foregrounding the situated nature of learning (Lave & Wenger, 1991). This follows contemporary studies of problem-solving, which began in the 1980s with laboratory experiments (Schoenfeld, 1985) and continue to today with classroom studies that emphasize fostering sense-making (Lampert, 2003). The transition from studying problem-solving in a laboratory to studying sense-making in the classroom was largely motivated by a theoretical shift to situative perspectives, which requires accounting for students' competence as interwoven with their social and material contexts (e.g., Gresalfi, Martin, Hand, & Greeno, 2009).

However, much of the recent research on problem-posing primarily consist of methodologies (e.g., Singer, Ellerton, & Cai, 2013) that limit our understanding of the mediating role of the social and material environment by backgrounding learners' reactions and reasoning in favor of analysis of the types of problems posed (Singer, Ellerton, Cai, Leung, 2011). For example, studies of problem posing often begin by giving students a streamlined story or a static representation and ask students to pose problems that would come from those resources. While this allows for clean analyses of the range of problems that are posed, it is distant from the ecology of problem-solving of which problem-posing is a part.

In this study, we ground our approach to problem-posing in Hiebert and colleagues' (1996) argument that mathematical problems arise as learners notice, wonder, inquire, and pursue resolutions (p. 12). Following this logic, we take a phenomenological perspective, considering problem-posing that occurs when children engage in *episodes of making,* or episodes of activity in which participants created something with the MOAS tools. By conceptualizing episodes of making (EOMs) as problem-posing at MOAS, we foreground children's perspectives and sensemaking as mediated by affordances of the mathematical-tools (Wertsch, 1998).

Methods

Data Collection

Following situative studies of families' interactions "in the wild" (Goodwin, 2006), video recordings serve as our primary data source. Videos comes from head-mounted GoPro™ cameras that captured participants' talk, gestures, and object manipulation (Figure 2) in order to support inferences on attention and interest. Over the 10 days of data collection, we recruited 345 children, aged 4 through 17, to participate. The average duration of participants' time at MOAS was 26 minutes (sd = 0.007). The majority of our participants were between the ages of 7 to 12 (n = 279), and thus we focused our preliminary analyses on this age band.

[INSERT FIGURE 2]

Data Analysis

Because we conjectured that sustained engagement at exhibits might support increasingly complex EOMs, we focused on children in our age band who stayed at an exhibit longer than was typical (above the median exhibit stay, n=171) at more than two exhibits (n=62). Narrowing our data in this way allowed us to identify contrasts in how problem-posing was mediated differently across exhibits. We examine participants' activity at two exhibits with contrasting designs that sustained children's engagement for much of their time at MOAS (Table 1): *Pattern Machine* (Figure 1a) and *Pentagons* (Figure 1b). This reduced our data set to 17 participants.

[INSERT TABLE 1]

Using an open-coding approach (Strauss & Corbin, 1990), we began by documenting students' EOMs by locating both the objectives of an episode (sometimes seen in participants' own words, or in the coherent activity of an episode), and the end of an episode (seen in a new goal being stated, or often clearing of work; see Table 2 for examples of this in our codebook). We then examined participants' activity within these episodes (Jordan & Henderson, 1995) to understand how participants' problem-posing and solving emerged in interaction with the tools. Examining both within-child contrasts across exhibits as well as across-children contrasts within each exhibit allowed us to develop a nuanced perspective on the nature of mathematical problem-posing in informal contexts designed for engaging mathematics.

[INSERT TABLE 2]

Results

The Emergence of Problem-Posing and The Influence of Tool Affordances

Our analysis revealed that problem-posing was emergent and often built on participants' prior EOMs within each exhibit. In addition, tool affordances influenced participants' EOMs, and thus also influenced the types of problems posed and opportunities for encountering mathematical concepts. In this section, we discuss the affordances for engagement in problem-posing and encountering mathematical concepts in relation to tool affordances.

Each exhibit's affordances influenced how many EOMs participants engaged, as well as how long they persisted within EOMs (Table 3). While the nature and quantity of EOMs varied across the exhibits, the average duration of participants' engagement at each exhibit was quite similar ($M_{pentagons}$ = 8.58 minutes; $M_{pattern\ machine}$ = 7.79 minutes). Both exhibits supported engagement in problem-posing and solving, but in different ways. As described below, the *Pattern Machine* supported participants to pose many problems, and usually to solve them rather quickly. The *Pentagons,* on the other hand, supported problem-posing that required more persistent efforts to realize a satisfactory end product.

