Automatic Generalized Quadrilateral Surface Meshing in Computational Electromagnetics by Discrete Surface Ricci Flow

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***Abstract*—We present methods for the automated generation of high-order generalized quadrilateral surface meshes for computational electromagnetics (CEM) using discrete surface Ricci flow and parametric domain iterative refinement. We offer a brief overview of discrete surface Ricci flow as it pertains to this work and demonstrate its usefulness for higher-order quadrilateral mesh generation. We present extremely-high-order surface meshes, automatically generated by this method using minute numbers of quadrilateral elements with exceptionally high geometrical orders, demonstrating the method's robustness on both simple and complicated surfaces. The examples of the NASA almond, one of the most popular benchmarking examples, and a more complicated fighter jet model, with as few as four elements of up to 50th geometrical order, are first of a kind, showing great promise for higher order CEM.**

Keywords—automated surface meshing, Ricci flow, higher-order methods, large-domain modeling, mesh refinement, method of moments, quadrilateral elements, computational electromagnetics

I. Introduction

Higher-order methods are of growing interest in computational electromagnetics (CEM). Although such techniques have shown great promise reducing the system dimension for method of moments (MoM) solvers [1], the complexity of the needed higher-order quadrilateral meshes has limited the promised applicability of these techniques from true large-domain modeling. The methodology for generating such meshes is often left out of scope [2], semi-manual, or, at best, unable to effectively generate large-domain elements [1]. Overall, meshing is probably the most challenging and restricting component of higher order CEM, and an open problem of great relevance.

The work presented in this paper seeks to resolve this issue, offering a robust generalized quadrilateral meshing approach able to fully utilize the geometric flexibility of high-order elements to represent arbitrary surfaces with high fidelity. The advocated approach requires little, if any, user input and can be used to convert an existing triangular, low-order quadrilateral, or point cloud surface description into a high-quality generalized quadrilateral mesh of arbitrary element count and arbitrary order. We present meshing results for a relatively simpler surface, the NASA almond (a standard CEM benchmark target), and a complicated fighter jet surface. The results are extremely promising.

II. Meshing by Ricci Flow with Iterative Refinement

Popularized by its role in Perelman's 2006 proof of the Poincaré conjecture [3], Ricci flow offers a mathematical framework for diffusing irregularities in the metric of a Riemannian manifold. In the context of this work, surface Ricci flow, by the discrete formulation described in [4], allows the generation of a conformal (angle-preserving) mapping between a surface of choice, and a homeomorphic (or non-homeomorphic, given a suitable cut graph) surface of prescribed Gaussian curvature, here constituting a parametric domain for the mesh. For instance, this allows the NASA almond to be mapped to the unit sphere or cut and mapped to the plane. Information on the prescribed surface can then be conformally mapped back to the original surface. In our application, this information comprises element sample points.

By choosing a simple prescribed surface on which these sample points can be easily defined and manipulated, we can exert a high degree of control over the resultant re-mapping. However, the mapping produced by Ricci flow preserves only angles, not relative areas, so simply mapping a uniform grid of sample points from the prescribed surface to the surface of choice produces poor results for our application, leading to wide discrepancies in mesh fidelity between minimally-warped and highly-warped portions of the mesh.

To overcome this, we instead construct a non-uniform grid of sample points. Beginning with a single element, and iteratively splitting rows and columns of the sample grid where high degrees of warping are encountered, we can produce an accurate, sample-efficient mesh that captures detail in even the most warped regions of the surface.

III. Results and Discussion

We show extremely-high-order quadrilateral meshing results for two illustrative examples. To demonstrate the proposed method on a well-known CEM test case, we have applied adaptive Ricci flow meshing to the NASA almond, shown in Fig. 1. The presented mesh contains 32 10^{th} order quadrilateral elements and was generated by mapping a triangular surface mesh of the almond to the plane using one surface cut.

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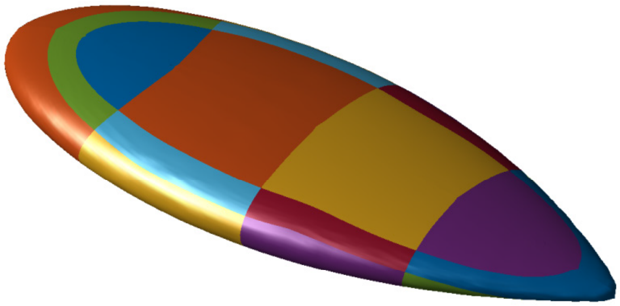

Fig. 1. NASA almond modeled by 32 10^{th} order elements using adaptive sampling.

