



Two-Mode Threshold Graph Dynamical Systems for Modeling Evacuation Decision-Making During Disaster Events

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Abstract. Recent results from social science have indicated that neighborhood effects have an important role in an evacuation decision by a family. Neighbors evacuating can motivate a family to evacuate. On the other hand, if a lot of neighbors evacuate, then the likelihood of an individual or family deciding to evacuate decreases, for fear of looting. Such behavior cannot be captured using standard models of contagion spread on networks, e.g., threshold models. Here, we propose a new graph dynamical system model, 2MODE-THRESHOLD, which captures such behaviors. We study the dynamical properties of 2MODE-THRESHOLD in different networks, and find significant differences from a standard threshold model. We demonstrate the utility of our model through agent based simulations on small world networks of Virginia Beach, VA. We use it to understand evacuation rates in this region, and to evaluate the effects of the model and of different initial conditions on evacuation decision dynamics.

1 Introduction

Background. Extreme weather events displaced 7 million people from their homes just in the first six months of 2019 [23]. With the rise in global warming, the frequency of these events is increasing and they are also becoming more damaging. Just in 2017–2018, there were 24 major events. In 2017, there was a total of 16 weather events that together costed over \$306 billion, according to NOAA. In 2018, there were eight hurricanes, out of which two were category 3 or higher and caused more than \$50 billion in damages.

Motivation. Timely evacuation is the only action that can reduce risk in many of these events. Although more people are exposed to these weather events, technological improvements in weather prediction, early warning systems, emergency management, and information sharing through social media, have helped

keep the number of fatalities fairly low. During Hurricane Fani [17], a record 3.4 million people were evacuated in India and Bangladesh and fewer than 100 fatalities were recorded [23]. However, in many disaster events, e.g. Hurricane Sandy, the fraction of people who evacuated has been much lower than what local governments would like.

The decision to evacuate or not is a very complex one and depends on a large number of social, demographic, familial, and psychological factors, including forecasts, warnings, and risk perceptions [13, 14, 19, 25, 26]. Two specific factors have been shown to have an important effect on evacuation decisions. First, peer effects, i.e., whether neighbors and others in the community have evacuated, are important. Up to a point, this has a positive impact on the evacuation probability of a household, i.e., as more neighbors evacuate, a household becomes more likely to evacuate. Second, concerns about property, e.g., due to looting, if a lot of people have already left, counteracts the first effect. Therefore, this has a negative impact on the evacuation probability. An important public policy goal in disaster planning and response is to increase the evacuation rates in an affected region, and understanding how this happens is crucial.

Summary of Results. There is a lot of work on modeling peer effects, e.g., the spread of diseases, information, fads and other contagions [1, 5, 7]. A number of models have been proposed, such as independent cascade [15], and different types of threshold models (e.g., [6, 24]). These are defined on a network, with each node in state 0 or 1 (0 indicating non-evacuating, 1 indicating a node has been influenced, e.g., is evacuating), and a rule for a node to change state from 0 to 1. For instance, in a τ -threshold model, a node switches from state 0 to state 1 if τ -fraction of its neighbors are in state 1. All prior models only capture the first effect above, i.e., as the number of effected neighbors increases, a node is more likely to switch to state 1. Here, we propose a new threshold model, referred to as 2MODE-THRESHOLD, which inhibits a transition from state 0 to 1 if a sufficiently large fraction of a family's neighborhood is in state 1, and demonstrate its use in a large scale study. Our results are summarized below.

1. Dynamics of the 2MODE-THRESHOLD model (results in Sects. 2 and 3). We introduce and formalize evacuation decision making as a graph dynamical system (GDS) [21] using 2MODE-THRESHOLD functions at nodes. We study theoretically the dynamics of 2MODE-THRESHOLD in different networks, and show significant differences from the standard threshold model that has no drop off. Specifically, we find that starting at a small set of nodes in state 1, the diffusion process does not go beyond a constant fraction of the network. System configurations in which more nodes are 1's (e.g., the all 1's vector of node states) are also fixed points, but our results imply that one cannot reach such fixed points with lots of 1's from most initial configurations that have a small number of 1's.

