Data Decoding Analysis of Next Generation GNSS Signals

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BIOGRAPHIES

Lorenzo Ortega received the Eng. degree in Telecommunication Engineering in 2014, and the M.Sc degree in Telecommunication and Signal Processing in 2016, both from Zaragoza University. In 2016, he coursed the second year of International Master of Electronics Systems for Embedded and Communicating Applications at the INP-ENSEEIHT (University of Toulouse) as an exchange student. Currently he is a PhD student at ENSEEIHT (University of Toulouse) and studies methodologies to optimize the signal structure of the new generation of Galileo. His thesis is funded by CNES (French Space Agency) and Thales Alenia Space.

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Charly Poulliat received the Eng. degree in Electrical Engineering from ENSEA, Cergy-Pontoise, France, and the M.Sc. degree in Signal and Image Processing from the University of Cergy-Pontoise, both in June 2001. From Sept. 2001 to October 2004, he was a PhD student at ENSEA/University Of Cergy-Pontoise/CNRS and received the Ph.D. degree in Signal Processing for Digital Communications from the University of Cergy-Pontoise. From 2004 to 2005, he was a post-doctoral researcher at UH coding group, University of Hawaii at Manoa. In 2005, he joined the Signal and Telecommunications department of the engineering school ENSEA as an Assistant Professor. He obtained the habilitation degree (HDR) from the University of Cergy-Pontoise in 2010. Since Sept. 2011, he has been a full Professor with the National Polytechnic Institute of Toulouse (University of Toulouse, INP-ENSEEIHT). His research interests are signal processing for digital communications, error control coding and resource allocation.

Marie-Laure Boucheret received the Eng. degree in Electrical Engineering from ENST Bretagne, Toulouse, France, and the M.Sc. degree in Signal Processing from the University of Rennes, both in June 1985. In June 1997, she received the Ph.D. degree in Communications from TELECOM ParisTech, and the "Habilitation Ãă diriger les recherches" in June 1999 from INPT University of Toulouse. From 1985 to 1986 she has been a research engineer at the French Philips Research Laboratory (LEP). From 1986 to 1991, she has been an engineer at Thales Alenia Space, first as a project engineer (TELECOM II program) then as a study engineer at the transmission laboratory. From 1991 to 2005 she was a Associated Professor then a Professor at TELECOM ParisTech. Since March 2005 Marie-Laure Boucheret is a Professor at the National Polytechnic Institute of Toulouse (ENSEEIHT - University of Toulouse). She is also with the Signal and Communication group of the IRIT Laboratory.

Aubault-Roudier is a radionavigation engineer in the navigation/location signals department in CNES, the French Space Agency, where she is involved in the optimization of GNSS signals. Marion Aubault-Roudier graduated as an electronics engineer in 2011 from ENAC (Ecole Nationale de lâĂŹAviation Civile) in Toulouse, France. She received her PhD in 2015 from the Department of Mathematics, Computer Science and Telecommunications of the INPT

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Hanaa Al-Bitar is a GNSS systems engineer at Thales Alenia Space France. She received her Ph.D in RadioNavigation in 2007 form the ENAC, in Toulouse, France, in the field of GNSS receivers. She joined TAS-F in 2012. Her main activities focus on GNSS signal processing, and SBAS Land Earth Stations signal processing and design.

ABSTRACT

Error correcting schemes are fundamental in the new generation of data navigation signals. Thanks to those, the system has the capability to correct possible data navigation errors, which potentially induces delays in first fix of the receiver. In the GNSS receivers, those error correcting schemes use the Log Likelihood Ratio (LLR) as the input of the decoding algorithm. Until now, the LLR was always computed under the Gaussian assumption and considering perfect Complete State Information (CSI), which does not hold in most of the real scenarios. Then; in this paper we proposed several methods to compute the LLR, considering a set of realitic scenarios and considering that perfect CSI is not available at the receiver. We test the proposed LLRs for several new generation GNSS signals.

I INTRODUCTION

One of the most challenging issues in designing the new GNSS signals (i.e., GPS L1C and Galileo E1-B) was to introduce the capability to correct the data navigation message errors. An error in the data navigation message involves that the frame information data is not reliable, which potentially induces a delay in the time to acquire or track a position's device. In order to introduce the capability to correct errors, additional redundant data, generated by a channel coding mechanism is included within the data navigation message. Moreover, an error correcting implementation at the receiver chain must be addressed [5].

The main motivation of this work is to analyze data decoding of the next generation of GNSS signals. Those signals seek to improve various figures of merit on the different phases of navigation solution. Indeed, those signals (i.e, GPS L1C, Galileo E1-B, as well as several proposal of the new Galileo acquisition signal) propose the addition of a channel coding scheme in order to enhance the robustness of the data decoding process. Thank to that, a reduction of the Time To First Fix (TTFF) is achieved, especially in challenging or hostiles GNSS environments.

In modern channel coding theory, the Log Likelihood Ratio (LLR) is computed as the input to the error correcting algorithm. The LLR is a statistical test to compare the goodness of fit between the probabilities of receiving a positive or a negative logic bit. In the GNSS receiver the LLR is typically used under the Gaussian channel assumption [14], resulting in

$$\mathrm{LLR} = \frac{2y_n}{\sigma_n^2} \tag{1}$$

where y_n is the normalized sample after the match filter and σ_n^2 is the instantaneous noise variance, which is considered known. However, the assumption does not hold in real situations, where the channel variance has to be estimated from a carrier-to-noise-density ratio (C/N_0) estimates [6]. Moreover, in some scenarios such as the urban or the intentional interference environment, the Gaussian distribution does not model well the received data distribution [3]. For those reasons, in this paper we reformulate the problem of computing the LLR under more realistic channel assumptions and we provide simulation result for several GNSS signals.

This paper is organized as follow: Section II presents the channel coding schemes of several GNSS signals. Section III presents a more realistic subset of scenarios along with solution to compute the LLRs considering perfect Complete State Information (CSI) [14]. Section IV present the proposed solution to compute the LLRs considering that no perfect CSI is available at the receiver. Section V presents and compares the error correcting performance in terms of Clock and Ephemerides Data error rate of the channel coding schemes presented in Section II, between the LLRs solution presented in section III (considering perfect CSI) and the LLRs solution presented in section IV (when no CSI is available at the receiver) Conclusion are finally drawn in Section VI.

II NAVIGATION SIGNALS

In this section, we present the channel coding schemes which protect the CED of several GNSS signals, of interest in this work due to their channel coding schemes:

