Physical and Virtual Implementation of Closed-loop Designs for Model Updating

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Abstract. A recently proposed virtual implementation of output feedback based on signal processing eliminates the practical overhead associated with physical operation in closed loop. Additionally, the virtual implementation facilitates realization of multiple closed-loop systems from a single test in open loop, allows for complex gains, and removes the constraint of closed-loop stability. Care must, however, be exercised in the design of the closed-loop systems, as the errors in these are governed by the intrinsic approximations in the open-loop identification. The present paper offers an examination of this item when the closed-loop systems are designed for parameter estimation in updating of numerical models of structural systems. The differences between physical realization and the proposed virtual implementation are discussed, and the pivotal points outlined are demonstrated in the context of a numerical examination with a structural system.

Keywords: Model updating, Output feedback, Virtual implementation, Parameter estimation, Closed-loop eigenstructure

1 Introduction

Model updating through parameter estimation is a well-known discipline used in many different application areas [1]. Within structural and mechanical engineering, model updating is commonly used to calibrate a numerical model of a physical structure for use in structural design, control, health monitoring, response prediction, and so forth [2]. In this context, a typical updating approach is to minimize the discrepancy between the poles from the numerical model, $\mathscr{M}$, and target poles estimated from the physical system, $\mathscr{P}$. Here, an obvious issue is that no unique solution exists for the inverse problem when the dimensionality of $\mathscr{M}$ is larger than the number of identifiable poles from $\mathscr{P}$ [3,4]. This will often be the case, at least when using poles as targets, in structural and mechanical engineering applications, where a limited amount of poles can be identified.

Several approaches have been discussed to increase the pole target space. Examples include (1) testing the structure in different configurations by adding known perturbations [5, 6] and (2) using output feedback to design and test multiple closed-loop systems [3, 4, 7]. With reference to the second option, it has recently been shown that the practical overhead associated with closed-loop testing can be eliminated by use of a virtual approach, in which multiple closed-loop systems can be computed based on a single open-loop realization [8–11]. The virtual approach also removes the restrictive stability constraint [7], enables the use of complex gains because the control forces do not have to be physically delivered [11], and allows an increase in the target space based on a single closed-loop system [12]. The scope of the present paper is to examine, in terms of model updating, the basic applicability of the virtual approach by comparing it to the physical counterpart, where it is noted that the latter refers to real-time closed-loop testing.

The remainder of this paper is organized as follows: the fundamentals of output feedback and the virtual approach are briefly described in section 2. The implementation of output feedback—including gain computation and selection—for model updating is outlined in section 3. In section 4, numerical examples are presented to demonstrate the performance of the virtual and physical approaches, and lastly, in section 5, some concluding remarks close the paper.

2 Output feedback

Let $\mathscr{P}$ be described as a linear, time-invariant system in discrete time with the direct transmission term being zero or subtracted from the measurements, then

$$x(k+1) = A_d x(k) + B_d u(k) \tag{1a}$$

$$y(k) = C x(k), \tag{1b}$$

where $x(k) \in \mathbb{R}^{n\times 1}$ is the state, $u(k) \in \mathbb{R}^{r\times 1}$ the control input, $y(k) \in \mathbb{R}^{m\times 1}$ the output while $A_d \in \mathbb{R}^{n\times n}, B_d \in \mathbb{R}^{n\times r}$, and $C \in \mathbb{R}^{m\times n}$ are the system matrices. In this paper, it is assumed that $\{A_d, B_d\}$ is controllable and $\{A_d, C\}$ is observable.

Considering dynamic output feedback, the control input, $u(k)$, is the output of a discrete-time, finite-dimensional linear time-invariant system driven by $y(k)$, which is formulated as

$$x_f(k+1) = A_f x_f(k) + B_f y(k) \tag{2a}$$

$$u(k) = C_f x_f(k) + D_f y(k) + v(k) \tag{2b}$$

for some excitation v and coefficient matrices $A_f \in \mathbb{C}^{q\times q}, B_f \in \mathbb{C}^{q\times m}, C_f \in \mathbb{C}^{r\times q}$, and $D_f \in \mathbb{C}^{r\times m}$. Augmenting Eq (2a) with Eq. (1a), using Eqs. (1b) and (2b), yield

$$\begin{Bmatrix} x(k+1) \\ x_f(k+1) \end{Bmatrix} = \begin{bmatrix} A_d + B_d D_f C & B_d C_f \\ B_f C & A_f \end{bmatrix} \begin{bmatrix} x(k) \\ x_f(k) \end{bmatrix} + \begin{bmatrix} B_d \\ 0 \end{bmatrix} v(k), \tag{3}$$

which is referred to as the compensator. As seen in Eq. (3), letting A_f, B_f and C_f equal 0 yields static output feedback, and the compensator can thus be adapted for both static and dynamic output feedback.