2. Agent based simulation and application (results in Sect. 4). We develop an agent-based modeling and simulation (ABMS) of the 2MODE-THRESHOLD model on a realistic small world network in the region of Virginia Beach, VA. This region has a population of over 450,000, and households are geographically situ-

ated based on land-use data, with a real geo-location which invokes the concept of neighbors and long range connections [4]. We add edges between households based on the Kleinberg small world (KSW) model [16]. Our ABM enables us to capture heterogeneities in the modeling of the evacuation decision-making process. This includes not only heterogeneities in families, but also differences in (local) neighborhoods of families as represented in social networks. We use it to understand the evacuation rates in this region, and evaluate the effects of different initial conditions (e.g., number of seeds) [seeds are families who are highly risk averse] on evacuation decision dynamics. For example, including the effects of looting can reduce evacuation rates by 50%.

Novelty and Implications. Models of type 2MODE-THRESHOLD have not been studied before. Our ABM approach can help (i) understand how planners and managers can more effectively convince families that are in harms way to evacuate; (ii) understand the effects of families' social networks on evacuation decisions [10,25,26]; and (iii) establish down-stream conditions after the evacuation decision has been made, to support additional types of analyses. For example, results from these studies can be used to forecast traffic congestion (spatially and temporally) during the exodus [19], and to determine places where shelters and triage centers should be established.

2 Evacuation Decision-Making Model

2.1 Motivation from Social Science

Our model is motivated by the analysis of a survey in the counties affected by Hurricane Sandy in the northeastern United States by [13], which is briefly summarized here. The goal of this survey was to assess factors driving evacuation decisions [20]. The survey was at a pretty large scale, with over 1200 individuals, and a response rate of 61.93%. A Binomial Logit model was applied to the survey data and tested for the factors associated with households' evacuation behaviors [13]. The results indicate that a respondent's employment status, consideration of neighbors' evacuation behavior, concerns about neighborhood criminal activities or looting, access to the internet in the household, age, and having flood insurance, each plays a significant role in a respondent's decision to evacuate during Hurricane Sandy. Noteworthy was the influence of neighbors' evacuation behaviors, and concerns about looting and criminal behavior. Neighbors' evacuations had a statistically significant and positive effect on evacuation probability but concerns about criminal and looting behavior had a significant negative effect—implying that if too many neighbors leave, then the remaining households are less likely to evacuate.

2.2 A Graph Dynamical Systems Framework

A *graph dynamical system* (GDS) is a powerful mathematical abstraction of agent based models, and we use it here to develop a model of evacuation behavior, motivated by the survey analysis described above. A GDS \mathcal{S} describes the

evolution of the states of a set of agents. Let $\mathbf{x}^t \in \{0, 1\}^n$ denote the vector of agent states at time t , with $x_v^t = 1$ indicating that agent v has evacuated. $x_v^t = 0$ means that agent v has not evacuated at time t . A GDS \mathcal{S} consists of two components: (1) an interaction network $G = (V, E)$, where V represents the set of agents (in our case, the households which are deciding whether or not to evacuate), and E represents a set of edges, with $e = \{u, v\} \in E$ if agents u and v can influence each other; and (2) a set $\mathcal{F} = \{f_v : v \in V\}$ of *local* functions $f_v : \{0, 1\}^{deg(v)} \rightarrow \{0, 1\}$ for each node $v \in V$, which determines the state of node v in terms of the states of $N(v)$, the set of neighbors of v . Given a vector \mathbf{x}^t describing the states of all agents at time t , the vector \mathbf{x}^{t+1} at the next time is obtained by updating x_v^{t+1} using its local function $f_v(\cdot)$. We say that a state vector \mathbf{x}^t is a *fixed point* of \mathcal{S} if the node states do not change, i.e., $\mathbf{x}^{t+1} = \mathbf{x}^t$.