[INSERT TABLE 3]

Revisions and boundedness. The easy-to-revise nature of the *Pattern Machines,* along with their bounded 9x9 grid, facilitated participants' ability to pose numerous problems and thus engage in many EOMs. Students often posed problems that involved the design of snowflakes, checkerboards or diagonals, and sometimes representations such as hearts, animals, or names. Participants often revised their representations to make them better fit their archetypes or to play with variations.

The small tiles at the *Pentagons* exhibit led to a different pace of activity. The tiles could be put together to fill up an entire table and so some participants spent their time covering space through a process of gap-filling. When participants made designs with the pentagon tiles, they rarely made large revisions or re-attempted a EOM by starting over, as was common at the *Pattern Machine,* likely because it required more effort and time, and because moving the tiles did not have the same unique aesthetic appeal as clicking the *Pattern Machine's* buttons.

Possible combinations. Because the pentagon tiles fit together in many ways, including ways that produced multiple curvatures, participants were able to make more intricate designs than those possible on the *Pattern Machines*. Thus, participants spent more time posing problems that resulted in complex and aesthetically pleasing designs at the *Pentagons*. More frequently than at the *Pattern Machine,* participants at the *Pentagons* exhibit showed their finished products to nearby peers, parents, or volunteers, indicating a kind of pride in what they had produced.

Affordances for Encountering Mathematical Concepts

The affordances of the tools at each exhibit influenced the mathematical concepts that participants encountered. These mathematical concepts emerged (though were rarely explicitly named) as the design of the exhibits pushed back against participants' expectations. We consider this push-back from the tools to elicit a kind of tacit problem-posing as participants work to overcome trouble in accomplishing their EOMs. As described below, the *Pattern Machine* afforded encounters with mathematical ideas of symmetry and curvature because it constrained participants' ability to achieve them. Similarly, the *Pentagons* afforded encounters with spatial orientation and patterning because finding tiles whose orientation and color both fit the pattern was often constrained by the dual-colored tile.

Grids: symmetry and curvature. While both the *Pattern Machine* and the *Pentagons* afforded symmetric designs, the 9x9 grid of the *Pattern Machine* included a true middle, and its square shape cued participants to produce rotational, vertical, and horizontal symmetries. Participants would often use two hands to create patterns on the *Pattern Machine*, mirroring the activity of their right and left hands. When participants recruited the symmetry of their own bodies as resources, they often created symmetric designs unproblematically. However, when only one hand was used, participants sometimes struggled. Thus, the *Pattern Machine's* design seemed to afford producing symmetry, but often ended up constraining it when the EOM was representational. Because symmetry became a source of trouble for some participants, the

Pattern Machine afforded problem-posing around the mathematical concept of symmetry as they worked to repair their designs.

Radial symmetry was also a frequent feature of participants' designs at the *Pentagons* exhibit. In fact, the most common EOM at the pentagon station was a pinwheel made out of curved pentagons. The pentagon tiles also made it easier for participants to produce symmetric designs with curvature. The *Pattern Machine's* constraints for curvature (its grid-like nature) seemed to afford problem-posing around curvature when they experienced trouble creating representations that met their prototypes (e.g., a heart). Creating curvature and symmetry were far less problematic at the *Pentagons,* and thus participants likely engaged in less sensemaking with these concepts.

Scholarly Significance: Problem-Posing and Problem-Solving as Reflexive Practices

This analysis has explored mathematical problem-posing in informal contexts that foreground agentic interaction with mathematical-tools. For the most part, current discussions of tool affordances are largely missing from problem-posing literature. Furthermore, while much of the problem-posing literature separates problem-posing from problem-solving — much as problem-solving literature separates problem-solving from problem-posing — our analyses suggest that disentangling these aspects of mathematical inquiry may remove ecological validity, because the processes are so clearly reflexive. Thus, this study bridges a gap in the field's understanding of problem-posing by examining mathematical activity as a process of problem-posing and solving, mediated by mathematical-tools. Because our analysis remained close the phenomena, this work contributes a nuanced perspective on problem-posing in informal contexts.