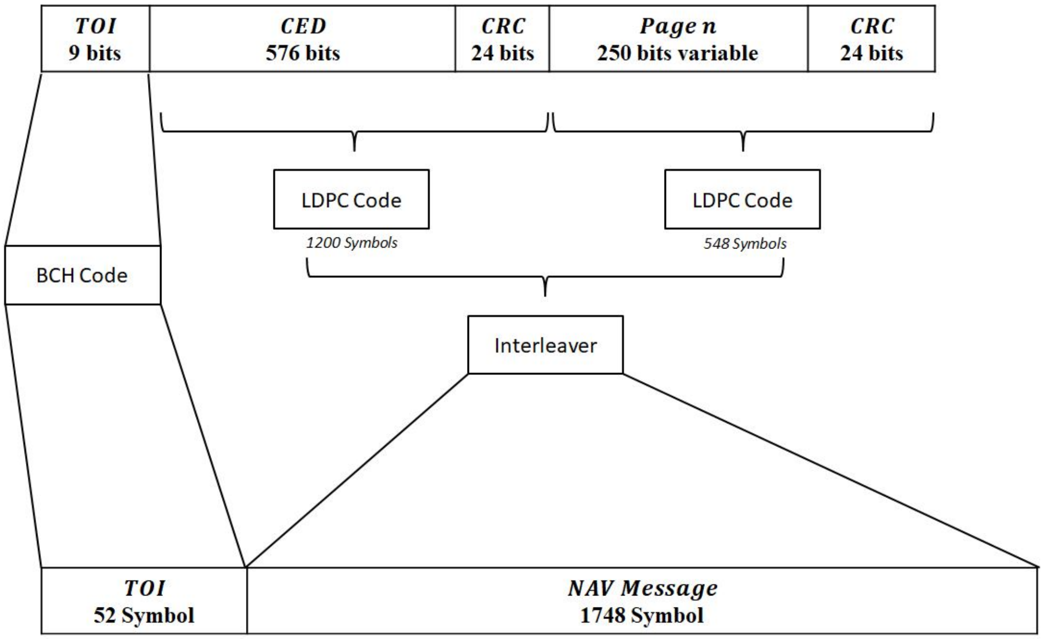

Figure 1: GPS L1C Message Structure

- **GPS L1C navigation message [1]**: The message modulated onto the GPS L1C signal consists of a set of consecutive frames, where the complete data message set is broadcasted to users. This signal is emitted in the new generation of GPS III satellites. A frame is divided into three subframes of various lengths. The first subframe consists of 9 bits of Time of Interval (TOI) data. The subframe 2 is composed of 600 bits of non-variable clock and ephemeris data with Cyclic Redundancy Check (CRC). The content of subframe 3 nominally varies from one frame to the next and is identified by a page number; the size of the block is 250 bits. Considering this navigation message, we are interested on the GPS L1C subframe 2, where it is allocated the CED. Thus, the subframe 2 is encoded by two encoders in serial form. The outer code is a cyclic code called CRC-24Q which provided the integrity of the data message. The inner encoder is an irregular LDPC code of rate 1/2 which provides error correction. Moreover, The 9-bit TOI data of subframe 1 are encoded with a BCH code and Subframe 3 data are encoded using a 24-bit CRC code and an irregular Low Density Parity Check (LDPC) Forward Error Correction (FEC) code with a parity check matrix of size 274×548. Encoded data from subframe 2 and 3 are then block-interleaved (48-36) in order to struggle the burst errors. The resulting 1800 symbols represent one message frame, which are then broadcast at rate 100 symbols per second. Figure 1 gives the structure of the described GPS L1C message.

  At the receiver, the computed LLR is used a the input of the LDPC decoder. After the LDPC decoding process, the CRC-24Q decoder is computed in order to verify the received solution.

- **Galileo E1-OS navigation message [2]**: The Galileo E1-OS navigation message is stored inside the I-NAV message. The structure of the I/NAV message is presented in [2]. It is composed by 15 nominal pages, each one with a duration of 2 seconds, represents the 30 seconds duration of the I/NAV subframe structure illustrated in Figure 2. Within the subframe structure, pages 1, 2, 11 and 12 are used to store the 4 CED information words. Therefore, every 30 seconds, 4 CED information words are provided by the I/NAV message. Each nominal page is subdivided in 2 subpages. Each subpage has 120 bits, which are encoded by a rate one-half convolutional code with polynomial generators in octal representation given by $(171, 133)$ [2]. At the output of the convolutional encoder, 240 data symbols are interleaved by a 30×8 block-interleaver. Finally, 10 bits of synchronization are added at the beginning of the data frame to achieve synchronisation to the page boundary. At the receiver, each page is decoded independently. First, the synchronisation pattern allows the receiver to achieve synchronisation to the page boundary. Each page is then de-interleaved by a 8×30 block-interleaver and decoded by a Viterbi algorithm [7]. Finally, the CRC is computed and compared with the CRC field. In order to retrieve the CED, pages 1, 2, 11 and 12 must be CRC validated.

- **Evolution Galileo E1-OS navigation message [15] [12]**: Thanks to the flexibility of the Galileo navigation message structure, an *Evolution* of the I/NAV message was proposed. This optimization seeks to reduce the TTD and improving the CED robustness under the precondition to keep the backward compatibility with the current I/NAV message structure. Thus, pages 8 and 9 (refers to Figure 2) were selected to store the redundant data generated by the extra Reed Solomon (RS) outer coding channel method, considered as Forward Error Correction 2 (FEC2). With these considerations in mind, a general outer channel coding $(n, k) = (6, 4)$ structure can be defined in order to generate those extra redundant bits, where n is equivalent to the total number of available bits (redundant + information bits) and k is the number of information bits. In order to keep backward compatibility, systematic information bits are stored in pages 1, 2, 11 and 12 while redundant bits are stored in pages 8 and 9. Remark that the RS codes are characterizes by the MDS property, then CED can be decoded when a number $k = 4$ of pages are retrieved.

- **Galileo E1D with Protograph Root LDPC code [11]**: The Galileo E1D signal is an hypothetical future Galileo signal which is designed to improve the performance of several point of the Galileo system. The CED is encoded by two encoders in serial form. The outer code is a cyclic code called CRC-24Q which provided the integrity of the data message. The inner encoder is a Protograph Root LDPC code of rate 1/2 which provides error correction. The Protograph Root LDPC code are an optimized version of the Root LDPC codes [4] through the Protograph EXIT chart optimization algorithm [9]. Those codes are characterized by the Maximum Distance Separable (MDS) full diversity properties over the block fading channel and under the Belief Propagation (BP) decoding algorithm [4]. Those properties are essential in order to retrieved the CED faster and thus reduce the TTFF. At the receiver, the computed LLR is used a the input of the LDPC decoder. After the LDPC decoding process, the CRC-24Q decoder is computed in order to verify the received solution.

- **Galileo E1D with Rate Compatible Root LDPC codes [10]**: It is a second proposal for the E1D signal. The CED is encoded by two encoders in serial form. The outer code is a cyclic code called CRC-24Q which provided the integrity of the data message. The inner encoder is a Rate compatible Root LDPC code of rate 1/3 which provides error correction. Those codes are an extension of the Root LDPC code proposed in [4] and are characterized by the Maximum Distance Separable (MDS) full diversity properties over the block fading channel and under the Belief Propagation (BP) decoding algorithm [4]. Moreover, those codes have the rate compatible property, which allows to improve the error correction capability by combining several retrieved blocks at the receiver. At the receiver, the computed LLR is used a the input of the LDPC decoder. After the LDPC decoding process, the CRC-24Q decoder is computed in order to verify the received solution.

III SYSTEM MODEL AND LLR COMPUTATION WITH PERFECT CSI

- **1st channel**: Let's consider the following channel model. We represent the transmitted message as a binary vector $\mathbf{u} = [u_1, \ldots, u_K]^\top$ of K bits. This message is encoded into a codeword $\mathbf{c} = [c_1, \ldots, c_N]^\top$ of length $N > K$ and mapped to BPSK symbols $x_n = \mu(c_n) \in \{-1, 1\}$, where n represents each symbol index and we impose $\mu(0) = 1$ and $\mu(1) = -1$. Then the symbol is spread by a pseudo-random noise (PRN) sequence that can be expressed in vector form as $\mathbf{p}_n \in \mathbb{R}^L$. L corresponds to the number of chips of the PRN sequence. Then, the transmitted symbol per coded bit is given by

$$\mathbf{x}_n = x_n \cdot \mathbf{p}_n \in \mathbb{R}^L, \quad n = \{1, \ldots, N\} \tag{2}$$

with the convention that we define column vectors.

The transmission channel is modeled as a binary-input AWGN noise channel with zero-mean and variance σ_n^2. Then, the received symbol sequence is modeled as

$$\mathbf{y}_n = \mathbf{x}_n + \mathbf{w}_n \in \mathbb{R}^L, \quad n = \{1, \ldots, N\} \tag{3}$$

where $\mathbf{w}_n \sim \mathcal{N}(0, \sigma_n^2 \mathbf{I})$. That is, the noise power remains constant to the entire codeword.