Let $\tilde{x}(k) = \{x(k)^T \ x_f(k)^T\}^T$, then Eq. (3) can, as shown in [9], be rewritten as

$$\tilde{x}(k+1) = \tilde{A}\tilde{x}(k) + \tilde{B}\tilde{u}(k) \tag{4}$$

with the output

$$\tilde{u}(k) = \tilde{G}\tilde{C}\tilde{x}(k) + \begin{Bmatrix} 0 \\ v(k) \end{Bmatrix} \tag{5}$$

and

$$\tilde{A} = \begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 0 & B_d \\ I & 0 \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} 0 & I \\ C & 0 \end{bmatrix}, \quad \tilde{G} = \begin{bmatrix} A_f & B_f \\ C_f & D_f \end{bmatrix}. \tag{6}$$

As shown, the compensator can be viewed as a static output feedback system with the control law in Eq. (5) and the system matrices $\tilde{A} \in \mathbb{R}^{(n+q)\times(n+q)}$, $\tilde{B} \in \mathbb{R}^{(n+q)\times(r+q)}$, $\tilde{C} \in \mathbb{R}^{(m+q)\times(n+q)}$ and $\tilde{G} \in \mathbb{C}^{(r+q)\times(m+q)}$.

The virtual implementation of the compensator is achieved by using the relation between the open- and closed-loop transfer matrices, $H(z)$ and $\mathscr{H}(z)$, which in a system governed by positive static output feedback with the gain G is defined by [13]

$$\mathscr{H}(z) = (I - H(z)G)^{-1}H(z), \tag{7}$$

from which it follows that the closed-loop system can be identified from the open-loop realization. In order to incorporate the compensator, allowing for dynamic feedback, an open-loop transfer matrix is defined as [9]

$$H_C(z) = \begin{bmatrix} \frac{1}{z}I & 0 \\ 0 & H(z) \end{bmatrix}, \tag{8}$$

which complies with the dimensions of the compensator model. The compensator transfer matrix is found by substituting Eq. (8) into Eq. (7), yielding

$$\tilde{H}(z) = \begin{bmatrix} I - \frac{1}{z}A_f & -\frac{1}{z}B_f \\ -H(z)C_f & I - H(z)D_f \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{z}I & 0 \\ 0 & H(z) \end{bmatrix}. \tag{9}$$

By use of different gains, it is, in principle, possible to generate as many compensators as required from just a single open-loop realization. Worth of explicit note is that when the identification of the open-loop system is conducted in the frequency domain, the implementation of the virtual compensator follows directly. If, however, the system is identified in the time domain, one must transform to the z-domain in order to calculate Eq. (9). An approach for this is provided in [8], and it is based on mapping observer Markov parameters to $H(z)$. Furthermore, it should be noted that the inverse z-transformation filters unstable poles, hence eliminating the stability constraint.

3 Model updating using closed-loop systems

The parameters to be estimated are gathered in $\theta \in \mathbb{R}^s$, and the updating is formulated as the following constrained optimization problem:

$$\begin{aligned} \underset{\theta \in \mathbb{R}^s}{\text{argmin}} \quad & ||\Lambda_{\mathscr{M}}(\theta) - \Lambda_{\mathscr{P}}|| \\ \text{subject to} \quad & \forall i \in [1, \text{s}] : \alpha_i \leq \theta_i \leq \beta_i, \end{aligned} \tag{10}$$

where $\alpha_i, \beta_i \in \mathbb{R}$ are lower and upper bounds on θ_i and

$$\Lambda_{\mathscr{P}} = [\Lambda_{\mathscr{P}_1} \; \cdots \; \Lambda_{\mathscr{P}p}] \in \mathbb{C}^{v \times p} \tag{11}$$

$$\Lambda_{\mathscr{M}}(\theta) = [\Lambda_{\mathscr{M}_1}(\theta) \; \cdots \; \Lambda_{\mathscr{M}p}(\theta)] \in \mathbb{C}^{v \times p} \tag{12}$$

are, respectively, v target poles and the corresponding model-predicted poles for each of the p gains. The idea is to ensure that the system to be solved in the optimization scheme is (over)determined, which is done by stacking the columns of $\Lambda_{\mathscr{P}}$ and $\Lambda_{\mathscr{M}}(\theta)$ into vectors with $vp \geq s$ rows.

