Market Power in Electric Power Distribution Systems

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***Abstract*—The roll-out of distributed energy resources (DERs) challenges the current operating practice of distribution utilities due to increased volatility and uncertainty of nodal power injections. Furthermore, many DERs are independently-owned and, therefore, will likely be operated without cooperation with a local utility. This combination of uncertainty and self-interest can motivate DERs to act strategically in future distribution (retail) markets to maximize their payoff. Without assuming a particular distribution market design, this paper investigates engineering mechanisms for strategic behavior by independently-owned DERs in the context of radial and voltage-constrained distribution systems. We derive conditions that identify an exercise of market power by an independently-owned DER and propose a market power metric that accounts for AC power flow physics and uncertainty of nodal power injections. The case study illustrates a range of effects of market power on distribution system operations.**

***Index Terms*—Chance constraints, Distributed energy resources, Distribution system, Market power, Radial Network**

I. Introduction

The penetration of small-scale distributed energy resources (DER) is generally regarded as a positive development to modernize the current electric power grid. However, this development is not without its challenges that put distribution utilities under pressure to deal with the stochasticity and limited controllability of DERs, leading to difficulties with voltage control, minimizing power losses, and managing various distribution system assets [1], [2], [3]. In the current practice of distribution utilities, these difficulties are overcome by imposing control policies on the output of DERs. These control policies use local measurements, available at the point of DER interconnection, and react to dynamically changing operating conditions in the distribution network without the intervention of utilities. Such policies are useful to pursue a variety of utility-centric objectives, e.g. maintain a desired power factor value, control voltage magnitudes within an acceptable range, reduce power losses, prevent reverse power flows, etc [2], [3], [4], [5]. However, as distribution systems become more transactive, i.e. DERs and electricity consumers gain the ability to transact electricity bypassing the utility, independently-owned DERs are likely to forgo these restrictive control policies and will control their power output to maximize their payoff.

This paper studies how technical peculiarities of distribution systems can be exploited by strategically acting DERs to increase its payoff by exercising *market power*. The notion of market power refers to the ability of a firm to profitably alternate market outcomes for a good or service above its marginal cost. In practice, however, the majority of distribution systems are operated by a monopoly (local utility), i.e. there is no competitive electricity supply or distribution market per se. As a result, this paper focuses on technical means that can be exploited by strategically acting DERs in distribution systems of the future, regardless of the market design choices that will be made by federal and local policymakers at a later date. In other words, this paper defines market power in a conditional manner, where the payoff is directly proportional to the amount of good or services provided by a firm. This assumption is consistent with the current rate design practice of US distribution systems, where DERs are remunerated based on coarse-grained tariffs in a proportion to their in-feed.

Previous efforts to understand market power in the context of electricity supply have been exclusively concentrated on wholesale (transmission) electricity markets. The effects of market power on wholesale competition among generation companies are well-documented, e.g. [6], [7], and extend to traditional generation companies [8] and large-scale aggregators of DERs [9]. While the studies in [6], [7], [8], [9] holistically explore means that can be used by strategic actors to favorably affect prices and quantities cleared in the wholesale market, they invoke transmission-specific assumptions (e.g. only active power supply, demand, and power flows are considered) that do not necessarily hold for distribution systems, which are notoriously reactive power and voltage constrained. Furthermore, [10] and [11] describe metrics to numerically characterize market power of a strategic actor. The market power metric proposed in [11] notably departs from the previous efforts due to two factors. First, unlike ex-post market power metrics surveyed in [10], it is a functional metric that can be internalized with optimal power flow and unit commitment models. Second, it is extended to deal with AC power flow constraints using semidefinite AC power flow relaxations. Nevertheless, the functional metric in [11]

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is limited to manipulations with active power outputs of a strategic actor.

While the body of literature pertaining to transmission market power is extensive, the analogous body for distribution systems is sparse. To the best of the authors knowledge, the economic and technical effects of exercising market power facilitated by the use of DERs has not been investigated. Such considerations are especially important in the formation of a distribution (retail) electricity market, which is under discussion in many US state jurisdictions. Due to lack of a consensus distribution market design, this paper studies the technical mechanisms of exercising of market power in distribution systems, leaving economic aspects outside of the scope of this paper. In addition to lacking proper market designs, the emphasis on technical mechanisms in this paper is motivated by the need to account for unique technical features of distribution systems that makes the use of the transmission market power metrics described in [10], [11] somewhat impractical.