While we do not claim that problem-posing would look identical in other contexts, we posit that understanding mathematical problems as emerging from tensions between children's exploration and the constraints of mathematical tools is a productive way to begin re-imagining engagement in the discipline. Rather than conceptualizing mathematical problems as existing within textbooks, we can conceptualize mathematical problems as something learners generate in interaction. By expanding our knowledge of how different designs for mathematical tools and environments can afford mathematical problem-posing, we can begin to make mathematics learning more accessible and meaningful.

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Figures

(a) (b) (c) (d)

Figure 1. The mathematical tools at the (a) *Pattern Machine* exhibit, (b) the *Pentagons* exhibit, (c) the *Tiles and Patterns* exhibit, and (d) the *Eggs and Crate* exhibit.

Figure 2. Child with video capturing device.

Tables

Table 1

Selection criteria for the two featured exhibits.

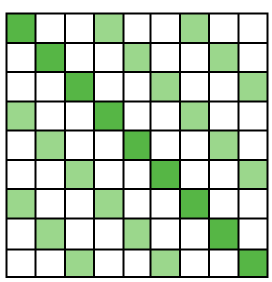
Selection Criteria	Pattern Machine	Pentagons
% of time at MOAS (*M* for *n* = 171)	27%	35%
Mathematical Design Features	Discrete 9x9 array	Irregular pentagons that tile the plane in many ways

Table 2

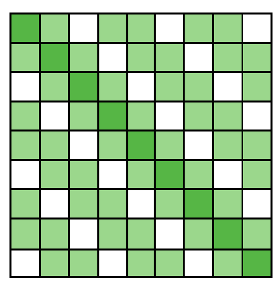
Codebook excerpt for goal coding at the Pattern Machine *exhibit.*

Code	Description
Diagonals	diagonal lines alternating pushed down/ popped up, usually started with the center diagonal (longest diagonal) and moved out to the sides, sometimes this resulted a checkerboard if they did not leave space between the diagonals.

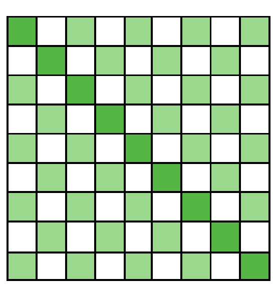
Diagonals, don't result in checkerboard



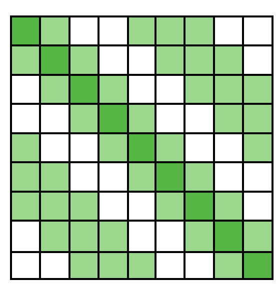
Diagonals that do not result in checkerboard, thick



Diagonals, result in checkerboard



Diagonals, thick



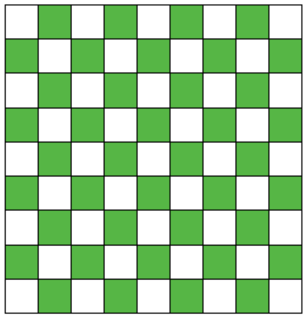
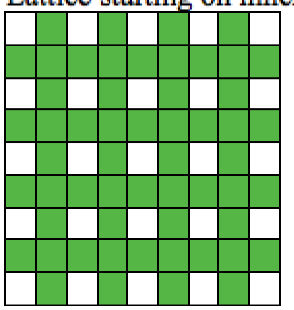
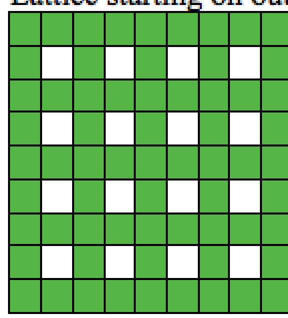
Checkerboard	alternating buttons pushed down/up over the entire pattern machine 
Lattice	Made rows and columns, skipping every other. The lattice that starts on the outside looks exactly the same as the “windows” code but was constructed differently (by rows and columns rather than by squares and +’s). Lattice starting on inner rows  Lattice starting on outer rows 

Table 3

Number and duration of goals at focal exhibits.

EOM Duration Measures (min)	Pattern Machine	Pentagons
Average	1.6	3.2
Standard deviation	2.8	2.5
Median	0.9	2.4
Minimum	0.2	0.4
Maximum	22.4	11.3
Total number of EOMs	75	37
Average EOMs per participant	4.4	2.2