A new perspective on seismic intensity measures (IMs)

M. Grigoriu
Cornell University
Ithaca, NY USA

ABSTRACT: The usefulness of current intensity measures (IMs) and fragilities are assessed in a setting in which the probability law of the seismic ground acceleration process is known. It is shown that typical demand parameters and IMs are weakly dependent so that fragilities defined as functions of these measures provide limited information for seismic design.

1 INTRODUCTION

Intensity measures (IMs) are intended to provide sufficient information on seismic ground accelerations to predict the seismic performance of arbitrary structural systems. These measures are used to construct current fragilities, i.e., probabilities that structural systems enter specified damage states conditional on IM values, and develop computational tools for performance based earthquake engineering. To be useful, IMs have to be *efficient*, i.e., structural demand parameters D conditional on IMs have small variances, and *sufficient*, i.e., the distributions of the conditional random variables $D|\text{IM}$ are completely defined for given IMs (1, 3). If IMs are efficient, the distributions of the conditional random variables $D|\text{IM}$ can be estimated satisfactorily from relatively small sets of structural responses. If IMs are sufficient, the conditional random variables $D|(\text{seismic hazard})$ and $D|\text{IM}$ will have similar properties so that fragilities will characterized accurately seismic performance of structural systems.

Efficiency and sufficiency of IMs have been studied extensively. Yet, these properties cannot be quantified precisely since the distributions of IMs and demand parameters are not known due to the limited information on the seismic acceleration process $A(t)$. Concepts of the information theory (1) and benchmark studies (3) have been used to rate IMs. These studies recognize that sufficient IMs may not exist and that resulting ratings of IMs depend on the particular information metrics and benchmark studies used in the analysis.

To overcome these difficulties, we represent the seismic acceleration processes $A(t)$ by the seismological model in (4) so that the probability law of $A(t)$ is known. Following current practice, the IM is the pseudo-acceleration response spectrum $S_a(T)$ of $A(t)$.

It is shown that the IM $S_a(T)$ and the demand parameter D are weakly dependent. Qualitative arguments and quantitative metrics based on the multivariate extreme value theory (MEVT) are used to assess the relationship between $S_a(T)$ and D. It is concluded that current fragilities provide limited information on the seismic performance of structural systems. We believe that fragilities need to be defined as functions of the parameters of the probability law of $A(t)$, e.g., current fragilities can be replaced with fragility surfaces of the type introduced in (2).

2 AN ILLUSTRATION

Denote by $X_{\text{sdof}}(t)$ and $X(t)$ responses to a seismic ground acceleration process $A(t)$ of a single degree of freedom (SDOF) linear oscillator with damping ratio ζ and period T and of an arbitrary structural system. Generally, $X(t)$ is a vector-valued process. For simplicity we consider real-valued demand parameters, e.g., an interstory displacement, a floor acceleration, or any other functional of $X(t)$. In this study, the demand parameter is defined by $D = \max_{0\leq t\leq\tau} \left|h\big(X(t)\big)\right|$, where τ denotes the duration of the seismic event and h maps X(t) into a real-valued response. As stated, the IM is the pseudo-spectral acceleration $S_{\text{a}}(T) = (2\,\pi/T)^2 \max_{0\leq t\leq\tau} |X_{\text{sdof}}(t)|$.

The random variables $S_a(T)$ and D are dependent as functionals of the same input, the seismic ground acceleration process $A(t)$. Yet, the dependence between these random variables is likely to be weak since the stochastic processes $X_{\text{sdof}}(t)$ and $X(t)$ are likely to have very different properties as solutions of simple linear and complex nonlinear random vibration problems to $A(t)$. For example, if

$A(t)$ is Gaussian, $X_{\text{sdof}}(t)$ and $X(t)$ are Gaussian and non-Gaussian processes with very different frequency bands. This qualitative observation suggests that the demand D and the conditional random variable $D|S_a(T)$ have similar properties and, as a result, current fragilities are of limited practical use. This intuition is confirmed by quantitative metrics presented in the following examples and later in the study.

