Geometrically Conformal Quadrilateral Surface-Reconstruction for MoM-SIE Simulations

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Abstract—**We present a higher order quadrilateral surface remeshing approach for application to the method of moments. We briefly introduce the motivation for high-quality quadrilateral meshes for accurate and efficient application of cutting edge integration and singularity extraction approaches. Results are demonstrated for simple surfaces and compared with a common quadrilateral remeshing algorithm.**

Keywords—computational electromagnetics, computational geometry, integral equations, method of moments, quadrilateral remeshing, surface reconstruction.

I. Introduction

One of the key challenges in numerical methods, geometric discretization, profoundly affects accuracy of a given simulation. An unnecessarily fine discretization (for a given frequency) produces an unnecessarily large number of unknowns, and therefore a correspondingly large linear system. Conversely, accurate geometric modeling of curved surfaces is essential for an overall optimal higher order solution [1]. In addition, even integration procedures strongly depend on the geometric discretization of a structure. Namely, improved singularity extraction and integration techniques have recently shown promise toward further improving the method of moments (MoM) in the surface integral equation (SIE) formulation based on higher order quadrilateral modeling; these novel approaches, however, are subject to increasing error as patch corner angles exceed ~120 degrees [2], [3]. Existing quadrilateral remeshing techniques, meanwhile, often neglect the corner angles of elements. A method, therefore, to efficiently remesh surfaces to quadrilaterals with concern for electrical size and corner angles provides benefits in terms of improving the applicability of these new integration schemes.

This paper focuses on the application of a geometrically conformal (angle preserving, explicitly biholomorphic) quadrilateral remeshing approach for computational electromagnetics problems to address these concerns. In particular, we provide examples in comparison to a non-conformal approach.

II. Conformal Remeshing

To produce a quadrilateral mesh with the desired properties, we begin, in general, with a point cloud describing the surface. The construction begins with a triangle meshing; Giesen, et al. provides a triangle meshing algorithm which offers several attractive guarantees, namely preservation of topology [4]. Following the reconstruction with an isotropic refinement assists in the parameterization.

Many methods exist to parameterize a surface, conformal or otherwise. However, for our application we require a conformal mapping. Conformal mappings such as discrete Ricci-flow [5], least squares conformal [6], and discrete conformal [7], each provide a means to map a surface to a parametric domain, ideally preserving the angles of the mapped surface. In this paper we show results using discrete-conformal mapping. Barycentric coordinates, unique on triangles, are then used to lift a sample pattern from a parametric domain to the original surface to produce vertices of the completed quadrilateral mesh. Crucially, as Barycentric coordinates depend on the corner angles of the triangles, conformality is preserved.

This method of remeshing provides several key advantages, especially for the higher order MoM-SIE. With a precomputed parameterization, additional remeshing is computed inexpensively; when coupled with adaptive sampling of the parametric domain, this permits a rapid means to adjust the electrical size of the patches for a given frequency. Additionally, curvilinear meshing arises naturally; given a separate sampling density for the patches and the interpolation points, quadrilateral-curvilinear patches can be generated identically to the linear case, with vertices used instead as geometric sample points for interpolation.

III. Results and Discussion

To demonstrate the viability of our implementation, we show results for two geometries: spherical and toroidal. For each case, we present several tiers of sampling density in the parametric domain to simulate a frequency-dependent remeshing.

As a comparison, we test against the 4 to 8 Subdivision method included in MeshLab [8].

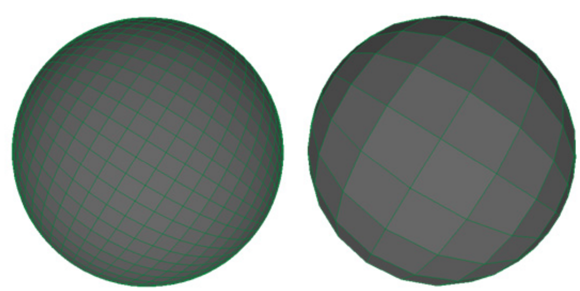

Fig. 1. Sphere quadrilateral remeshing for 2000 sample points (left) and 200 sample points (right).

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We observe in Fig. 1 that a coarser sampling preserves the general constraints on the corner angles of the patches. Patch size decreases at the seams of the surface to maintain conformality. Note also that the quadrilateral remeshing does not exactly replicate the boundary of the original triangular surface mesh, with small distortions occurring; however, this effect is realized primarily for very coarse sampling or abrupt changes in curvature and is largely inconsequential.