The uniqueness of distribution systems arises from multiple features. First, unlike meshed transmission systems with strict power flow limits, distribution systems are mostly radial and their operation is typically voltage-constrained rather than limited by power flow ratings. Second, unlike conventional generation resources, DERs have a greater ability to control both active/reactive power outputs and terminal voltage due to the inverter-based interface to the system. Using oversized, yet affordable inverters, DERs can enhance their ability to exercise market power by cutting off downstream electricity consumers from upstream providers (e.g. utility or other DERs).

This paper focuses on the special means to exercise market power by independently-owned DERs in a radial distribution system with voltage constraints and analyzes its sensitivity to different distribution system operating parameters and the effects of uncertainty. Motivated by [11], we propose a new functional metric to characterize the ability of strategically acting DERs to levy advanced inverter-based controls for exercising market power.

II. Market Power in a Radial Network

A. Modeling Preliminaries

1) Notations: We consider a radial distribution system, as schematically shown in Figure 1, with the set of nodes (buses) $\mathcal{N}$ and the set of edges (lines) $\mathcal{E}$, where the root node is indexed as 1 and $\mathcal{N}^+ := \mathcal{N} \setminus \{1\}$ is the set of all non-root nodes. The root node is modeled as an infinite bus and is referred to as the substation. Each non-root node has only one ancestor node, denoted as $\mathcal{A}_i$, and one set of children nodes, denoted as $\mathcal{C}_i$. Also, each non-root node has an active and reactive power demand (d_i^P and $d_i^Q, i \in \mathcal{N}$), and its voltage magnitude ($v_i \in [v_i^{\min}, v_i^{\max}], \forall i \in \mathcal{N}$) for some upper and lower voltage constraints, $v_i^{\min}$ and $v_i^{\max}$. For the sake of simplicity, we introduce $u_i = v_i^2$. Should a node also have a controllable DER, the active and reactive power generation is modeled as g_i^P and g_i^Q and the power output limits are given as $g_i^P \in [0, g_i^{\text{P,max}}]$ and $g_i^Q \in [-g_i^{\text{Q,max}}, g_i^{\text{Q,max}}]$.

The active and reactive power flows along the lines are denoted by f_i^P and $f_i^Q, i \in \mathcal{E}$, respectively, where i is the index of the *downstream* node of the line, i.e. the receiving end of line i. Conversely, the *upstream* node of i will be the sending node. Each line has resistance r_i, reactance x_i, and apparent power flow limit $S_i^{\max}, i \in \mathcal{E}$. For convenience of notations, vectors $R = \{r_i, \forall i \in \mathcal{E}\}$ and $X = \{x_i, \forall i \in \mathcal{E}\}$ are the vectors of line resistances and reactances.

The uncertainty of nodal power injections at node i is modeled using the normally distributed error $\epsilon_i^P \sim N(0, \sigma_i)$ and aggregated in random vector $\boldsymbol{\epsilon}^P = \epsilon_i^P, \forall i \in \mathcal{N}^+$. The system-wide uncertainty is defined as $\tilde{\boldsymbol{\epsilon}}^P = \sum_{i \in \mathcal{N}^+} \epsilon_i^P$. Notations $\tilde{\boldsymbol{\epsilon}}^Q$ and $\boldsymbol{\epsilon}^Q$ are derived by analogy, see [12]. The **bold** characters $\boldsymbol{f}_i^P, \boldsymbol{f}_i^Q, \boldsymbol{g}_i^P, \boldsymbol{g}_i^Q, \boldsymbol{u}_i$ denote random variables associated with their deterministic namesakes $f_i^P, f_i^Q, g_i^P, g_i^Q, u_i$, respectively, under the influence of $\tilde{\boldsymbol{\epsilon}}^P$ and $\tilde{\boldsymbol{\epsilon}}^Q$.