Let $X(t)$, $0 \leq t \leq \tau$, be the displacement of a Duffing oscillator with parameters (ν_0, ζ, β) which is at rest at the initial time and is subjected to a ground acceleration process $A(t)$. Then, $X(t)$ satisfies the differential equation

$$\ddot{X}(t) + 2\,\zeta\,\nu_0\,\dot{X}(t) + \nu_0^2\left(X(t) + \beta\,X(t)^3\right) = -A(t), \tag{1}$$

with initial conditions $X(0) = 0$ and $\dot{X}(0) = 0$. If $\beta = 0$ and $\nu_0 = 2\,\pi/T$, then $X(t) = X_{\text{sdof}}(t)$ is the displacement of a linear oscillator with damping ratio ζ and period T. Otherwise, $X(t)$ is the response of a simple oscillator with cubic nonlinearity. The random variables $S_{\text{a}}(T) = (2\,\pi/T)^2 \max_{0\leq t\leq \tau} |X_{\text{sdof}}(t)|$ and $D = \max_{0\leq t\leq\tau} |X(t)|$ are dependent as functionals of the seismic ground acceleration process $A(t)$. The degree of dependence between these two random variables depends on the intensity of the ground motion.

For small seismic excitation, the contribution of the cubic nonlinearity $\nu_0^2\,\beta\,X(t)^3$ to the displacement $X(t)$ of the Duffing oscillator is insignificant so that $X(t)$ will be similar to the displacement $X_{\text{sdof}}(t)$ of the associate linear oscillator ($\beta = 0$). The random variables $S_{\text{a}}(T)$ and D are strongly dependent so that $S_{\text{a}}(T)$ is a very good IM. For large seismic excitations, the cubic nonlinearity $\nu_0^2\,\beta\,X(t)^3$ contributes to $X(t)$ so that $X(t)$ and $X_{\text{sdof}}(t)$ have different properties. For example, if $A(t)$ is a Gaussian process, then $X_{\text{sdof}}(t)$ and $X(t)$ are Gaussian and non-Gaussian processes. The dependence between $S_{\text{a}}(T)$ and D is likely to be weak so that $S_a(T)$ can be an unsatisfactory IM.

These qualitative arguments are consistent with the scatter plots of $\left(S_a(T)/(2\,\pi/T)^2, D\right)$ in Fig. 1 which have been obtained from $n = 500$ independent samples of this random vector for $\nu_0 = 2\,\pi$, $\zeta = 0.05$, $\beta = 3$, $\tau = 20$, and a stationary Gaussian band-limited white noise (BLWN) $A(t)$ with mean 0, variance 1, and frequency band $[0, 10]$. The top, middle, and bottom panels are for ground accelerations $A(t)$ scaled by 1, 5, and 10. For small ground excitations corresponding to a scale factor of 1 (left panel), the dependence between $S_a(T)/(2\,\pi/T)^2$ and D is nearly perfect. The differences between the responses $X(t)$ and $X_{\text{sdof}}(t)$ are negligible. For large ground excitations corresponding to a scale factor of 10 (right panel), the dependence between $S_a(T)/(2\,\pi/T)^2$ and D is weaker. The middle panel corresponds to moderate earthquakes, the scale factor is 5. It represents a transition between the extreme cases in the left and right panels.

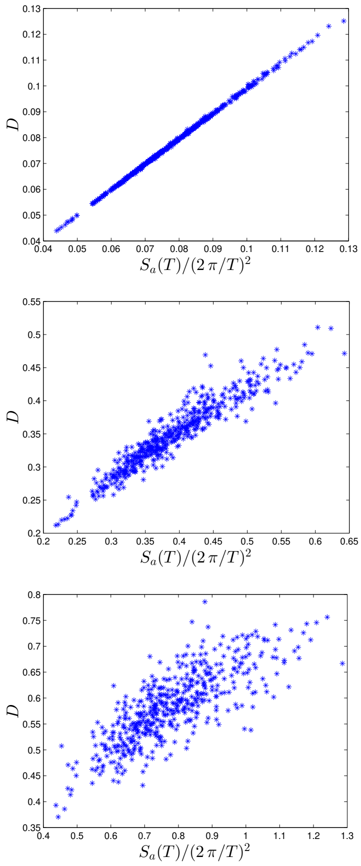

Figure 1: Scatter plots of $n = 500$ samples of $\left(S_a(T)/(2\,\pi/T)^2, D\right)$ for $\beta = 3$ and a stationary Gaussian BLWN $A(t)$ with mean 0, variance 1, and frequency band $[0, 10]$ scaled by 1, 5, and 10 (top, middle, and bottom panels)

We note that even in this favorable setting for IMs (the Duffing oscillator is a conservative SDOF), the predictive capability of $S_a(T)$ deteriorates with the magnitude of the structural responses. For example, the correlation coefficients between the top m samples of the demand parameter D and the corresponding samples of $S_a(T)$ are 0.8107, 0.7684, 0.6418, 0.4381, and 0.3408 for $m = 10{,}000$, 9,000, 1,200, 600, and 150. The estimates are based on 10,000 samples of $\left(S_a(T)/(2\,\pi/T)^2, D\right)$ corresponding to a scale factor of 10.