Thus, LLR associated to the n-th symbol is defined as [8]

$$\mathcal{L}_n = \ln\left(\frac{P(c_n = 0|y_n)}{P(c_n = 1|y_n)}\right) = \ln\left(\frac{P(x_n = 1|y_n)}{P(x_n = -1|y_n)}\right). \tag{4}$$

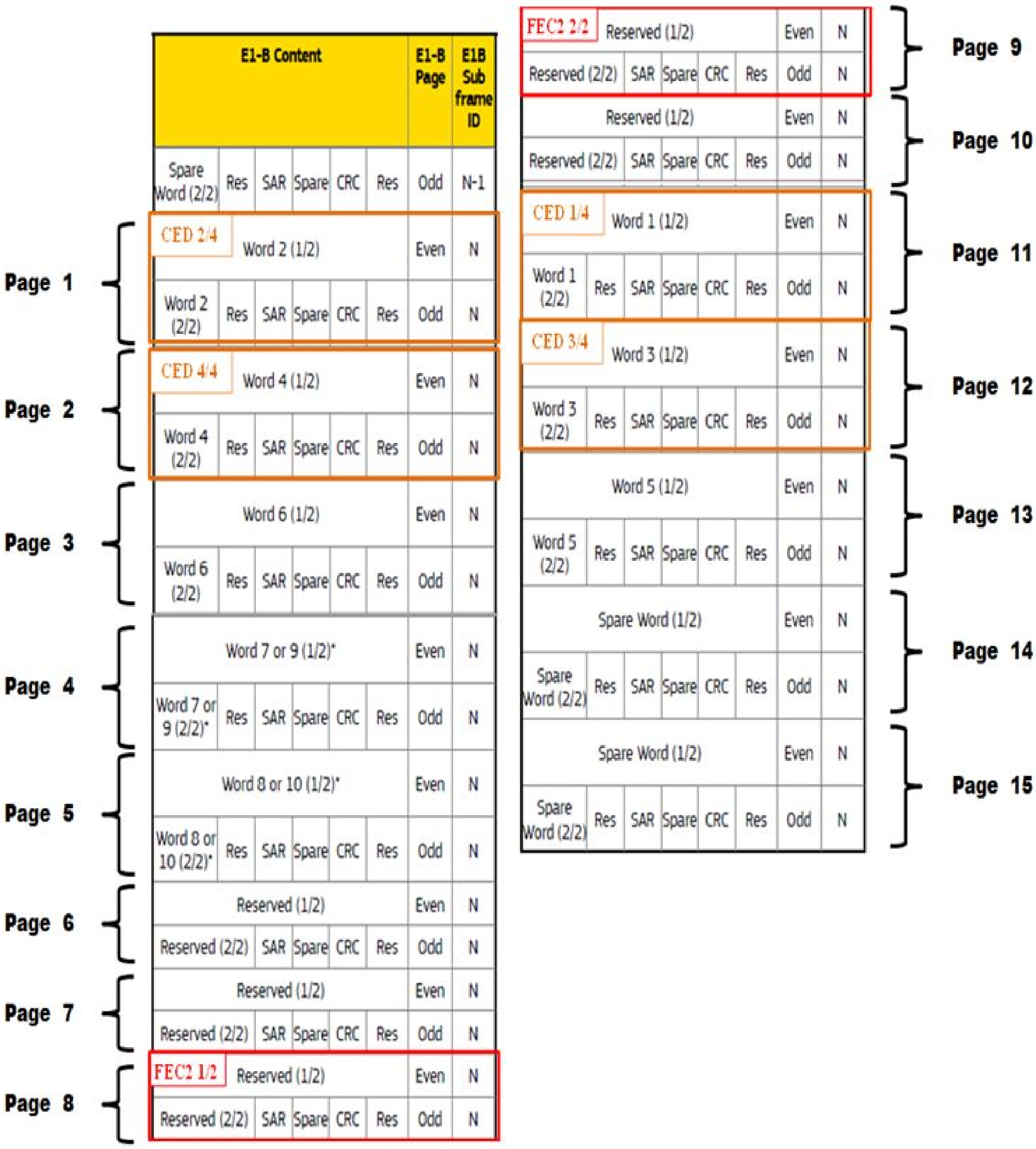

Figure 2: I/NAV E1-OS Nominal Subframe Structure

Obtained LLRs are then used to feed the input of the soft input channel decoder. When perfect CSI is assumed, the LLR can be trivially computed as

$$\mathcal{L}_n = \frac{2}{\sigma_n^2}\mathbf{y}_n^T\mathbf{p}_n \ , \tag{5}$$

which explicitly assumes that the noise variance is perfectly known at the receiver. In practice, this assumption does not hold and σ_n^2 has to be estimated. Then, having access to a point estimator as the C/N_0 estimator, the mismatched LLR is

$$\mathcal{L}_n = \frac{2}{\hat{\sigma}_n^2}\mathbf{y}_n^T\mathbf{p}_n \ , \tag{6}$$

where $\hat{\sigma}_n^2$ is the noise variance estimate at the n-th symbol through the C/N_0 estimator. Remark that variance can be computed through:

$$\sigma_n^2 = 10^{-(E_s/N_0)_n/10} \tag{7}$$

and

$$E_s/N_0 = C/N_0 - 10\log_{10}(\frac{1}{A_{data}^2}T_s) \tag{8}$$

where A_{data}^2 is the power of the data component and T_s is the symbol period.

- **2nd channel**: The transmission channel can be modeled as Gaussian noise (Jamming interference) with is integrated along with the Gaussian thermal noise at the receiver input. We represent the transmitted message as a binary vector $\mathbf{u} = [u_1, \cdots, u_K]$ of K bits. This message is encoded into a codeword $\mathbf{c} = [c_1, \cdots, c_N]$ of length $N > K$ and mapped to BPSK symbols $x_n = \mu(c_n) \in \{-1, 1\}$, where n represents each symbol and we impose $\mu(0) = 1$ and $\mu(1) = -1$. Modeling the transmission channel with additional real-valued a AWGN with noise variance σ^2 and a AWGN jamming with affect to the entire codeword with a noise variance σ_I^2, then the received symbol sequence is:

$$y_n = x_n + w_n + w_{I,n} \in \mathbb{R}, \quad n = \{1, \dots, N\}, \tag{9}$$

where $w_n \sim \mathcal{N}(0, \sigma^2)$ and $w_{I,n} \sim \mathcal{N}(0, \sigma_I^2)$ are the statistical models for the noise and the jamming. Let's denote $w_{n+I} = w_n + w_{I,n}$, then $w_{n+I} \sim \mathcal{N}(0, \sigma^2 + \sigma_I^2)$

Thus, LLR associated to the n-th symbol is defined in equation 4. Obtained LLR are used to feed the input of the decoder. Considering perfect CSI, the LLR expression simplifies to:

$$\mathcal{L}_n = \frac{2}{\sigma^2 + \sigma_I^2} \cdot y_n \ , \tag{10}$$

which explicitly implies that the variance σ^2 and σ_I^2 are known. In practice, this assumption does not hold and even if σ^2 is known, σ_I^2 remains unknown and must be estimated. Considering now the GNSS system, because of the low data rate [**?**], the variance can be throug a point estimator such as the $C/(N_0 + I)$ estimator. Thus, denoting $\sigma_{T+I}^2 = \sigma^2 + \sigma_I^2$, σ_{T+I}^2 can be computed symbol by symbol through

$$\sigma_{(T+I)_n}^2 = 10^{-(E_s/(N_0+I))_n/10} \tag{11}$$

where $(E_s/(N_0 + I))_n$ is in decibels (dB) scale and $(N_0 + I)$ represents the noise and interference density. Then, the computed LLRs simplify to

$$\mathcal{L}_n = \frac{2}{\sigma_{(T+I)_n}^2} \cdot y_n \ , \tag{12}$$

which explicitly implies that variance $\sigma_{(T+I)_n}^2$ is known symbol a symbol. Considering a real scenario, where no perfect CSI is available at the receiver, point estimators can lead to a symbol-wise noise variance estimation. Then, the computed LLRs yields to

$$\mathcal{L}_n = \frac{2}{\hat{\sigma}_{(T+I)_n}^2} \cdot y_n \ , \tag{13}$$

where $\hat{\sigma}_{(T+I)_n}^2$ represents the $\sigma_{(T+I)_n}^2$ estimation.