2) Distribution power flow: To represent AC power flows in the distribution system, we adopt the LinDistFlow power flow approximation [13] that allows for modeling active/reactive power flows and voltage magnitudes in a computationally tractable manner at the expense of neglecting power losses:

$$\sum_{j \in \mathcal{C}_i} f_j^P = f_i^P - d_i^P + g_i^P, \quad \forall i \in \mathcal{N}^+ \tag{1}$$

$$\sum_{j \in \mathcal{C}_i} f_j^Q = f_i^Q - d_i^Q + g_i^Q, \quad \forall i \in \mathcal{N}^+ \tag{2}$$

$$u_i = u_{\mathcal{A}_i} - 2(r_i f_i^P + x_i f_i^Q), \quad \forall i \in \mathcal{N}^+. \tag{3}$$

$$v_i^{2,\min} \le u_i \le v_i^{2,\max}, \quad \forall i \in \mathcal{N}, \tag{4}$$

where (1)-(2) compute active and reactive flows in each line, (3) computes nodal voltage magnitudes and (4) enforces the nodal voltage limits. In the following, the power flow limits are not enforced as typically distribution systems have sufficient line flow capacity.

Each DER is constrained as follows:

$$0 \le g_i^P \le g_i^{P,\max}, \quad \forall i \in \mathcal{G} \tag{5}$$

$$-g_i^{Q,\max} \le g_i^Q \le g_i^{Q,\max}, \quad \forall i \in \mathcal{G} \tag{6}$$

$$\left(g_i^P\right)^2 + \left(g_i^Q\right)^2 \le \left(S_i^{\text{inv}}\right)^2, \quad \forall i \in \mathcal{E}, \tag{7}$$

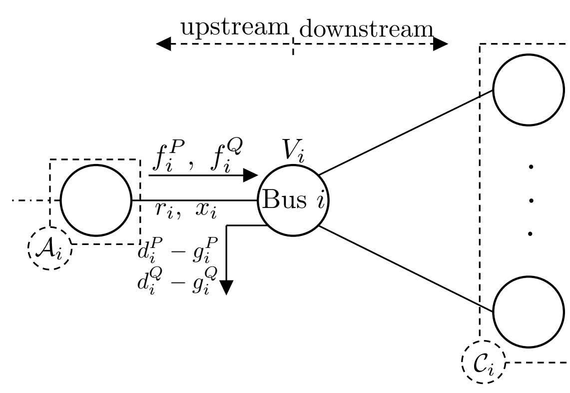

Fig. 1. Main notations in a radial distribution system.

where (5)-(6) are power capacity limits on the DER and (7) represents the apparent power flow limit (S_i^{inv}) on the inverter that connects the DER to the system. Note that (5)-(7) make it possible to represent various DER technologies.

B. Proposed Market Power Metric

The radial topology, as in Figure 1, enables the DER located at node i to exercise market power over the downstream consumers by blocking the power supply from the utility to downstream nodes.

Consider (3) and note that, similarly to utility-centric control policies [2], [3], [4], [5], DER at node i can manipulate its power output to adjust u_i at its terminals. In contrast to [2], [3], [4], [5], in this case the DER would aim to block the power supply from node $\mathcal{A}_i$. As follows from (3), under the assumption of neglecting the effect of voltage angle differences (typically small in distribution systems) between nodes i and $\mathcal{A}_i$, the power supply can be blocked if $u_i - u_{\mathcal{A}_i} = -2(r_i f_i^P + x_i f_i^Q) = 0$. In turn, this expression can be broken down in two independent market power conditions:

$$u_i(g_i^P, g_i^Q) = u_{\mathcal{A}_i}, \quad \forall i \in \mathcal{N} \tag{8}$$

$$f_i^P = -\frac{x_i}{r_i} f_i^Q, \quad \forall i \in \mathcal{E}, \tag{9}$$

where $u_i(\cdot)$ denotes the terminal voltage of the DER at node i as a function of its power outputs g_i^P and g_i^Q. When either of these constraints is satisfied, the player at node i cuts off all children node from the power flowing into the substation. Player i then becomes the sole provider of electricity to all nodes downstream of it, and is free to charge the downstream nodes a price higher than i's marginal cost of producing electricity given their situational monopoly. While (8) explains the market power exercising mechanism, (9) offers insights on possible power flow conditions that are caused by this behavior. First, (9) has a trivial solution when $f_i^P = f_i^Q = 0$, i.e. there is no power flow in line i. Second, if $f_i^P \neq 0$ and $f_i^Q \neq 0$, the reactive and active power flows must be in the opposite direction for exercising market power due to the minus sign in (9). The opposite direction of reactive and active power currently occurs in some distribution systems and is anticipated to become more common as more DERs are integrated, [1].