3 DEPENDENCE METRICS

We use two statistical tools, correlation coefficients and concepts of the multivariate extreme value theory (MEVT), to examine the relationship between the random variables D and $S_a(T)$ and assess the capability of current fragilities to predict accurately the seismic performance of structural systems.

The correlation coefficients are attractively simple but rather crude metrics for the dependence between random variables. For example, suppose $X_1 \sim N(0, 1)$ is a standard Gaussian variable with mean 0 and variance 1. The non-Gaussian random variable

$X_2 = X_1^2 - 1$ has mean 0, variance 2, and is uncorrelated to X_1 since $E[X_1 X_2] = E[X_1^3] - E[X_1] = 0$ (the Pearson correlation coefficient). Yet, $\text{Var}[X_2 \mid X]_1 = 0$ since, given X_1, X_2 is known. Other correlation models also view X_1 and X_2 as nearly uncorrelated, e.g., estimates of the Spearman and Blomqvis correlation coefficients based on 100,000 independent samples of X are 0.0062 and 0.0014.

This simple example indicates that correlation coefficients are inadequate metrics for the dependence between the components of the two dimensional random vector $X = \big(X_1 = S_a(T), X_2 = D\big)$. Moreover, we are particularly interested in the likelihood that X_1 and X_2 are simultaneously large, rather than the overall relationship between these random variables which is provided by correlation coefficients. This interest is justified by the fact that large demand parameters D are likely to cause extensive damages or even structural failure.

The multivariate extreme value theory (MEVT) is employed to quantify the dependence between simultaneously large values of $X_1 = S_a(T)$ and $X_2 = D$. We present a heuristic description of concepts of the MEVT and computational tools relevant to our discussion. A rigorous discussion on MEVT concepts can be found in (5). Let $\{x^{(k)}\}$, $k = 1, \ldots, n$, be n independent samples of $X = (X_1, X_2)$. We use these samples to quantify the dependence between simultaneously large values of X_1 and X_2.

Suppose $X_1, X_2 > 0$ almost surely (a.s.), i.e., $P(X_i > 0) = 1$ for $i = 1, 2$, and the marginal distributions of $X = (X_1, X_2)$ coincide, i.e., $F_1 = F_2$, so that the components of X have similar scales. In polar representation $X = (X_1, X_2) = \big(V\,\cos(\Theta), V\,\sin(\Theta)\big)$, where $V = \|X\|$ is a norm in $\mathbb{R}^2$ and $\Theta = \tan^{-1}(X_2/X_1)$. The polar representation of the samples $\{x^{(k)}\}$ of X is $x^{(k)} = (x_1^{(k)}, x_2^{(k)}) = \big(v^{(k)}\,\cos(\theta^{(k)}), v^{(k)}\,\sin(\theta^{(k)})\big)$, $k = 1, \ldots, n$, with the previous notations. Set $v_0 > 0$ and relatively large. Samples of X with distance to origin $v^{(k)} > v_0$ are of interest as they correspond to simultaneously large values of X_1 and X_2. The selection of v_0 is critical to assure that the right tails of the components of X are accurately represented (5).

The histogram $s(\theta)$ of the subset of $\{x^{(1)}, \ldots, x^{(n)}\}$ with $v^{(k)} > v_0$ has the support $[0, \pi/2]$ and is referred to as *angular measure*. If most of the mass of $s(\theta)$ is concentrated at $\theta = 0$ and $\pi/2$, extremes of X_1 and X_2 are nearly independent. If most of the mass of $s(\theta)$ is concentrated at θ away from $\{0, \pi/2\}$, large values of X_1 and X_2 are strongly dependent. Histograms $s(\theta)$ between these two limit cases describe various degrees of dependence between large components of X.

For illustration, suppose $X_i = \lambda\, G_0^2 + (1-\lambda)\, G_i^2$, $i = 1, 2$, where G_0, G_1, and G_2 are independent standard Gaussian variables. The components X_1 and X_2 of X follow the same distribution. They are perfectly dependent for $\lambda = 1$ and independent for $\lambda = 0$. The left and right panels of Fig. 2 show scatter plots for