- **3rd channel**: The transmission channel model a jammer device which broadcasts a Gaussian interference which disrupts some percentage of the codeword symbols. We represent the transmitted message as a binary vector $\mathbf{u} = [u_1, \cdots, u_K]$ of K bits. This message is encoded into a codeword $\mathbf{c} = [c_1, \cdots, c_N]$ of length $N > K$ and mapped to BPSK symbols $x_n = \mu(c_n) \in \{-1, 1\}$, where n represents each symbol and we impose $\mu(0) = 1$ and $\mu(1) = -1$. Modeling some percentage $P \in [0, 1]$ the transmission channel with additional real-valued AWGN with instantaneous noise variance σ^2 and a AWGN jamming with instantaneous noise variance σ_I^2, then the received symbol sequence is:

$$y_n = \begin{cases} x_n + w_n \in \mathbb{R}, & n \in \mathbb{Q}, \\ x_n + w_n + w_I \in \mathbb{R}, & n \in \mathbb{S}, , \end{cases} \tag{14}$$

where $w_n \sim \mathcal{N}(0, \sigma^2)$ and $w_I \sim \mathcal{N}(0, \sigma_I^2)$ are the statistical models for the noise and jamming. $\mathbb{Q}$ is the set of bits not affected by the jamming noise and $\mathbb{S}$ is the set of bits harmed with the jamming. Remark that $\frac{|\mathbb{S}|}{|\mathbb{Q}|+|\mathbb{S}|} = P$

Considering perfect CSI, for each n-th symbol of the codeword the LLR expression simplified to:

$$\mathcal{L}_n = \begin{cases} \frac{2}{\sigma^2} \cdot y_n \ , & n \in \mathbb{Q}, \\ \frac{2}{\sigma^2+\sigma_I^2} \cdot y_n \ , & n \in \mathbb{S} \end{cases} \tag{15}$$

which explicitly implies that the variance σ^2 and σ_I^2 are known. In real scenarios, σ^2 and σ_I^2 are unknown then must be estimated, yielding equation (15) to:

$$\mathcal{L}_n = \begin{cases} \frac{2}{\hat{\sigma}^2} \cdot y_n \ , & n \in \mathbb{Q}, \\ \frac{2}{\hat{\sigma}^2+\hat{\sigma}_I^2} \cdot y_n \ , & n \in \mathbb{S} \end{cases} \tag{16}$$

Remark that estimate simultaneously σ^2 and σ_I^2 is an extremely complex work.

- **4th channel**: An urban fading channel (in this paper we consider a 2-state Prieto channel [13], where the PLL is capable to perfectly track the phase of the signal) is presented. Then, we assume the transmission of a binary message vector $\mathbf{u} = [u_1, \cdots, u_K]$ of K bits. Using a binary error correcting code of rate $R = K/N$, this message is then encoded into a binary codeword $\mathbf{c} = [c_1, \cdots, c_N]$ of length $N > K$ and mapped to binary phase shift keying (BPSK) symbols $x_n = \mu(c_n) = 1 - 2.c_n \in \{-1, 1\}$, $\forall n = 1 \cdots N$. Modeling the transmission channel as an uncorrelated fading channel with additional real-valued additive white Gaussian noise (AWGN) with noise variance σ^2, the received symbol sequence is then given by

$$y_n = h_n \cdot x_n + w_n \in \mathbb{R}, \quad n = \{1, \ldots, N\}, \tag{17}$$

where both w_n and h_n are identically and independently distributed (i.i.d.) random variables such that $w_n \sim \mathcal{N}(0, \sigma^2)$ and $h_n \sim p(h)$ respectively.

By definition, the expression of the LLR for the n-th symbol is given for the case of perfect CSI (h_n is perfectly known) as [14]:

$$\mathcal{L}_n = \ln \left(\frac{P(c_n = 0|y_n, h_n)}{P(c_n = 1|y_n, h_n)} \right) = \ln \left(\frac{P(x_n = 1|y_n, h_n)}{P(x_n = -1|y_n, h_n)} \right) . \tag{18}$$

Assuming that $, \forall n = 1 \cdots N, c_n$ are i.i.d., Equation (18) can also be written as

$$\mathcal{L}_n = \ln \left(\frac{P(y_n|x_n = 1, h_n)}{P(y_n|x_n = -1, h_n)} \right) , \tag{19}$$

considering perfect CSI, the LLR simplifies to

$$\mathcal{L}_n = \frac{2}{\sigma^2} h_n \cdot y_n, \tag{20}$$

which explicitly implies that the variance σ^2 and the fading gain h_n are known. In real environments, that does not hold on and both parameters must be estimated.

IV CLOSE FORM LLR APPROXIMATION WITH NO PERFECT CSI

- **Solution to the 1st channel:** Considering no perfect CSI, the variance is unknown, it should be modeled as a random variable characterized by a Probability Density Function (PDF). Then, identifying that our knowledge on the variance is due to an estimate of $(Es/N0)_n$ (refers to equation 7) that was assumed to be Gaussian, the variance is characterized by a Log-Normal PDF whose mean and variance are estimated by the Narrowband Wideband Power Ratio (NWPR) algorithm [6]. Then, based on the bayesian inference, we reformulate the problem of obtaining the LLR values by first computing the joint PDF of symbols and variance, which is then marginalized in order to compute the desired LLR. In order to compute the marginalized distribution, we impose a conjugate prior formulation in order to obtain an analytic closed form solution that can be computed as a function of the receiver correlator output. Thus, the LLR approximation expression yields to:

$$\mathcal{L}_n = -(a_n + \frac{1}{2}) \left[\ln \left(\frac{1}{b_n} + \frac{(y_n - 1)^2}{2} \right) - \ln \left(\frac{1}{b_n} + \frac{(y_n + 1)^2}{2} \right) \right] \tag{21}$$

where $\hat{a}$ and $\hat{b}$ can be approximated by the close form values:

$$\hat{a} \approx 1/\sigma_\lambda \qquad \hat{b} \approx \sigma_\lambda^2 e^{\mu_\lambda + \frac{\sigma_\lambda^2}{2}} \ , \tag{22}$$

with the details to compute μ_λ and σ_λ based on the mean μ_{E_s/N_0} and variance $\sigma^2_{E_s/N_0}$ estimated by the NWPR algorithm.

$$\mu_\lambda = \left(\mu_{E_s/(N_0)} \log_e(10)\right)/10 \tag{23}$$

$$\sigma_\lambda = \left(\sigma_{E_s/(N_0)}/10\right) \log_e(10) \tag{24}$$

- **Solution to the 2nd channel:** Considering no perfect CSI, the variance joint variance between σ^2 and σ_I^2, denoted as σ^2_{T+I} is unknown and it can be modeled as a random variable characterized by a PDF. Then, identifying that our knowledge on the variance is due to an estimate of $(E_s/(N_0 + I))_n$ (refers to equation 10) that was assumed to be Gaussian, the variance is characterized by a Log-Normal PDF whose mean and variance are estimated by the NWPR algorithm. As it was already proposed for the first scenario, based on the Bayesian theory, we reformulate the problem obtaining the LLR values by first computing the joint PDF of symbols and variance, which is then marginalized in order to compute the desired LLR. In order to compute the marginalized distribution, we impose a conjugate prior formulation in order to obtain an analytic closed form solution that can be computed as function of the receiver correlator output. Thus, the LLR approximation expression yields to:

$$\mathcal{L}_n = -(a_n + \frac{1}{2}) \left[\ln \left(\frac{1}{b_n} + \frac{(y_n - 1)^2}{2} \right) - \ln \left(\frac{1}{b_n} + \frac{(y_n + 1)^2}{2} \right) \right] \tag{25}$$

where $\hat{a}$ and $\hat{b}$ can be approximated by the close form values:

$$\hat{a} \approx 1/\sigma_\lambda \qquad \hat{b} \approx \sigma_\lambda^2 e^{\mu_\lambda + \frac{\sigma_\lambda^2}{2}} \ , \tag{26}$$

with the details to compute μ_λ and σ_λ based on the mean $\mu_{E_s/(N_0+I)}$ and variance $\sigma^2_{E_s/(N_0+I)}$ estimated by the NWPR algorithm.

$$\mu_\lambda = \left(\mu_{E_s/((N_0+I))} \log_e(10)\right)/10 \tag{27}$$

$$\sigma_\lambda = \left(\sigma_{E_s/((N_0+I))}/10\right) \log_e(10) \tag{28}$$

- **Solution to the 3rd channel:** Considering the pulsed jamming channel, in this paper we propose a solution when powerful Gaussian interference disrupts a few symbols in the codeword ($P \approx 0.05 - 0.4$). In that case, heavy tails appears in the observation distribution and the Gaussian model does not fit properly anymore.