Given the current state of inverter technolologies [5], the only limiting factor of the DER at node i to set its terminal voltage $u_i(g_i^P, g_i^Q)$ to a desired level , e.g. as in (8), is the maximum inverter capacity (S_i^{inv}) as given by (7). Therefore, it is natural to *exploit S_i^{inv} as a quantifiable measure of market power* of the DER at node i and motivates the following optimization for each node i:

$$\left\{ \min \ S_i^{\text{inv}} \right. \tag{10}$$

$$\text{Eq. (1)} - \text{(7)} : \text{[Optimal power flow constraints]} \tag{11}$$

$$\left. \text{Eq. (8) or (9): [Market power condition]} \right\}, \forall i \in \mathcal{N}^+. \tag{12}$$

Considering strictly positive values of S_i^{inv} for a physically realistic model, the objective is concave, and due to the quadratic constraint in equation (7), the problem presented in (10) - (12) is a quadratically constrained quadratic program (QCQP).

The optimized value of S_i^{inv} can be interpreted as the minimum inverter capacity that the DER at node i needs for exercising market power at that node. Similarly to [11], the optimization in (10)-(12) internalizes optimal power flow constraints and, therefore, *proposed metric S_i^{inv} also has a functional interpretation*, i.e. as a function of distribution system conditions.

C. Market Power and Voltage Limits

In addition to the power output of the DER and the maximum capacity of its inverter, the optimization in (10)-(12) includes voltage limits in (4) enforced by the utility. From the utility perspective, these constraints are "hard" and, therefore, they can be used as a reference for analyzing the effect of exercising market power by the DERs. This interplay can be inferred from (8).

Consider $\Delta u_i = u_{\mathcal{A}_i} - u_i$. As per (4), one can assert that $(v_{\mathcal{A}_i}^{\min})^2 \leq u_i \leq (v_{\mathcal{A}_i}^{\max})^2$ and $(v_i^{\min})^2 \leq u_i \leq (v_i^{\max})^2$. Since the nominal voltages at neighboring distribution nodes are approximately the same, i.e. $u^0 := u_i^0 \approx u_{\mathcal{A}_i}^0$, the nominal operating voltage about which the grid typically operates, it is convenient to represent the voltage limits at nodes i and $\mathcal{A}_i$ relative u^0. This leads to $(1-\kappa)u^0 \leq u_{\mathcal{A}_i} \leq (1+\kappa)u^0$ and $(1-\kappa)u^0 \leq u_i \leq (1+\kappa)u^0$, where $\kappa \in [0,1]$ is a constant that defines the acceptable range given by the limits in (4). Normally parameter κ varies from 0.05 to 0.20 for different systems, [3]. Given these notations, the upper and lower bounds on $\Delta u_i \in [\underline{u}_i, \overline{u}_i]$ are:

$$\overline{u}_i = \max[u_{\mathcal{A}_i}] - \min[u_i] = (1+\kappa)u^0 - (1-\kappa)u^0 = 2\kappa \tag{13}$$

$$\underline{u}_i = \min[u_{\mathcal{A}_i}] - \max[u_i] = (1-\kappa)u^0 - (1+\kappa)u^0 = -2\kappa. \tag{14}$$

Notably, the market condition in (8) corresponds to the case with $\Delta u_i = 0$, i.e. the center of the range for $\Delta u_i \in [-2\kappa, 2\kappa]$ computed in (13)-(14). Since bounds $\underline{u}_i$ and $\overline{u}_i$ depend on κ, it emphasizes that the strictness of voltage limits relative to their nominal value affect the ability to act strategically. Tighter voltage limits will make it easier for strategic DERs to exercise market power.