The 2mode-Threshold Local Functions: Modeling Evacuation Behavior. The 2MODE-THRESHOLD function $f_v(\cdot)$ will be probabilistic, and will depend on the *probability of evacuation*, in order to capture the qualitative aspects of the results of [13]. This is shown in Fig. 1a and specifies the probability of evacuation p_e for agent v_i as a function of the fraction η_1 of neighbors of v_i in state 1. We have $p_e = p_{e,max}$ for $\eta_1 \in (\eta_{min}, \eta_c]$, and $p_e = 0$ for $\eta_1 \in [0, \eta_{min}]$ and $\eta_1 > \eta_c$. In this paper, we primarily focus on $\eta_{min} = 0$. Specifically, this captures the following effects: (i) peer (neighbor) influence can cause families to evacuate and (ii) if too many of a family’s neighbors evacuate, there are not enough neighbors remaining behind to dissuade potential looters, so a family reduces its probability of evacuation. The first effect makes $p_e = p_{e,max}$ for $\eta_1 > 0$, and the second effect results in p_e dropping to zero at $\eta_1 = \eta_c$. Note that the special case where $p_e = p_{e,max}$ for $\eta_1 > \eta_{min} = 0$ is a probabilistic variant of the η_{min} -threshold function (e.g., [6]); we will sometimes refer to this as the “regular probabilistic threshold” model, and denote them by RP-THRESHOLD. This model is shown in Fig. 1b. These are models that can be assigned to any agent; in GDS, an agent is a node that resides in a networked population.

Network Models. We describe the models for the contact network $G = (V, E)$, which is the other component of a GDS \mathcal{S} . A node $v_i \in V$ represents a family, or a household. Edges represent interaction channels, for communication and observations. Edges are *directed*: a directed edge $(v_j, v_i) \in E$, with $v_i, v_j \in V$, means that family v_j *influences* family v_i . We use the population model developed in [4] for representing the set V of households.

Edges are specified using the Kleinberg small world (KSW) network approach [16], and there are two types of edges: short range and long range. Short range edges (v_j, v_i) represent either (i) a family v_i speaks with (is influenced by) another family v_j about evacuation decisions, or (ii) a family v_i observes v_j ’s home and infers whether or not a family v_j has evacuated. A long-range edge represents a member of one family v_i interacting with a member of family v_j at work. Each edge has a label of distance between homes, using (lon, lat) coordinates of each home. Thus, the KSW model has the following parameters: the node set V and their attributes, the short-range distance d_{sr} over which short-range edges are placed between nodes, and the number q of long range

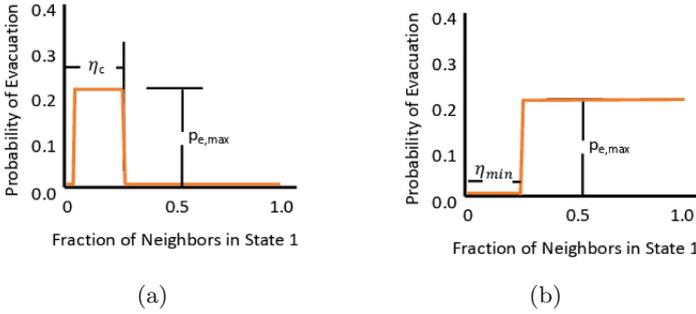


Fig. 1. Dynamics models—probability of evacuation curve—for probability p_e of evacuation for a family versus the fraction η_1 of its neighbors in state 1 (i.e., evacuating). (a) The 2MODE-THRESHOLD model: the evacuation probability is $p_e = 0$ for $\eta_1 = \eta_{min} = 0$ and for $\eta_1 > \eta_c$. The maximum probability is $p_e = p_{e,max}$ in the interval $(\eta_{min}, \eta_c]$. (b) The RP-THRESHOLD model: this curve is similar to the previous curve, except that $p_e = p_{e,max}$ for $\eta_1 > \eta_{min}$. This is a special case of 2MODE-THRESHOLD, but is a variation of the regular probabilistic threshold model [6, 21, 24]. As an illustration, if an agent has 50% of its neighbors in state 1, then the model in (a) shows that $p_e = 0$, while (b) shows that $p_e = p_{e,max} > 0$. An example with values for these parameters is given in the text.

edges incident on each node v_i . For each node v_i , (i) short range edges (v_j, v_i) are constructed, where $d(v_j, v_i) \leq d_{sr}$; and (ii) q long range edges (v_k, v_i) are placed at random, with probability proportional to $1/d(v_k, v_i)^\alpha$, for a parameter α . Note that for each short range edge (v_j, v_i) , there is a corresponding edge (v_i, v_j) . See [16] for details.