Then, the *LLR approximation* proposed to approximate the observable distribution by an equivalent Laplacian model. Thus, modeling the transmission channel with additional real-valued additive Laplacian noise:

$$y_n = x_n + w_L \tag{29}$$

where $w_L \sim \mathcal{L}(0, 2 \cdot c^2)$ and c^2 is the variance of the approximate Laplacian model. Considering no perfect CSI, c is unknown and can be modeled as a random variable. Then, identifying that our knowledge on the variance is due to the C/N_0 estimator that was assumed to be Gaussian, the random variable variance is characterized by a Log-Normal PDF whose mean and variance are estimated by the NWPR algorithm. Remark that c can be directly computed symbol per symbol from $E_s/(N_0)$.

$$c = \frac{1}{\sqrt{2}} 10^{-(E_s/(N_0))/20} \tag{30}$$

Thus, based on the Bayesian inference , we compute the LLR values by first computing the joint PDF of symbols and variance, which is then marginalized in order to compute the desired LLR. In order to compute the marginalized distribution, we impose a conjugate prior formulation in order to obtain an analytic closed form solution that can be computed as a function of the receiver correlator output. Thus, the LLR approximation expression yields to:

$$\mathcal{L}_n = -(a+1) \left[\ln \left(\frac{1}{b} + |y_n - 1| \right) - \ln \left(\frac{1}{b} + |y_n + 1| \right) \right] \tag{31}$$

where $\hat{a}$ and $\hat{b}$ can be approximated by the close form values:

$$\hat{a} \approx 1/\sigma_\rho \qquad \hat{b} \approx \sigma_\rho^2 e^{\mu_\rho + \frac{\sigma_\rho^2}{2}} \ , \tag{32}$$

with the details to compute μ_ρ and σ_ρ based on the mean $\mu_{E_s/(N_0+I)}$ and variance $\sigma^2_{E_s/(N_0+I)}$ estimated by the NWPR algorithm.

$$\mu_\rho = \left(\mu_{E_s/(N_0+I)} \log_e(10) \right) /20 \tag{33}$$

$$\sigma_\rho = \left(\sigma_{E_s/(N_0+I)}/20 \right) \log_e(10) \tag{34}$$

- **Solution to the 4th channel:** In the last channel model, we provide a solution based on the statistical knowledgde of the CSI. Then, considering perfect knowledge of σ^2 and the mean μ_h and variance σ_h^2 of the PDF of the fading gain $p(h)$, that is assumed to follow a Gaussian variable. We reformulate the problem of obtaining the LLR values by first computing the joint PDF of symbols and fading gain, which is then marginalized in order to compute the desired LLR. In order to compute the marginalized distribution, we impose a conjugate prior formulation in order to obtain an analytic linear closed form solution that can be computed as a linear function of the receiver correlator output. Remark that in real scenarios, $p(h)$ will follow Rayleigh or Rice distributions [14], then computed mean and variance of the Gaussian distribution are those which minimize the difference between distributions.

  Thus, the LLR approximation expression yields to a linear close form expression:

$$\mathcal{L}_n = -\mu_h^2 \frac{\left(1 - \frac{y_n}{\mu_h}\right)^2}{2\left(\sigma^2 + \sigma_h^2\right)} + \mu_h^2 \frac{\left(-1 - \frac{y_n}{\mu_h}\right)^2}{2\left(\sigma^2 + \sigma_h^2\right)} = \frac{2 y_n \mu_h}{\left(\sigma^2 + \sigma_h^2\right)} \ . \tag{35}$$

V RESULTS

In order to verify the potential solutions, we simulate the aforementioned channels environments, compute the proposed LLR values for each of the channels and run the error correcting algorithm implemented at the receiver chain. The results are then compared with the LLR values computed under perfect CSI assumption.

In figure 3, it is illustrated the CED error rate as a function of the C/N_0 considering the navigation signals: the GPS L1C navigation message; the E1-OS navigation message; the Evolution E1-OS navigation message; the E1D with Protograph Root LDPC code and the E1D with rate compatible Root LDPC code over the 1st channel and considering that the LLR values are computed: under perfect CSI (black curves / equation 5); with instantaneous estimation of the variance through the NWPR algorithm (black curves/ equation 6) and with the proposed analytic solution (blue curves/ equation 21). Simulations curves show that the proposed method to compute the LLR provides the same solution as that provide by the perfect CSI assumption, improving the results with respect the instantaneous estimation of the variance. Moreover, the provided solution seems to be independent of the order of the NWPR filter, as it is illustrated in figure 4. However, the order of the filter has an effect on the solution where the LLR are computed considering the instantaneous estimation of the variance.

In figure 5, it is illustrated the CED error rate as a function of the $C/(N_0 + I)$ over a Gaussian jamming channel where the jammer device is broadcasting an interference of power $J = 2dB$. We consider the navigation signals: the GPS L1C navigation message; the E1-OS navigation message; the Evolution E1-OS navigation message; the E1D with Protograph Root LDPC code and the E1D with rate compatible Root LDPC code over the 1st channel and considering that the LLR values are computed: under perfect CSI (black curves / equation 10); with instantaneous estimation of the variance through the NWPR algorithm (red curves/ equation 13) and with the proposed analytic solution (blue curves/ equation 25). Simulations curves show that the proposed method to compute the LLR provides the same solution as that provide by the perfect CSI assumption, improving the performance with respect to the LLR solution provide by the instantaneous estimation of the variance.

In figure 6, it is illustrated the CED error rate as a function of the $C/(N_0)$ over a pulsed jamming channel where the jammer device is broadcasting an interference over 10% of the symbols with a power P_{eq}

$$P_{eq} = 10\log_{10}\left(10^{\frac{\frac{P_{G\_Jam}}{10}}{P}}\right) dB \tag{36}$$

where P_{G_Jam} is the equivalent power broadcasts by a jammer device which disrupts the entire codeword with a Gaussian interference and P it is the percentage of disrupted symbols. This scenario is modelled with $P_{G\_Jam} = 2dB$ and $P = 0.1$. We consider the navigation signals: the GPS L1C navigation message; the E1-OS navigation message; the Evolution E1-OS navigation message; the E1D with Protograph Root LDPC code and the E1D with rate compatible Root LDPC code over the 3rd channel and considering that the LLR values are computed: under perfect CSI (black curves / equation 15); with instantaneous estimation of the variance (equation 30) through the NWPR algorithm (red curves/ equation 13) and with the proposed analytic solution (blue curves/ equation 31). Moreover, we compute also CED error rate considering the AWGN with perfect CSI scenario. The effect of the pulsed jamming is illustrated by comparing the AWGN with perfect CSI scenario (green curves) with the Pulsed Jamming with perfect CSI scenario (black curves). A loss in error correction performances around 0.5-0.7 dB it is shown for the different navigation signals. Simulations curves also illustrate the error correction loss when no CSI is available at the receiver. Considering the LLR solution based on the instantaneous estimation of the variance: 4dB for an error probability of 10^{-1} are loss with the GPS L1C navigation message; 5 dB for an error probability of 10^{-1} are loss with the E1-OS navigation message; 3.5 dB for an error probability of 10^{-1} are loss with the Evolution E1-OS navigation message and 3 dB for an error probability of 10^{-1} are loss with the E1D with rate compatible Root LDPC code. Considering the LLR proposed solution, an enhance of the error correcting performance with respect the previous solution is reached: 3 dB for an error probability of 10^{-1} are improve with the GPS L1C navigation message; 3 dB for an error probability of 10^{-1} are improve with the E1-OS navigation message; 3 dB for an error probability of 10^{-1} are improve with the Evolution E1-OS navigation message and 2 dB for an error probability of 10^{-1} are improve with the E1D with rate compatible Root LDPC code.