D. Market Power under Uncertainty

Thus far nodal power injections have been treated deterministically, i.e. as known to the utility with perfect accuracy. In practice, this assumption does not hold and, therefore, the proposed market power metric and the optimization in (10)-(11) are extended using the chance-constrained framework that captures the effect of uncertain nodal injections on power flows, [12], [14].

Leveraging the chance-constrained optimal power flow model for distribution systems from [12], the chance-constrained equivalent of (10)-(11) is given as:

$$\left\{ \min \ S_i^{\text{inv}} \right. \tag{15}$$

$$\sum_{i \in \mathcal{G}} \alpha_i = 1 \tag{16}$$

$$\text{Eq. (1)} - \text{(2)} \tag{17}$$

$$\boldsymbol{u}_i = u_{\mathcal{A}_i} - 2a_{*i}^T\Big(R \circ A(\boldsymbol{\epsilon}^P - \alpha\tilde{\boldsymbol{\epsilon}}^P) + X \circ A(\boldsymbol{\epsilon}^Q - \alpha\tilde{\boldsymbol{\epsilon}}^Q)\Big), \quad \forall i \in \mathcal{N}^+ \quad (18)$$

$$\boldsymbol{f}_i^P = f_i^P + a_{i*}(\boldsymbol{\epsilon}^P - \alpha\tilde{\boldsymbol{\epsilon}}^P), \quad \forall i \in \mathcal{E} \quad (19)$$

$$\boldsymbol{f}_i^Q = f_i^Q + a_{i*}(\boldsymbol{\epsilon}^Q - \alpha\tilde{\boldsymbol{\epsilon}}^Q), \quad \forall i \in \mathcal{E} \quad (20)$$

$$\boldsymbol{g}_i^P = g_i^P + \alpha_i\boldsymbol{\epsilon}^P, \quad \forall i \in \mathcal{G} \quad (21)$$

$$\boldsymbol{g}_i^Q = g_i^Q + \alpha_i\boldsymbol{\epsilon}^Q, \quad \forall i \in \mathcal{G} \quad (22)$$

$$\mathbb{P}(g_i^{P,\min} \le \boldsymbol{g}_i^P \le g_i^{P,\max}) \ge (1-2\eta_g), \quad \forall i \in \mathcal{G} \quad (23)$$

$$\mathbb{P}(g_i^{Q,\min} \le \boldsymbol{g}_i^Q \le g_i^{Q,\max}) \ge (1-2\eta_g), \quad \forall i \in \mathcal{G} \quad (24)$$

$$\mathbb{P}((v_i^{\min})^2 \le \boldsymbol{u}_i \le (v_i^{\max})^2) \ge (1-2\eta_v), \quad \forall i \in \mathcal{N}, \quad (25)$$

$$\left.\boldsymbol{u}_i^2 = \boldsymbol{u}_{\mathcal{A}_i}^2 \ \text{ or } \boldsymbol{f}_i^P = -\frac{x_i}{r_i}\boldsymbol{f}_i^Q\right\}, \forall i \in \mathcal{N}^+. \quad (26)$$

where ($\circ$) denotes the Schur-product. Eq. (16) ensures that controllable DERs have sufficient active power margins to compensate for the uncertainty imbalance. The voltage magnitudes and power flows affected by uncertainty are computed in (18)-(20), while the output of controllable DERs under uncertainty is modelled in (21) and (22).

Decision variable α_i is optimized to set a participation factor of the controllable DER at node i in active power regulation and $\alpha = \{\alpha_i, \forall i \in \mathcal{N}^+\}$ denotes the vector of participation factors. Lastly, the matrix, A, has dimensions $l \times b$ where $l := |\mathcal{E}|$ is the number of edges, $b := |\mathcal{N}^+|$ is the number of nodes, and is formed leveraging power transmission distribution factors as in [15] to map the change of load at every node to the change of edge power flow. Since the network is radial each edge has to carry the complete net load of all its downstream nodes, and as such A is formed as

$$a_{ij} \triangleq \begin{cases} 1, & \text{if line } i \text{ is part of the path from root bus to bus } j \\ 0, & \text{else} \end{cases} \quad \forall i \in \mathcal{E},\ \forall j \in \mathcal{N}^+.$$

Thus $a_{i*} : 1 \times |\mathcal{B}|$ corresponds to the i-th row of A.

