Combinatorial Auctions with Interdependent Valuations: SOS to the Rescue

ALON EDEN, Tel-Aviv University, Israel
MICHAL FELDMAN, Tel-Aviv University and Microsoft Research, Israel
AMOS FIAT, Tel-Aviv University, Israel
KIRA GOLDNER, University of Washington, USA
ANNA R. KARLIN, University of Washington, USA

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In this paper, we consider combinatorial auctions, where each agent has a value for every subset of items, and the goal is to maximize the social welfare, namely the sum of agent valuations for their assigned bundles. As a special case of general social choice settings, the VCG mechanism solves this problem optimally, *as long as the values are independent.*

There are many settings, however, in which the independence of values is not realistic. If the item being sold has money-making potential or is likely to be resold, the values different agents have may be correlated, or perhaps even common. A classic example is an auction for the right to drill for oil in a certain location [7].

The following model due to Milgrom and Weber [5], described here for single-item auctions, has become standard for auction design in such settings. These are known as *interdependent value settings* (IDV) and are defined as follows: (i) Each agent i has a real-valued, private *signal* s_i. The set of signals $\mathbf{s} = (s_1, s_2, \ldots, s_n)$ may be drawn from a (possibly) correlated distribution. (ii) The *value of the item* to agent i is a function $v_i(\mathbf{s})$ of the signals (or information) of *all* agents.

For single-item auctions, there are payments that yield an ex-post incentive-compatible mechanism if and only if the corresponding allocation rule is monotone in each agent's signal [6].

Maximizing efficiency in ex-post equilibrium, single parameter settings, is impossible unless the valuation functions $v_i(\mathbf{s})$ satisfy a technical condition known as the *single-crossing condition* [1, 4, 5].

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Author's email addresses: alonarden@gmail.com; michal.feldman@cs.tau.ac.il; fiat@tau.ac.il; kgoldner@cs.washington.edu; karlin@cs.washington.edu.

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Unfortunately, the single crossing condition does not generally suffice to obtain optimal social welfare in settings beyond that of a single item auction [3].

This paper addresses the following two issues related to social welfare maximization in the interdependent values model: (1) To what extent can the optimal social welfare be approximated in interdependent settings that do not satisfy the single-crossing condition? and (2) How far beyond the single item setting can we go? Is it possible to approximately maximize social welfare in *combinatorial auctions with interdependent values*? The first question was recently considered by Eden et *al.* [2] who gave approximation guarantees for valuations satisfying a relaxed version of single crossing.

We focus on *s*ubmodular over signals (SOS) valuations. *I.e.*, for all j, for any s_j, $\delta \geq 0$, and for any $\mathbf{s}_{-j}$ and $\mathbf{s}'_{-j}$ such that component-wise $\mathbf{s}_{-j} \leq \mathbf{s}'_{-j}$, it holds that

$$v_i(s_j + \delta, \mathbf{s}_{-j}) - v_i(s_j, \mathbf{s}_{-j}) \geq v_i(s_j + \delta, \mathbf{s}'_{-j}) - v_i(s_j, \mathbf{s}'_{-j}).$$

Many valuations considered in the literature on interdependent valuations are SOS (though this term is not used). The simplest (yet still rich) class of SOS valuations are *fully separable* valuation functions (see, e.g., [3]), where there are *arbitrary* (weakly increasing) functions $g_{ij}(s_j)$ for each pair of bidders i and j such that

$$v_i(\mathbf{s}) = \sum_{j=1}^{n} g_{ij}(s_j).$$

The main question considered herein is *to what extent can social welfare be approximated in interdependent settings with SOS valuations?* We provide the first welfare approximation guarantees for multi-dimensional combinatorial auctions, achieved by universally ex-post IC-IR mechanisms. Our main results are: (i) 4-approximation for any single-parameter downward-closed setting with single-dimensional signals and SOS valuations; (ii) 4-approximation for any combinatorial auction with multi-dimensional signals and *separable*-SOS valuations; and (iii) $(k + 3)$- and $(2\log(k) + 4)$-approximation for any combinatorial auction with single-dimensional signals, with k-sized signal space, for SOS and strong-SOS valuations, respectively. All of our results extend to a parameterized version of SOS, d-SOS, while losing a factor that depends on d.

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