In figure 7, it is illustrated the CED error rate as a function of the $C/(N_0)$ over an urban environment modeled through a 2-state prieto model for a vehicle speed of 40 km/h and an elevation angle of 40 degrees, where the PLL track perfectly the phase of the signal. We consider: the navigation signals: the GPS L1C navigation message; the E1-OS navigation message; the Evolution E1-OS navigation message and the E1D with rate compatible Root LDPC code over the 1st channel and considering that the LLR values are computed: under perfect CSI (black curves / equation 20) and with the proposed analytic solution (blue curves/ equation 35). Simulation curves show that the proposed solution to compute the LLR almost converge to the solution is computed considering CSI at the receiver. Considering the GPS L1C navigation message, the E1-OS navigation and the Evolution E1-OS navigation message, only a gap of 0.5 dB is found. This gap is reduced to 0.4 dB for the E1D with rate compatible Root LDPC code.

CONCLUSIONS

In this article, several error correcting schemes corresponding to new GNSS signals are presented. Those schemes are simulated under different scenarios with new proposed method to compute LLR. Simulations results show an enhancement in error probability performances for each of schemes under the different realistic scenarios of interest.

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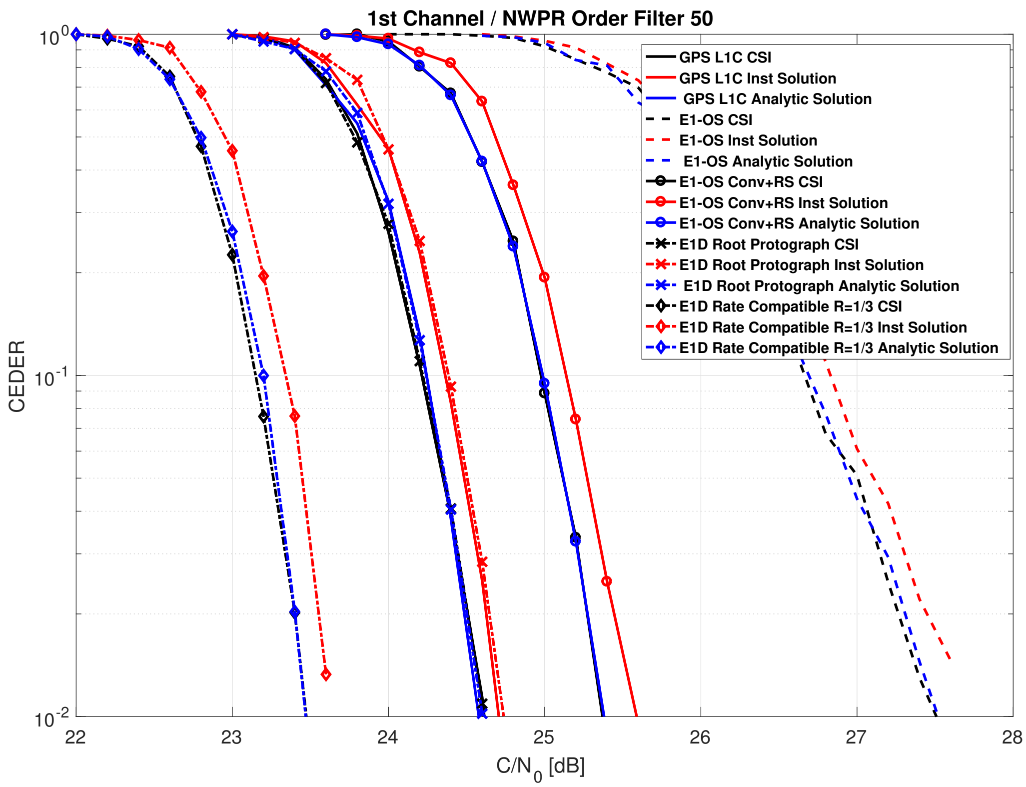

Figure 3: CED error rate over AWGN channel considering: the GPS L1C navigation message and LLR computed with CSI (black/solid curve); the GPS L1C navigation message and LLR computed with instantaneous estimation of σ_n^2 (red/solid curve); the GPS L1C navigation message and LLR computed with the proposed solution (blue/solid curve); the E1-OS navigation message and LLR computed with CSI (black/dash curve); the E1-OS navigation message and LLR computed with instantaneous estimation of σ_n^2 (red/dash curve); the E1-OS navigation message and LLR computed with the proposed solution (blue/dash curve); the Evolution E1-OS navigation message and LLR computed with CSI (black/solid-circle curve); the Evolution E1-OS navigation message and LLR computed with instantaneous estimation of σ_n^2 (red/solid-circle curve); the Evolution E1-OS navigation message and LLR computed with the proposed solution (blue/solid-circle curve); the E1D with Protograph Root LDPC code and LLR computed with CSI (black/dash-cross-point curve); the E1D with Protograph Root LDPC code and LLR computed with instantaneous estimation of σ_n^2 (red/dash-cross-point curve); the E1D with Protograph Root LDPC code and LLR computed with the proposed solution (blue/dash-cross-point curve); the E1D with rate compatible Root LDPC code and LLR computed with CSI (black/dash-diamond-point curve); the E1D with rate compatible Root LDPC code and LLR computed with instantaneous estimation of σ_n^2 (red/dash-diamond-point curve) and the E1D with rate compatible Root LDPC code and LLR computed with the proposed solution (blue/dash-diamond-point curve)