Example. Figure 1a shows an example of the 2MODE-THRESHOLD model with the parameters $p_{e,max} = 0.2$, and $\eta_c = 0.4$. Figure 1b shows a RP-THRESHOLD model. The purpose of this example is to illustrate the dynamics of these models on a network of five agents. In Fig. 2, \mathbf{x}^1 is the initial configuration with node 1 evacuated (in state 1), and nodes 2, 3, 4, and 5 not evacuated (in state 0). Nodes 2 and 3 have $\eta_1 = 1/3 < \eta_c = 0.4$, and so for both of them, the evacuation probability is $p_e = 0.2$. Nodes 4 and 5 have $\eta_1 = 0$, so $p_e = 0$ for them. Therefore, the probability that the state vector is \mathbf{x}^2 at the next time step (see Fig. 2) is $p_{e,max}(1 - p_{e,max}) = 0.2 \cdot 0.8 = 0.16$, since only node 2 switches to 1. With respect to the configuration \mathbf{x}^2 , nodes 3, 4, and 5 have $\eta_1 = \frac{2}{3}$, 1 and 0, respectively. Therefore, $p_e = 0$ for all these nodes, and \mathbf{x}^2 is a fixed point of the \mathcal{S} with the 2MODE-THRESHOLD functions. However, for the regular probabilistic threshold model, with $\eta_{min} < 0.3$, \mathbf{x}^2 is not a fixed point, since nodes 3 and 4 both have $p_e = p_{e,max}$ (since they have $\eta_1 > \eta_{min}$). Therefore, in the regular probabilistic threshold model, the $\mathbf{x}^2 \rightarrow \mathbf{x}^3$ transition occurs with probability $p_{e,max}^2 = 0.04$.

Problems of Interest. We will refer to a GDS system $\mathcal{S}_{2m} = (G, \mathcal{F})$ in which the local functions are 2MODE-THRESHOLD functions as a 2MODE-THRESHOLD-

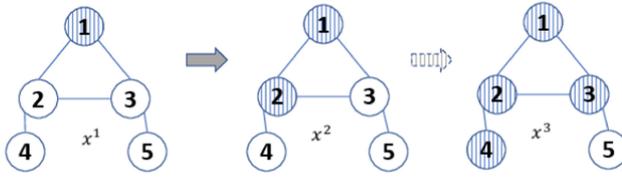


Fig. 2. An example showing the transitions in a \mathcal{S} on a graph with five nodes, and 2MODE-THRESHOLD local functions, with parameters $p_{e,max} = 0.2$ and $\eta_c = 0.4$. The figure shows a transition of the dynamics model from configuration \mathbf{x}^1 to \mathbf{x}^2 , with shaded nodes indicating evacuation. The $\mathbf{x}^1 \rightarrow \mathbf{x}^2$ transition occurs with probability $p_{e,max}(1 - p_{e,max}) = 0.16$. For the above parameters, \mathbf{x}^2 is a fixed point, and the node states do not change. However, if we had $\eta_c = 1$ (i.e., this is a regular probabilistic threshold), \mathbf{x}^2 is not a fixed point, and there can be a transition to configuration \mathbf{x}^3 with probability $p_{e,max}^2 = 0.04$ (indicated as a dashed arrow).

GDS. Our objective in this paper is to study the following problems on a \mathcal{S}_{2m} system:

- (1) How do the dynamical properties of 2MODE-THRESHOLD GDS systems differ from those of \mathcal{S} with RP-THRESHOLD model functions? Do they have fixed points, and what are their characteristics?
- (2) How do the number of 1’s in the fixed point depend on the initial conditions, and the model parameters, namely $p_{e,max}$ and η_c ? How can this be maximized?

We provide solutions to these problems next.

3 Analyzing Dynamical Properties in Different Network Models

It can be shown that any \mathcal{S}_{2m} converges to a fixed point in at most $n/p_{e,max}$ steps. \mathcal{S}_{2m} systems have significantly lesser levels of diffusion (i.e., number of nodes ending up in state 1), compared to the RP-THRESHOLD model, as we discuss below. Many details are omitted for space reasons.

