

EQUALITY ON A SCALE VS EQUAL SIGN IN A MATHEMATICAL EQUATION

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Introduction

Plenty of literature has highlighted elementary students' misinterpretation and difficulty in dealing with an equal sign at different grade levels (e.g., Jones, Inglis, Gilmore & Evans, 2013; Sherman & Bisanz, 2009; Stephens et al., 2013). Studies have suggested various tools to develop a relational understanding of equality (e.g., Ellis & Yeh, 2008; Jones & Pratt, 2006). Leavy, Hourigan, and McMahon (2013) reported evidence of a relational understanding of the equal sign, rather than an operational understanding, in elementary students' work when using a physical balance. In this study, we investigate how often elementary students use their relational understanding of equality using a balance to write a balanced equation of the given situation.

Data, Method, and Results

As part of the broader Measuring Early Mathematics Reasoning Skills (MMaRS) project, we conducted 32 cognitive interviews of elementary students (K-3) to confirm the ordering of skills on a learning progression focused on Properties of Operations. The interview protocol was aligned to the learning progression and included items designed to test students' understanding of various representations of equality. In this study, we focused on students' responses to three questions associated with three different skills on the learning progression. First, a picture of three apples on each side of a balance was displayed, and students were asked to write a number sentence to describe the relationship between the apples on the balance. Second, students were asked to write an equation to describe the balanced scale. Lastly, students were asked to identify if an equation, in the form of $a = a$, was true or not.

The analysis of 32 interviews revealed that there is a disconnection between students' understanding of equality on a pictorial scale and symbolic representation in the form of $a=a$. Figure 1 displays the number of students who answered correctly with the associated percentage out of 32 students. In 21 (65%) interviews, the interviewer asked student explicitly if the equation $3=3$ described the situation on the balanced scale, and only 5 (24%) students correctly associated the equation with the pictorial representation.

First: Pictorial representation of apples on a balanced Scale	Second: Write an equation ($a=a$) for the balanced scale	Identify a true equation of form $a=a$ shown on a card.
27 (84%)	0 (0%)	12 (37.5%)

Figure 1: Number of Correct Responses for Three Skills Out of 32

These results show a clear disconnection between pictorial and symbolic representations of equality among these elementary students. Students did not demonstrate a relational understanding of the equal sign. Rather, an operational understanding was evident, as 21 out of

32 (66%) students wrote an equation of format $a + b = c$ to describe the relationship between the apples on either side of the balance.

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