Constraints (23)-(25) are chance-constrained equivalents of (4)-(6), where parameters η_g and η_v define the tolerance to constraint violations and the factor of 2 is due to two-sided chance constraints. Constraint (26) defines the chance-constrained equivalent of the market power condition in (12).

The chance-constraints of (23)-(25) are a special case in that the random variables, ϵ, that are present via equations (18), (21) - (22) are all linear, and furthermore assumed to be normally distributed with zero-mean and known variance. We further assume here that each of the uncertainties are statistically independent of each other, which is reasonable so long as the uncertainty in the node of one agent does not affect the level of uncertainty present for another agent. With these assumptions then, such chance-constraints may be recast as equivalent deterministic Second-Order Conic (SOC) Constraints. To show this, consider a normally distributed random variable $\boldsymbol{\epsilon} \sim N(\mu, \sigma^2)$ such that the inequality $\mathbb{P}(\boldsymbol{\epsilon} \le \epsilon^{\max}) \ge \eta$ if and only if $\epsilon^{\max} \ge \mu + \boldsymbol{\Phi}^{-1}(1-\eta)\sigma$, where $\boldsymbol{\Phi}$ is the cdf of a standard normally distributed random variable. The chance constraints present in (18), (21) - (22) may be recast in a similar manner. For a complete proof and detailed derivation of these results, see [16], [12], [17]. Unfortunately, however, what we gain in tractability with this reformulation, we lose in simplicity, as the QCQP problem from the purely deterministic model in (10) - (12) is now a SOC Program (SOCP) due to the deterministic reform shown above. Fortunately SOCPs are handled easily by most commercially available solvers as well such as Gurobi.

Relative to the deterministic formulation in (10)-(12), the optimal solution of (15)-(26) is such that allows for the DER at node i to exercise market power in the presence of uncertain nodal injections and, therefore, the value of S_i^{inv} can be interpreted probabilistically. This probabilistic interpretation makes it possible to assess the impact of the underlying uncertainty assumptions (e.g. uncertainty distribution) and externalities (e.g. η_g and η_v).

III. Numerical Examples

The case study is carried out on the 33-node IEEE Distribution Test System, as illustrated in Figure 2, with modifications reported in [4]. All simulations are conducted using JuMP [18] and JuMPChance [19] packages for Julia v0.6.1.

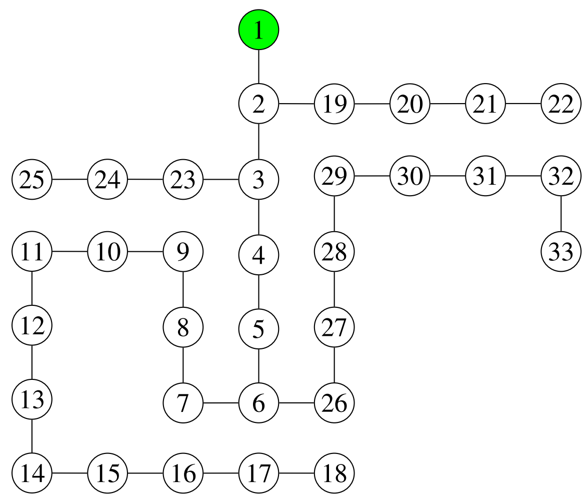

Fig. 2. An illustration of the 33-node IEEE Distribution Test System with the data described in [4], where the root node is in green.

A. Deterministic Case

The deterministic case assumes that all nodal power injections are known with perfect accuracy and computes the minimum inverter capacity at each node that will make it possible for the DER at that node to exercise market power. Figure 3 compares the results obtained with the market power condition given in (8)-(9) enforced individually. As expected, both market power conditions return identical results. Therefore, in the following only the market power condition in (8) is enforced since it represents a more physically accurate case when a strategic actor makes its decision based on locally available voltage information.