Lemma 1. *Consider a \mathcal{S}_{2m} with $G = K_n$ being a complete graph on n nodes. Starting at a configuration \mathbf{x}^0 with a single node in state 1, \mathcal{S}_{2m} converges to a fixed point with at most $(p_{e,max} + \eta_c)n$ nodes in state 1, in expectation. In contrast, in a regular probabilistic threshold system on K_n with $\eta_{min} = 0$, the system converges to the all 1’s vector as a fixed point.*

Proof. Consider a state vector \mathbf{x}^t with k nodes in state 1. Consider any node v with $\mathbf{x}^t_v = 0$. If $k \leq \eta_c n$, then, $\Pr[\text{node } v \text{ switches to } 1] = p_{e,max}$. Therefore, the expected number of nodes which switch to 1 is $p_{e,max}(n - k) \leq np_{e,max}$. If $k > \eta_c n$, for every node in state 0, the probability of switching to 1 is $p_e = 0$. Therefore, the expected number of 1’s in a fixed point is at most $np_{e,max} + n\eta_c$.

On the other hand, in a regular probabilistic threshold model, the system does not converge till each node in state 0 switches to 1 (since $p_e = p_{e,max}$ for all $\eta_1 > 0$).

We observe below that starting at an initial configuration with a single 1, \mathcal{S}_{2m} converges to a fixed point with at most a constant fraction of nodes in state 1. Note, however, that configurations with more than that many 1’s, e.g., the all 1’s vector, are also fixed points. The result below implies that those fixed points will not be reached from an initial configuration with a few 1’s.

Lemma 2. *Consider a \mathcal{S}_{2m} on a $G(n,p)$ graph with $p\eta_c \geq \frac{6}{\epsilon^2} \frac{\log n}{n}$, for any $\epsilon \in (0,1)$. Starting at a configuration \mathbf{x}^0 with a single node in state 1, \mathcal{S}_{2m} converges to a fixed point with at most $(1+2\epsilon)(\eta_c+p_{e,max})n$ nodes in state 1, in expectation. In contrast, in a regular probabilistic threshold system on K_n with $\eta_{min} = 0$, the system converges to the all 1’s vector as a fixed point.*

Proof. (Sketch) Let $deg(v)$ denote the degree of v . For a subset S , let $deg_S(v)$ denote the degree of v with respect to S , i.e., the number of neighbors of v in S . For any node v , we have $E[deg(v)] = np$. By the Chernoff bound [9], it follows that $\Pr[deg(v) > (1 + \epsilon)np] \leq e^{-\epsilon^2 np/3} \leq 1/n^2$. Consider a set S of size $\frac{1+\epsilon}{1-\epsilon}\eta_c n$. For $v \notin S$, $E[deg_S(v)] = |S|p$, and so $\Pr[deg_S(v) < (1 - \epsilon)|S|p] \leq e^{-\epsilon^2 |S|p/2} \leq 1/n^2$. For $|S| \geq \frac{1+\epsilon}{1-\epsilon}\eta_c n$, we have $(1 - \epsilon)|S|p \geq (1 + \epsilon)\eta_c np$. Putting these together, with probability at least $1 - 2/n$, we have $deg(v) \leq (1 + \epsilon)np$ and $deg_S(v) \geq (1 + \epsilon)\eta_c np \geq \eta_c deg(v)$, for all nodes v . Therefore, if \mathcal{S}_{2m} reaches a configuration with nodes in set S of size $\frac{1+\epsilon}{1-\epsilon}\eta_c n < (1 + 2\epsilon)\eta_c n$, with probability $1 - 2/n$, S is a fixed point. With probability $\leq 2/n$, S is not a fixed point, and the process converges to a fixed point with at most n 1’s, so that the expected number of 1’s in the fixed point is at most $|S| + 2 \leq (1 + 2\epsilon)\eta_c n$. On the other hand, consider the last configuration S' which has size $|S'| < (1 + 2\epsilon)\eta_c n$. Then, in expectation, at most $p_{e,max}n$ additional nodes switch to state 1, after which point, the configuration has more than $(1 + \epsilon)\eta_c n$ 1’s. Therefore, the expected number of 1’s in the fixed point is at most $(1 + 2\epsilon)(\eta_c + p_{e,max})n$.