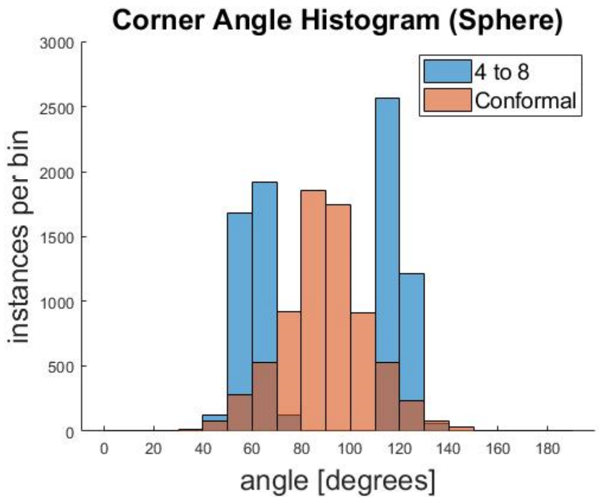

Fig. 2. Corner angle histogram for conformal remeshing and 4 to 8 subdivision for a sphere.

As seen in Fig. 2, the conformal remeshing distributes corner angles well within the desired range. In contrast, 4 to 8 subdivision—for the same number of patches—forces most angles outside this range. With further iterations of 4 to 8 subdivision, the distribution of corner angles improves, but at the expense of significantly greater element count compared to conformal remeshing.

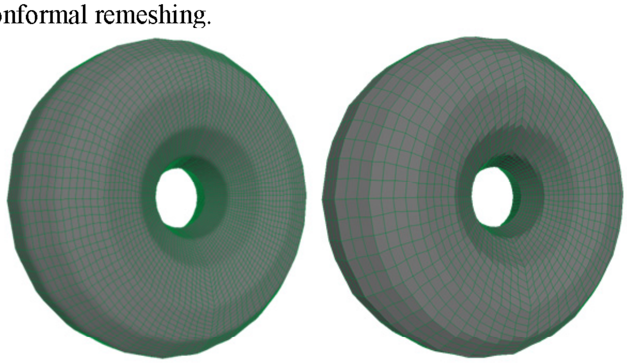

Fig. 3. Torus quadrilateral remeshing for 10000 sample points (left) and 4000 sample points (right).

As in Fig. 1, the torus of Fig. 3 exhibits similar behavior when varying sampling density. However, the change in quadrilateral size as a function of the curvature is much more apparent; note that in the center of the torus, the patch size shrinks rapidly to preserve conformality.

In terms of angle distribution, the histogram in Fig. 4 illustrates again the quality of the quadrilateral elements generated by a conformal remeshing. For the same number of elements, 4 to 8 subdivision produces many more nearly-degenerate quadrilateral elements, or elements with three collinear vertices.

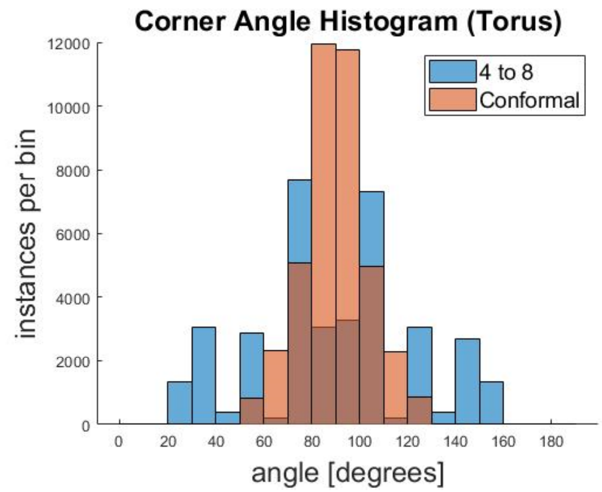

Fig. 4. Corner angle histogram for conformal remeshing and 4 to 8 subdivision for a torus.

By precomputing a conformal parameterization for a scatterer, new quadrilateral meshes with optimal corner angles can be constructed efficiently. The shape of the parent triangular mesh is also preserved well in the quadrilateral reconstruction. In principle, quadrilateral patches can be flat or curved, described by arbitrary geometric orders in parametric coordinates.

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