There are several ways to design the required gains, such as optimizing a cost function with specific goals [7, 14] or, as is currently being explored [10, 11], by generating random matrices. Here, we choose to simply generate gains as random real matrices using scheme 1 [11]. The real scalars $\bar{a}$ and $\bar{b}$ should be selected such that reasonable pole shifts occur while still, since the physical approach is included for comparison, retaining system stability.

One approach to investigate the error in the realization of the closed-loop poles, using the virtual dynamic approach with the gain $\tilde{G}$, is through the poles' sensitivity with respect to some parameter, g, of the gain, that is,

$$\frac{\partial \lambda_j}{\partial g} = \psi_j^T \frac{\partial \mathscr{A}_{cs}}{\partial g} \phi_j = \psi_j^T \tilde{B} \frac{\partial \tilde{G}}{\partial g} \tilde{C} \phi_j. \tag{13}$$

Here, ψ_j and ϕ_j are the jth left and right eigenvectors of the compensator state matrix, $\mathscr{A}_{cs}$, presented in Eq (3). In order to omit gains that cause undue error,

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Scheme 1 Generation of $\tilde{p}$ gains with $\bar{a}, \bar{b} \in \mathbb{R}$

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for $i = 1 : \tilde{p}$ **do**

Define $A_f^i = \bar{a}R^i$ where $R^i \in \mathbb{R}^{q \times q}$ with $\Re\left(R_{kl}^i\right) \sim \mathscr{N}(0,1)$

Define $B_f^i = \bar{a}P^i$ where $P^i \in \mathbb{R}^{q \times m}$ with $\Re\left(P_{kl}^i\right) \sim \mathscr{N}(0,1)$

Define $C_f^i = \bar{b}Q^i$ where $Q^i \in \mathbb{R}^{r \times q}$ with $\Re\left(Q_{kl}^i\right) \sim \mathscr{N}(0,1)$

Define $D_f^i = \bar{b}D^i$ where $D^i \in \mathbb{R}^{r \times m}$ with $\Re\left(D_{kl}^i\right) \sim \mathscr{N}(0,1)$

Define $\tilde{G}_i = \begin{bmatrix} A_f^i & B_f^i \\ C_f^i & D_f^i \end{bmatrix}$

end for

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we use the heuristic gain selection procedure proposed in [11], which circumvents calculation of the gain derivative. In particular, we use the initial model of the closed-loop system to define the metric

$$\gamma_i = \|\Psi_{\mathscr{M}_i}\tilde{B}\|\|\tilde{C}\Phi_{\mathscr{M}_i}\|, \tag{14}$$

where $\Psi_{\mathscr{M}_i}$ and $\Phi_{\mathscr{M}_i}$ are the left and right eigenvectors associated with the model-based poles using gain i, $\Lambda_{\mathscr{M}_i}$. Using this metric, the gain configurations yielding the lowest values are selected for use in the updating. Worth of explicit note is that since we base the metric on the initial models, pole path linearity is implicitly assumed.

4 Numerical examples

We consider a 10-DOF shear building model and a square plate model consisting of 16 elements in the context of model updating for damage characterization. For both examples, it is assumed that there is no pre-existing knowledge regarding the location of the damage, thus θ contains, respectively, all the inter-story stiffness values in the shear building example and all the moduli of elasticity in the plate example.

In the examples, we will use the terms *simulation model* and *virtual* and *physical nominal models* in order to refer to the model used for simulations and the numerical models to be updated. In both examples, the simulation model is drawn from the manifold containing the virtual and physical nominal models, which implies that in the absence of noise there is a set of parameters for which the models to be updated coincide with the simulation model. This will, of course, not be realizable in practice. The virtual and physical nominal models are updated using the "fmincon" algorithm in MATLAB® to solve the optimization problem in Eq. (10). The required system identification is carried out using the Eigensystem Realization Algorithm [15], where the output in both examples are contaminated with 2% white Gaussian noise.