4 Agent-Based Simulations and Results

Simulation Process. Inputs to the simulation are a social network (described below), a set of local functions that quantifies the evacuation decision making process of each node $v_i \in V$ (see Sect. 2), and a set of *seed* nodes whose state is 1 (i.e., these nodes are set to “evacuate” at the start of a simulation instance, at time $t = 0$). All other nodes at time $t = 0$ are in state 0 (the non-evacuating state). We vary a number of simulation input parameters, as discussed immediately below, across simulations. Each simulation instance or *run* consists of a particular set of seed nodes, and time is incremented in discrete timesteps, from $t = 0$ to t_{max} . Here, $t_{max} = 10$ days, to model the ten days *leading up* to hurricane arrival. At each timestep, nodes that are in state 0 may change to state 1, per the models in Sect. 2. At each $1 \leq t \leq t_{max}$, the state of the system

at time $t - 1$ is used to compute the next state of each $v_i \in V$ (corresponding to time t) *synchronously*; that is, all v_i update their states in parallel at each t . A simulation consists of 100 runs, where each run has a different seed set; the network and dynamics models are fixed in a simulation across runs. We present results below based on averaging the results of the 100 runs.

Social Networks. Table 1 provides the social networks (and selected properties) that are used in simulations of evacuation decision making. The network model of Sect. 2.2 was used to generate KSW networks for Virginia Beach, VA. Inputs for the model were $n = 113967$ families forming the node set V , with (lat, long) coordinates, $d_{sr} = 40$ m, $\alpha = 2.5$ (see [16]), and $q = 0$ to 16.

Simulation Parameters Studied. The input parameters varied across simulations are provided in Table 2.

Table 1. Kleinberg small world (KSW) networks [16] used in our experiments and their properties. The number n of nodes is 113967 for all graphs. The short range distance $d_{sr} = 40$ m and the exponent $\alpha = 2.5$ is for computing the probabilities of selecting particular long-range nodes with which to form long-range edges with each node $v_i \in V$. Column “No. LR Edges” ($= q$) means number of long-range edges incoming to each node v_i . There are five graph instances for every row. Average degree is d_{ave} and maximum degree is d_{max} , for in-degree and out-degree.

Network Class	No. LR Edges	Avg. In-Deg.	Max. In-Deg.	Avg. Out-Deg.	Max. Out-Deg.
KSW0	0	10.11	380	10.11	380
KSW2	2	11.71	382	11.71	381
KSW4	4	13.70	384	13.70	381
KSW8	8	17.70	388	17.70	382
KSW16	16	25.70	396	25.70	383

Table 2. Summary of the parameters and their values used in the simulations.

Parameter	Description
Networks	Networks in Table 1. We vary q per the table, from 0 to 16
Num. random seeds, n_s .	Number of seed nodes specified per run (chosen uniformly at random). Values are 50, 100, 200, 300, 400, and 500
Threshold model	The 2MODE-THRESHOLD model of Fig. 1a and the RP-THRESHOLD (i.e., classic) threshold model of Fig. 1b, in Sect. 2
Threshold range, η_c .	The range in relative degree over which nodes can change to state 1. Discrete values are 0.2 and 1.0. Note that $\eta_c = 1$ corresponds to the classic stochastic threshold model (Fig. 1b), whereas smaller values of η_1 correspond to the 2MODE-THRESHOLD model (Fig. 1a)
Maximum probability, $p_{e,max}$	The maximum probability of evacuation $p_{e,max}$ of Fig. 1. Discrete values are 0.05, 0.10, and 0.15

Basic Results and the Effects of Seeding. Figure 3b provides average fraction of population deciding to evacuate (Frac. DE) as a function of time for one

instance of the KSW2 category of networks. We use the 2MODE-THRESHOLD model with $p_{e,max} = 0.15$ and $\eta_c = 0.2$ (see Fig. 1a). A simulation uses a fixed value of number n_s of random seed nodes per run, but the set of nodes differs in each run (see legend). Other simulation parameters are in the figure. Error bars indicate the variance in results across 100 runs (i.e., simulation instances). The variance is very small (the bars cannot be seen in the plots, and are barely visible even under magnified conditions). Hence we say no more about the variance in output. As number n_s of random seeds increases from 50 to 500, the fraction deciding to evacuate f_{de} increases from about 0.02 to 0.1.