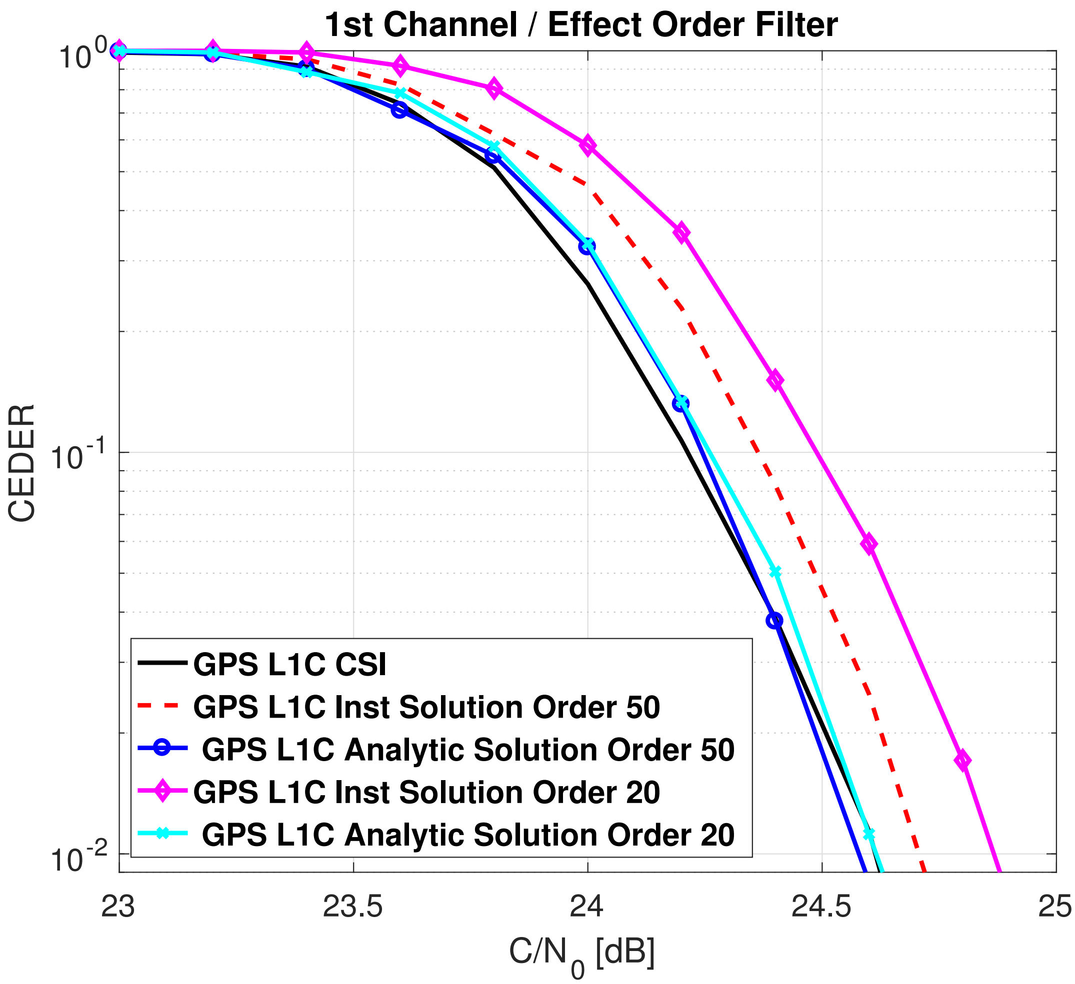

Figure 4: CED error rate over of the GPS L1C navigation message over AWGN channel considering: LLR computed with CSI (black/solid curve); LLR computed with CSI with instantaneous estimation of σ_n^2 and order of the filter 50 (red/dash curve); LLR computed with the proposed solution and order of the filter 50 (blue/solid-circle curve); LLR computed with CSI with instantaneous estimation of σ_n^2 and order of the filter 20 (magenta/solid-diamond curve); LLR computed with the proposed solution and order of the filter 20 (black/solid-cross curve);

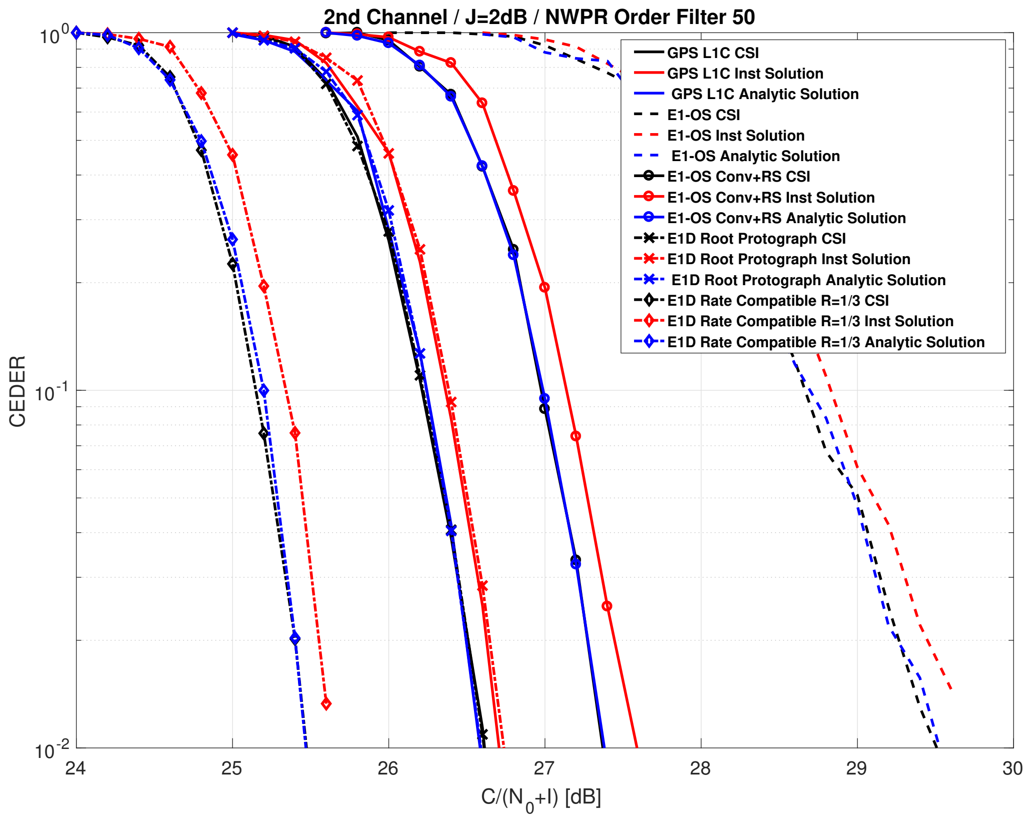

Figure 5: CED error rate over a Gaussian jamming channel where the jammer device is broadcasting an interference of power $J = 2dB$. We consider: the GPS L1C navigation message and LLR computed with CSI (black/solid curve); the GPS L1C navigation message and LLR computed with instantaneous estimation of σ^2_{T+I} (red/solid curve); the GPS L1C navigation message and LLR computed with the proposed solution (blue/solid curve); the E1-OS navigation message and LLR computed with CSI (black/dash curve); the E1-OS navigation message and LLR computed with instantaneous estimation of σ^2_{T+I} (red/dash curve); the E1-OS navigation message and LLR computed with the proposed solution (blue/dash curve); the Evolution E1-OS navigation message and LLR computed with CSI (black/solid-circle curve); the Evolution E1-OS navigation message and LLR computed with instantaneous estimation of σ^2_{T+I} (red/solid-circle curve); the Evolution E1-OS navigation message and LLR computed with the proposed solution (blue/solid-circle curve); the E1D with Protograph Root LDPC code and LLR computed with CSI (black/dash-cross-point curve); the E1D with Protograph Root LDPC code and LLR computed with instantaneous estimation of σ^2_{T+I} (red/dash-cross-point curve); the E1D with Protograph Root LDPC code and LLR computed with the proposed solution (blue/dash-cross-point curve); the E1D with rate compatible Root LDPC code and LLR computed with CSI (black/dash-diamond-point curve); the E1D with rate compatible Root LDPC code and LLR computed with instantaneous estimation of σ^2_{T+I} (red/dash-diamond-point curve) and the E1D with rate compatible Root LDPC code and LLR computed with the proposed solution (blue/dash-diamond-point curve)