Scheme 1 is used to generate 100 gains for dynamic output feedback with $q = 2$, where the metric described in section 3 is used to choose the 10 gains yielding the lowest value. Furthermore, the scheme is used to generate 10 gains that provide system stability for static output feedback. Four poles are selected from each gain configuration to form the target vector $\lambda_{\mathscr{P}} \in \mathbb{C}^{40}$, where the corresponding poles from the nominal models, $\lambda_{\mathscr{M}}(\theta) \in \mathbb{C}^{40}$, are taken as the ones yielding the lowest discrepancy to the identified target poles.

4.1 10-DOF shear building

The shear building illustrated in Fig. 1 is equipped with 3 displacement sensors and 1 actuator, and a 20% stiffness perturbation is introduced in the 6th floor. In the simulation, the structure is excited with white noise in the first floor, and the resulting displacements are measured with a sampling frequency of 100 Hz at the 1st, 5th and 9th floor for a duration of 5 minutes.

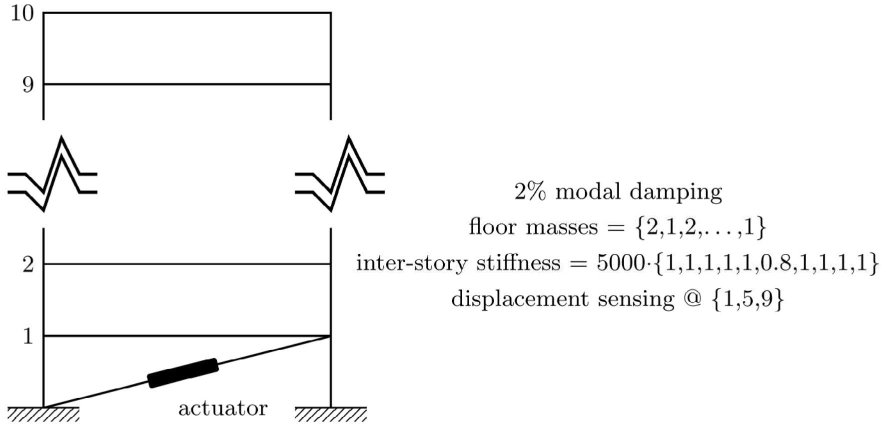

Fig. 1. Shear building simulation model with stiffness perturbation in the 6th floor, where the inter-story stiffness and the floor masses are in any consistent set of units.

The model updating scheme provides the results illustrated in Fig. 2, where the converged stiffness estimates, $\hat{k}_i$, of the updated model are normalized with respect to the true value of the stiffness components. The results show, qualitatively, that the performance of the virtual implementation is comparable to that of the physical. The parameters are estimated with a maximum absolute error of 0.42% and 0.49% using the virtual approach with static and dynamic feedback, respectively, and 0.48% using the physical approach. The mean absolute percentage error is 0.25% for the physical approach and, respectively, 0.19% and 0.24% for virtual static and dynamic feedback.

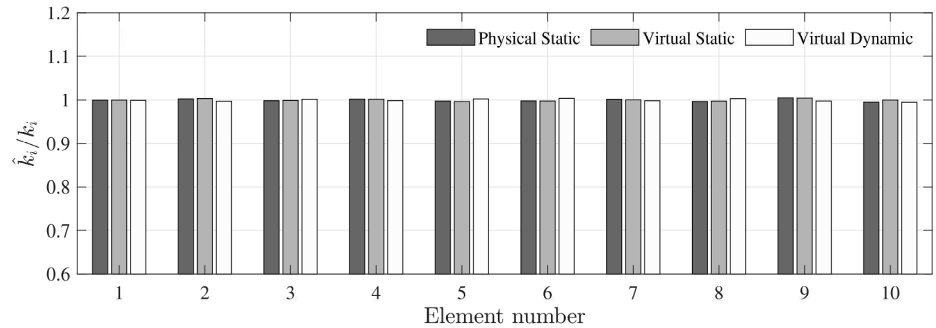

Fig. 2. Model updating results for the 10-DOF shear building with $\hat{k}$ and k being the converged estimate and true value of the stiffness components.