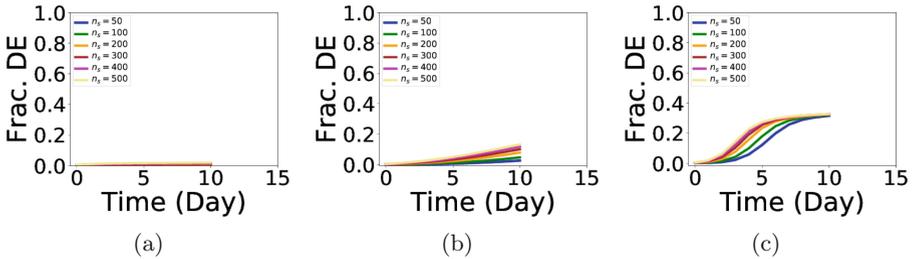


Fig. 3. Simulation results of fraction of population deciding to evacuate (Frac. DE) versus simulation time. All results use the 2MODE-THRESHOLD model of Fig. 1a, $p_{e,max} = 0.15$, $\eta_c = 0.2$, and n_s (numbers of random seeds) varies from 50 to 500 (see legend). Error bars denote variance. (The variance is very small.) (a) Results for one graph instance of network class KSW0 (i.e., $q = 0$ long range edges per node). (b) Results for one graph instance of network class KSW2 (i.e., $q = 2$ long range edges per node). (c) Results for one graph instance of network class KSW16 (i.e., $q = 16$ long range edges per node).

Effect of Graph Structure: Long Range Edges. The effect of number q of long range edges is shown across the three plots in Fig. 3 for the 2MODE-THRESHOLD model. For $q = 0$ (i.e., no long-range edges), the fraction of the population evacuating (Frac. DE) = $f_{de} \approx 0$. As q increases to 2 and then 16 long-range edges per node, f_{de} increases markedly. In particular, Fig. 3c shows how the spread of evacuation decisions has an upper bound in the 2MODE-THRESHOLD model: too many families have evacuated, so the remaining families do not evacuate over concerns of looting and crime. This effect of greater contagion spreading as q increases is the “weak link” phenomena [12], where long-range edges can cause remote nodes to change their state to 1 (i.e., evacuating), thus moving a “contagion” into a different region of the graph. Note that the speed with which the maximum of $f_{de} = 0.32$ is attained increases with n_s .

Effect of Dynamics Model: Maximum Evacuation Probability $p_{e,max}$. Figure 4 shows the effect of number $p_{e,max}$ of the 2MODE-THRESHOLD model. As $p_{e,max}$ increases from 0.05 (Fig. 4a) to 0.10 (Fig. 4b) to 0.15 (Fig. 4c), the fraction of population evacuating increases at smaller $p_{e,max}$, almost plateaus for all n_s

when $p_{e,max} = 0.1$, and increases its speed to plateau for the largest $p_{e,max}$. The values of $p_{e,max}$ were selected based the survey results [13] mentioned in Sect. 2.1.

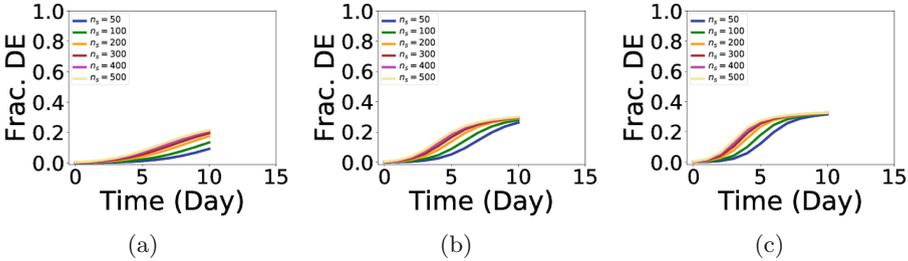


Fig. 4. Simulation results of fraction of population deciding to evacuate (Frac. DE) versus simulation time. All results use the 2MODE-THRESHOLD model of Fig. 1a with $\eta_c = 0.2$, and n_s (numbers of random seeds) varies from 50 to 500, for one instance of the KSW16 graph class, i.e., $q = 16$ long range edges per node (similar results for other graph instances). (a) Results for $p_{e,max} = 0.05$. (b) Results for $p_{e,max} = 0.10$. (c) Results for $p_{e,max} = 0.15$, is the same as Fig. 3c, reproduced for completeness.