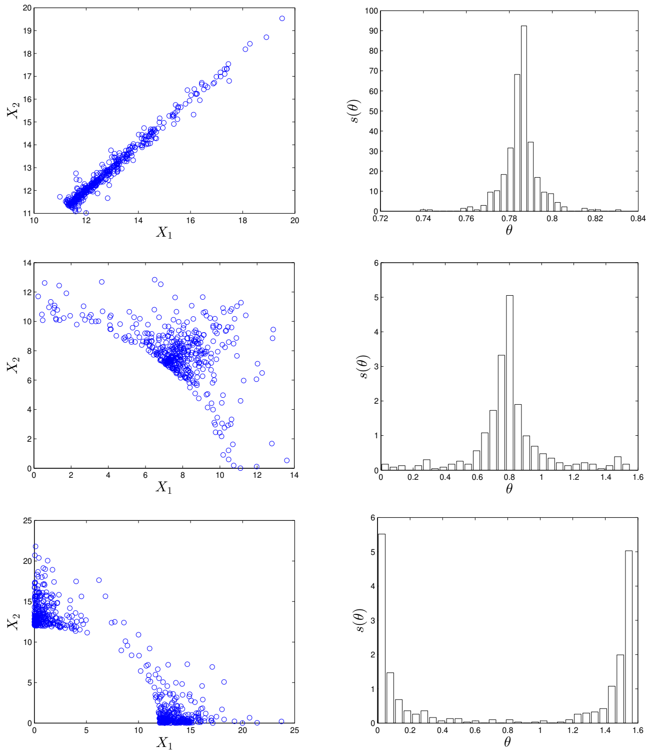

Figure 2: Scatter plots for samples of X with distance to the origin of the coordinate system larger than v_0 (left panels) and histograms of the angular measure $s(\theta)$ (right panels) for $\lambda = 0.9$, 0.5, and 0.1 (top, middle, and bottom panels)

samples of X with distance to the origin of the coordinate system larger than v_0, and histograms of the angular measure $s(\theta)$. The top, middle, and bottom panels correspond to $\lambda = 0.9$, 0.5, and 0.1 and thresholds $v_0 = 16$, 10, and 12. The resulting sample sizes of the vectors X used to construct $s(\theta)$ are 435, 448, and 585, respectively, and have been extracted from $n = 100,000$ independent samples of X. Large values of the components of X are strongly and weakly dependent for $\lambda = 0.9$ and 0.1. The angular measures $s(\theta)$ in the right panels quantifies the degree of dependence between the components of X. For $\lambda = 0.9$, the samples of X are aligned along the 45^o line and the angular measure is concentrated about $\pi/4 \simeq 0.7854$. The support of $s(\theta)$ is approximately $[0.74, 0.83]$. This shows that simultaneously large values of X_1 and X_2 are strongly dependent. For $\lambda = 0.5$, the samples of X are less concentrated along the 45^o line and $h(\theta)$ takes non-zero values in $[0, \pi/2]$. For $\lambda = 0.1$, the samples of Y cluster on the axis of the system of coordinates and most of the mass of $s(\theta)$ is concentrated is small vicinities of $\theta = 0$ and $\pi/2$. This means that large values of X_1 are likely to be associated with small values of X_2 and viceversa.

4 DEPENDENCE OF $S_A(T)$ AND D FOR BOUC-WEN MODELS

Suppose $X(t)$ is the displacement of a Bouc-Wen SDOF system defined by

$$\begin{aligned} &\ddot{X}(t) + 2\,\zeta\,\nu_0\,\dot{X}(t) + \nu_0^2\,\big(\rho\,X(t) \\ &\qquad + (1-\rho)\,W(t)\big) = -A(t), \quad \text{where} \\ &\qquad \dot{W}(t) = \gamma\,\dot{X}(t) - \alpha\,|\dot{X}(t)|\,|W(t)|^{\chi-1}\,W(t) \\ &\qquad - \beta\,\dot{X}(t)\,|W(t)|^{\chi}, \end{aligned} \tag{2}$$

ν_0 are ζ as in Eq. 1, α, β, γ, ρ, and χ are positive constants, and $A(t)$ denotes the seismic acceleration process. The parameters α, β, γ, ρ, and χ control the model behavior, e.g., the system is linear for $\rho = 1$ and, for $\rho \neq 1$, its nonlinear/hyteretic behavior depends strongly on β and γ. The following numerical results are for $\nu_0 = 2\,\pi$, $\zeta = 0.05$, $\alpha = 0.5$, $\beta = 5$, $\gamma = 3$, $\rho = 0.1$, and $\chi = 1$. The ground acceleration $A(t)$ is given by the specific barrier model (SBM) for a seismic event with magnitude $m = 5.0$, source-to-site distance $r = 185$km, and a rock site. It is a zero-mean stationary Gaussian process with spectral density in the top panel of Fig. 3. The middle panel shows $n = 500$ independent samples of $\big(X_1 := S_a(T), X_2 = D\big)$ for $\tau = 20$ seconds. The samples marked with circles denote the top 20 samples of $\big(S_a(T), D\big)$ which have been used to estimate the angular measure $s(\theta)$ shown in the bottom panel of the figure. The visual inspection of this scatter plot suggests that $S_a(T)$ and D are weakly dependent so that $S_a(T)$ is not likely to be a satisfactory IM for this Bouk-Wen oscillator. This observation is consistent with the angular measure $s(\theta)$ in the right panel of the figure. Since most of the mass of $s(\theta)$ is concentrated in small vicinities of $\theta = 0$ and $\theta = \pi/2$, simultaneously large values of $S_a(T)$ and D are unlikely. Hence, large values of D are not likely to be associated with large values of $S_a(T)$ which means that $S_a(T)$ is an unsatisfactory IM.