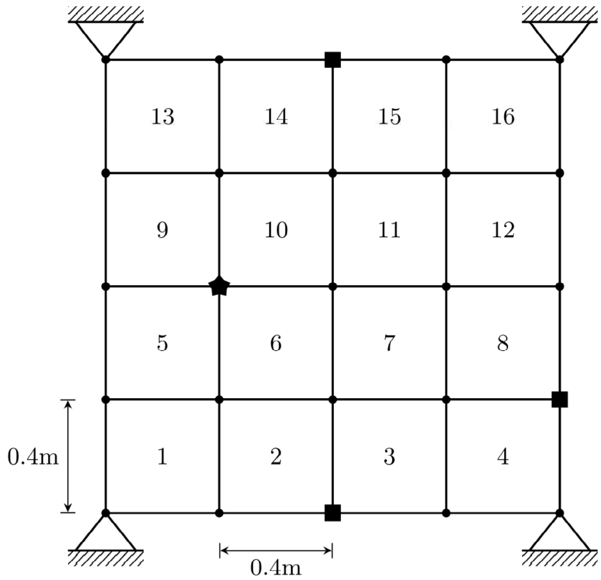

Fig. 3. Finite element plate simulation model with modulus of elasticity $E_i = E \; \forall i \in [2, 16]$ and $E_1 = 0.8E$. Out-of-plane input is applied at node 12 (★) and out-of-plane displacements are measured at node 3, 10, and 23 (■).

4.2 Plate model

We consider the square finite element plate model, depicted in Fig. 3, which consists of 16 four-noded plate elements, where each node has 1 translational and 2 rotational degrees of freedom. The plate is assigned a material model corresponding to typical structural steel, and classical damping is assumed such that each mode is assigned a damping ratio of $\zeta_i = 2\%$ in open loop. The elements

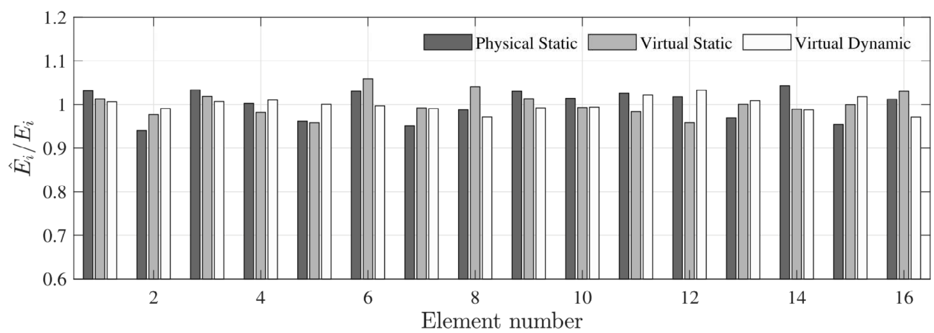

Fig. 4. Model updating results for the plate model with $\hat{E}$ and E being the converged estimate and true value of the moduli of elasticity of the elements.

have a side length of 0.4m and thickness of 0.003m and are all assigned an initial modulus of elasticity of 200 GPa in the nominal models, while the elements of the simulation model is assigned a modulus of elasticity of $E_i = 200$ GPa, except for element 1 where $E_1 = 160$ GPa. The system is excited with white noise in node 12, and the resulting displacements are measured at node 3, 10, and 23 with a sampling frequency of 100 Hz for a duration of 5 minutes.

The model updating scheme converges to the results illustrated in Fig. 4. As in the previous example, the results from the virtual approach show to be comparable to the results obtained using the physical approach. The parameters are estimated with a maximum absolute error of 6.0% and 3.3% using the virtual approach with static and dynamic feedback, respectively, and 5.9% using the physical approach. The mean absolute percentage error is 3% for the physical approach and, respectively, 2.2% and 1.3% for virtual static and dynamic feedback.

5 Conclusion

The paper addresses model updating by use of closed-loop system formulations. In particular, we explore the applicability of a recently proposed virtual approach—based on processing of open-loop signals to form closed-loop systems—by comparison with physical operation in closed loop.

Numerical examination of a shear building and a finite element plate show the performance of the physical and virtual approaches to be comparable. As such, the two examples suggest the virtual approach to be a viable alternative to physical closed-loop testing; an alternative that eliminates the practical overhead associated with the latter.

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