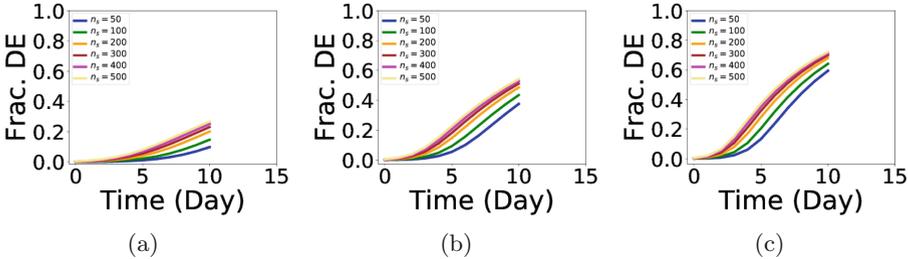


Fig. 5. Simulation results of fraction of population deciding to evacuate (Frac. DE) versus simulation time. All results use the RP-THRESHOLD model of Fig. 1b where looting and crime are not concerns, and n_s (numbers of random seeds) varies from 50 to 500, for one instance of the KSW16 graph class, i.e., $q = 16$ long range edges per node (similar results for other graph instances). (a) Results for $p_{e,max} = 0.05$. (b) Results for $p_{e,max} = 0.10$. (c) Results for $p_{e,max} = 0.15$. These results can be compared with corresponding plots from Fig. 4 for the 2MODE-THRESHOLD model.

Effect of Dynamics Model: Range of Relative Threshold for Transition to State 1. We compare results from the 2MODE-THRESHOLD (Fig. 4), with various values for $p_{e,max}$ and $\eta_c = 0.2$, against the RP-THRESHOLD model, with the same $p_{e,max}$ values, where $\eta_c = 1.0$ (Fig. 5). The corresponding plots, left to right in each figure, can be compared. As $p_{e,max}$ increases, the discrepancy between the two models increases: concern over looting dampens evacuation in

the 2MODE-THRESHOLD model. For $p_{e,max} = 0.15$, the RP-THRESHOLD model results in Fig. 5c reach $f_{de} > 0.6$, while the corresponding results for 2MODE-THRESHOLD model in Fig. 4c are only roughly one-half the values of f_{de} in Fig. 5c. Hence, the 2MODE-THRESHOLD model can produce a large difference (dampening) in the fraction of families evacuating. Therefore, ignoring the influence of looting and crime can cause a large overprediction of family evacuations.

5 Related Work

Many studies have identified factors that affect evacuation decision making. These include social networks and peer influence [18, 22], risk perceptions, evacuation notices, storm characteristics [2, 3, 8] and household demographics such as nationality, proximity to hurricane path, pets, disabled family members, mobile home, access to a vehicle etc. [11, 25].

Other studies use social networks and relative threshold models to model evacuation behavior. A *relative threshold* [6, 24] θ_i for agent v_i is the minimum fraction of distance-1 neighbors in $G(V, E)$ that must be in state 1 in order for v_i to change from state 0 and to state 1. Several studies [14, 25, 26] assign thresholds to agents in agent-based models (ABMs) of hurricane evacuation modeling. Stylized networks of 2000 nodes are used in [14] to study analytical and ABM solutions to evacuation. In [25], 12,892 families are included in a model of a 1995 hurricane for which 75% of households evacuated. They include three demographic factors in their evacuation model, in addition to the the peer influence that is captured by a threshold model. Small world and random regular stylized networks are used for social networks. Simulations of hurricane evacuation decision-making in the Florida Keys are presented in [26]. The simulations cover 24 hours, where the actual evacuation rate was about 53% of families. The social network is also a small-world network, with geospatial home locations, which is similar to our network construction method. In all of these studies, as the number of neighbors of a family v_i evacuates, the more likely it is that v_i will evacuate. Our threshold model differs: in our model, if too many neighbors evacuate, then v_i will not evacuate because of concerns over crime and looting.

6 Summary and Conclusions

We study evacuation decision-making as a graph dynamical system using 2MODE-THRESHOLD functions for nodes. This work is motivated by the results of a survey collected during Hurricane Sandy which shows that concerns about crime motivates families to stay in their homes. We study the dynamics of 2MODE-THRESHOLD in different networks, and show significant differences from the standard threshold model. Results obtained from this work can help determine the size and characteristics of non-evacuees which city planners can use for contingency planning.

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