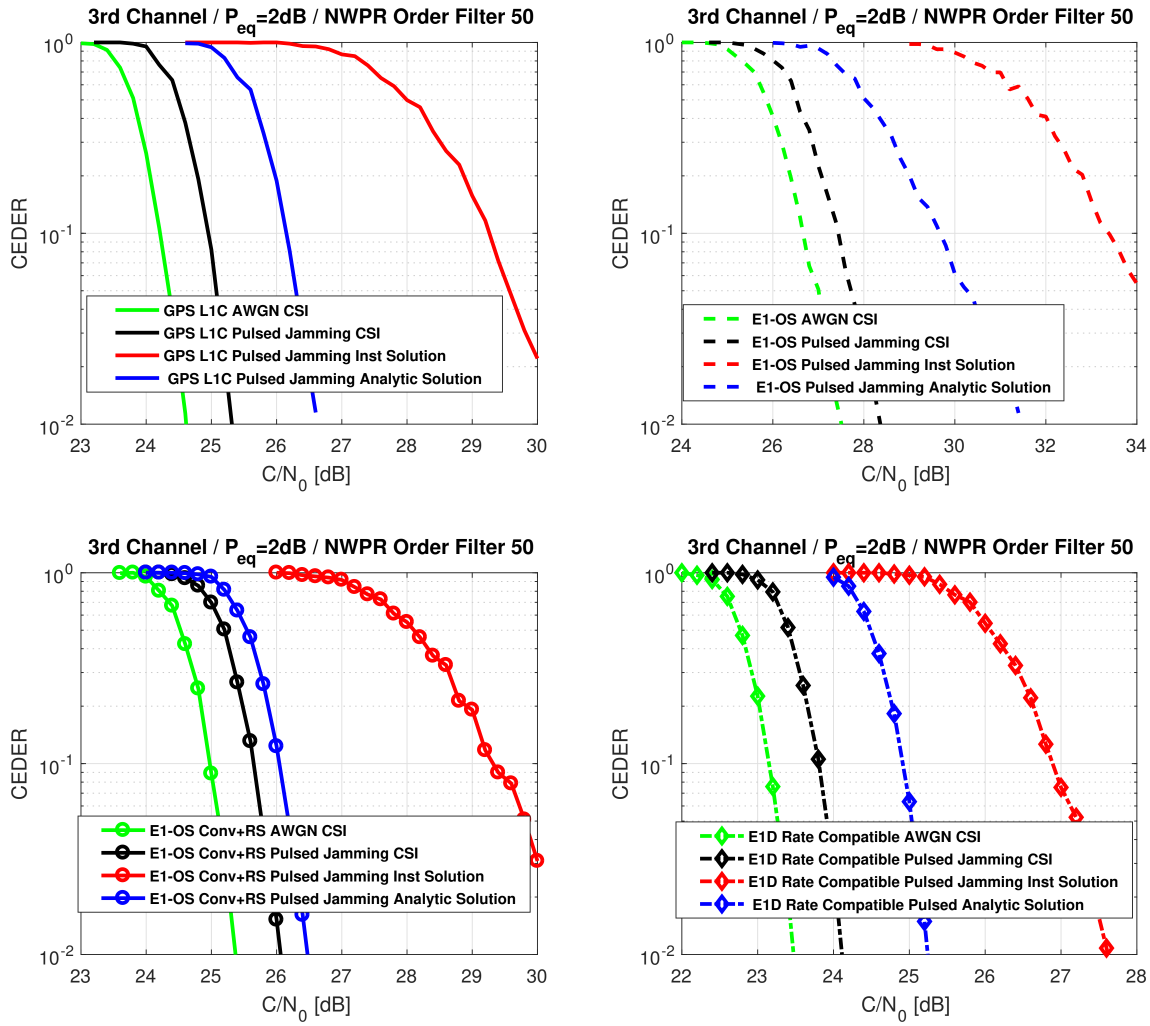

Figure 6: CED error rate over a pulsed jamming channel where the jammer device broadcast a Gaussian interfence which disrupt 10% of the symbols with an equivalent power $P_{G\_Jam} = 2dB$. We consider: the GPS L1C navigation message and LLR computed with CSI (black/solid curve); the GPS L1C navigation message and LLR computed with instantaneous estimation of the variance (red/solid curve); the GPS L1C navigation message and LLR computed with the proposed solution (blue/solid curve); the E1-OS navigation message and LLR computed with CSI (black/dash curve); the E1-OS navigation message and LLR computed with instantaneous estimation of the variance (red/dash curve); the E1-OS navigation message and LLR computed with the proposed solution (blue/dash curve); the Evolution E1-OS navigation message and LLR computed with CSI (black/solid-circle curve); the Evolution E1-OS navigation message and LLR computed with instantaneous estimation of the variance (red/solid-circle curve); the Evolution E1-OS navigation message and LLR computed with the proposed solution (blue/solid-circle curve); the E1D with rate compatible Root LDPC code and LLR computed with CSI (black/dash-diamond-point curve); the E1D with rate compatible Root LDPC code and LLR computed with instantaneous estimation of the variance (red/dash-diamond-point curve) and the E1D with rate compatible Root LDPC code and LLR computed with the proposed solution (blue/dash-diamond-point curve). Moreover, it is computed CED error rate over an AWGN channel with perfect CSI at the receiver considering the GPS L1C navigation message (green /solid curve); the E1-OS navigation message (green/dash curve);the Evolution E1-OS navigation message(green/solid-circle curve) and the E1D with rate compatible Root LDPC code (green/dash-diamond-point curve).

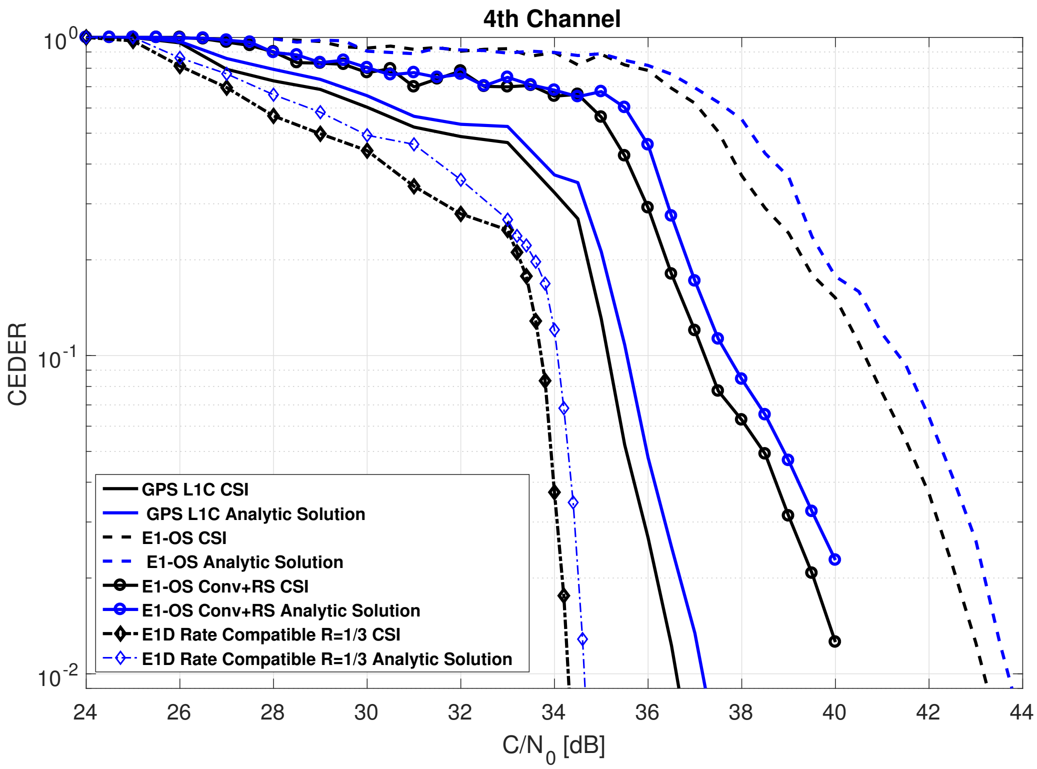

Figure 7: CED error rate over a urban fading channel modeled through the 2-state Prieto channel for a vehicle speed of 40 km/h and an elevation angle of 40 degrees, where the PLL is capable to perfectly track the phase of the signal and considering: the GPS L1C navigation message and LLR computed with CSI (black/solid curve); the GPS L1C navigation message and LLR computed with the proposed solution (blue/solid curve); the E1-OS navigation message and LLR computed with CSI (black/dash curve); the E1-OS navigation message and LLR computed with the proposed solution (blue/dash curve); the Evolution E1-OS navigation message and LLR computed with CSI (black/solid-circle curve); the Evolution E1-OS navigation message and LLR computed with the proposed solution (blue/solid-circle curve); the E1D with rate compatible Root LDPC code and LLR computed with CSI (black/dash-diamond-point curve) and the E1D with rate compatible Root LDPC code and LLR computed with the proposed solution (blue/dash-diamond-point curve)