In summary, the dependence between simultaneously large values of $S_a(T)$ and D is weak so that the conditional random variable $D|S_a(T)$ and the random variable D have similar distributions which implies that the fragility of the Bouc-Wen system under consideration is nearly independent of $S_a(T)$. This is of significant concern since the Bouc-Wen SDOF system is a simplistic model of realistic structures, large values of the demand parameter D and the intensity measure $S_a(T)$ are nearly independent, and large values of D are associated with excessive damage or even structural collapse.

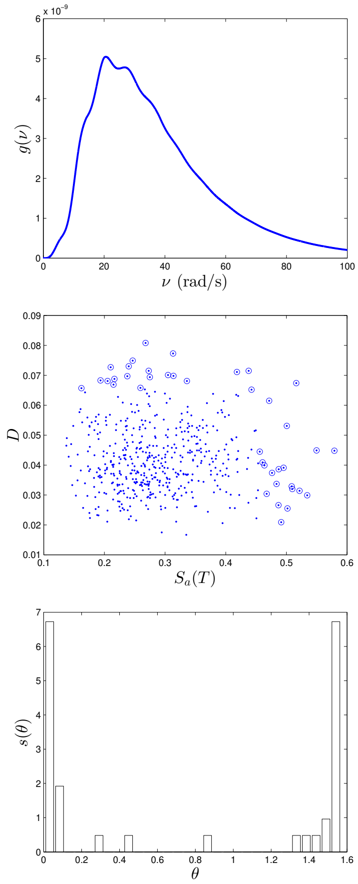

Figure 3: Spectral density of $A(t)$, $n = 500$ independent samples of $S_a(T)$ and D, and angular measure of $\big(S_a(T), D\big)$ (top, middle, and bottom panels)

5 CONCLUSIONS

It was shown that a demand parameter D of an SDOF Bouc-Wen system subjected to a seismic ground acceleration process $A(t)$ with known probability law is weakly dependent to ordinates of the intensity measure $S_a(T)$ particularly for large seismic events. This finding is at variance with the assumption that $S_a(T)$ captures sufficient information on the seismic ground acceleration process $A(t)$ such that demand parameters D of nonlinear, complex, multi-degree of freedom structures to $A(t)$ correlates satisfactorily with ordinates of $S_a(T)$. It strongly suggests to explore alternative methods to characterize the intensity of seismic events and construct fragilities, e.g., the parameters of the probability law of the seismic ground acceleration process, and develop alternative fragilities, e.g., fragility surfaces of the type proposed in (2).

REFERENCES

[1] H. Ebrahimian, F. Jalayer, A. Lucchini, F. Mollaioli, and G. Manfredi. Preliminary ranking of alternative scalar and vector intensity measures of ground shaking. *Bulletin of Earthquake Engineering*, 2015. DOI 10.1007/s10518-015-9755-9.

[2] C. Kafali and M. Grigoriu. Seismic fragility analysis: Application to simple linear and nonlinear systems. *Earthquake Engineering and Structural Dynamics*, 36:1885–1900, 2007.

[3] N. S. Kwong, A. K. Chopra, and R. K. McGuire. Evaluation of ground motion selection and modification proce-

dures using synthetic ground motions. *Earthquake Engineering and Structural Dynamics*, 44:1841–1861, 2015. DOI: 10.1002/eqe.2558.

[4] A. S. Papageorgiou and K. Aki. A specific barrier model for the quantitative description of the inhomogeneous faulting and the prediction of strong ground motion. Part I: Description of the model. *Bulletin of the Seismilogical Society of America*, pages 693–722, 1983.

[5] S. Resnick. *Heavy-Tail Phenomena: Probabilistic and Statistical Modeling*. Springer, New York, 2007.