To test the proposed method on a more-ambitious practical case, we attempted to represent a fighter jet using only four generalized quadrilateral elements. As with the almond, an existing triangular mesh of the jet was flattened to the plane using one cut, shown in Fig. 2 (a). Evident in Fig. 2 (a) is the root of the issue – extreme warping at the jet fins, wings, wing tip, and fuselage tip. The warping is most severe at the fin tips, as evidenced by Fig. 2 (b), showing the relative deformation for each quadrilateral element with uniform sampling.

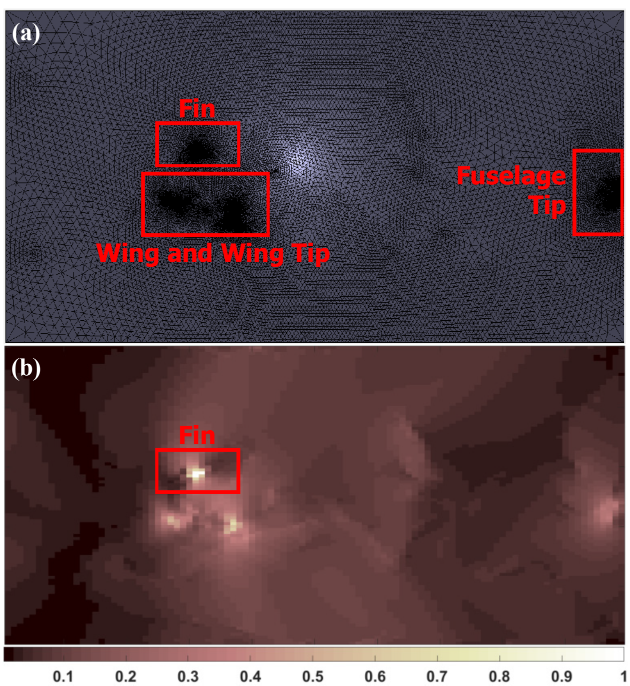

Fig. 2. (a) Triangle mesh of fighter jet cut and conformally flattened to the plane using discrete surface Ricci flow. Highly warped areas near the fin, wing, wing tip, and fuselage tip are boxed in red. (b) Relative deformation (normalized) for each quadrilateral element demonstrating high degree of warping at fighter jet fin when uniform sampling is used.

Figure 3 shows the resultant quadrilateral mesh, using four 50^{th} order elements and uniform sampling. The high degree of warping evident in Fig. 2 at the fin tip greatly reduces mesh fidelity in these regions when coupled with uniform sampling, causing drastic loss of information and failure to replicate the original surface. In Fig. 4, we use adaptive sampling to more-efficiently distribute element sample points over the fighter jet surface, leading to a high-quality reconstruction using the same element count and order.

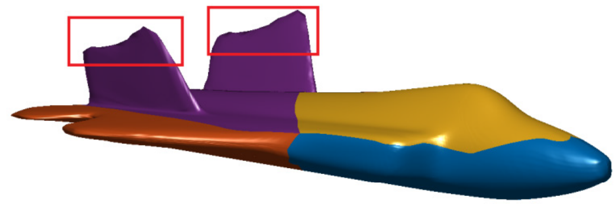

Fig. 3. Fighter jet modeled using four 50^{th} order elements, uniformly sampled. Low mesh fidelity caused by severe warping at the fin tips leads to loss of information and poor representation of original surface.

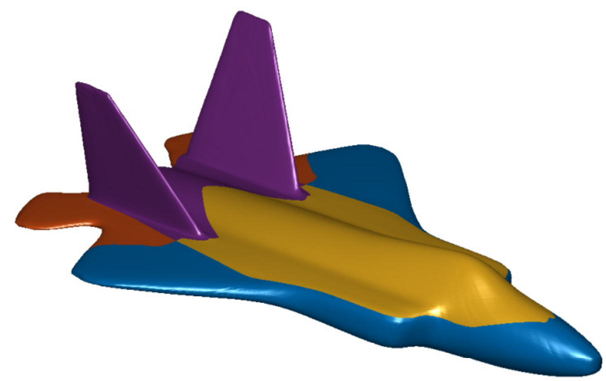

Fig. 4. Fighter jet modeled using four 50^{th} order elements, adaptively sampled. No mesh fidelity issues are present, and fin tips are accurately represented.

IV. Conclusions

We conclude that discrete surface Ricci flow, coupled with careful adaptive sampling in the parametric domain, offers a viable and robust approach to automatic generalized quadrilateral meshing. The presented high-fidelity extremely-high-order meshes (e.g., a very accurate fighter jet model with as few as four elements of as high as 50^{th} geometrical order) are the first of their kind and show great promise toward large-domain MoM modeling by high-order quadrilateral surface elements. Generalization to other types of elements frequently used in CEM is also possible.

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