The results displayed in Figure 3 demonstrate that the minimum inverter capacity needed for a strategic actor to

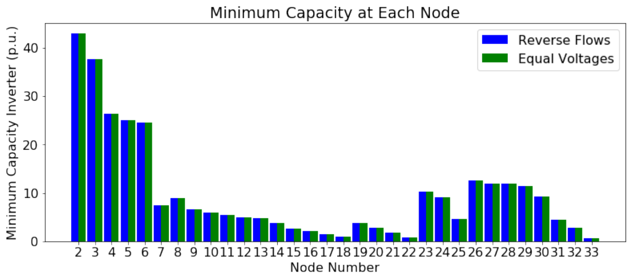

Fig. 3. The minimum inverter capacity needed at each node (S_i^{inv}) that makes it possible for a local DER to exercise a market power with the market power conditions enforced as in (8)-(9).

exercise market power depends on its location in the distribution system. In general, the inverter capacity monotonically decreases for more downstream nodes, i.e. the nodes with a greater electrical distance from the root node of the distribution network. Thus, strategic actors at downstream nodes would require a smaller capacity inverter. On the other hand, these actors will have less downstream load to deal with, thus undermining their supply potential. This observation represents an interesting economic trade-off between the minimum inverter capacity (thus, its capital cost) and the amount of downstream load that it can potentially supply (thus, its potential payoff).

B. Chance-Constrained Case

The chance-constrained case aims to explore the impact of uncertainty of nodal injections on the ability of strategically acting DERs to exercise market power. In the following simulations, parameters η_v and η_g are set to 0.005.

To evaluate the impact of uncertain nodal injections on the minimum inverter capacity needed to exercise market power at a given node of the distribution system, the simulations are performed for different values of parameter σ that represents the variance of uncertain nodal injections. The results presented in Figure 4 reveal that there is no significant change to the minimum inveter capacity needed under small values of variance. On the other hand, under larger values of the variance, the minimum capacity inverter needed to exercise market power increases. Similarly, it can be seen that the impact of uncertainty depends on the location of the inverter in the distribution system, where downstream nodes are more sensitive to changes in the variance.

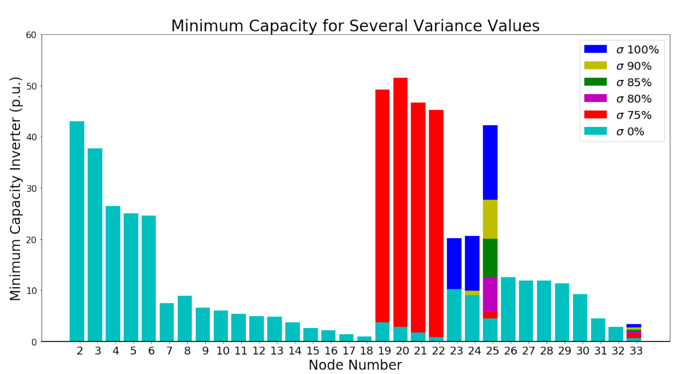

Fig. 4. The minimum inverter capacity needed for different values of variance (σ), whose values are given as percentages to the nominal (deterministic) nodal power injections.

From the practical viewpoint, these results suggest that rolling out DERs with a relatively high value of variance (e.g. photovoltaic resources) will create unfavorable conditions for exercising market power by strategic actors as it would require inverters with a significantly larger capacity.

C. Impact on the Voltage Profile

As discussed above, the ability to exercise market power is closely related to nodal voltages and, therefore, the simulations reported in Figure 4 are extended for different values of parameter κ in (4), where the upper and lower voltage limits are set to $(1+\kappa)u_i^0$ and $(1-\kappa)u_i^0$, as in Section II-C. To do so, we introduce a slack variable to the voltage magnitude constraint. This is necessary because the control of the DER is not up to a system operator and is controlled by an individual, who may choose to operate their resource in this manner. As such, while exercising such strategic behavior may lead to violations of the given acceptable range, this falls under the purview of the *system operator*, who may not be able to control how the individual who owns the DER chooses to operate it. In this respect, the following results represent how the network would respond to such behavior of an individual DER *if* the network were to be able to stand such wide voltage swings. As such, these results essentially give insight into how much of a control action a system operator would have to take to negate such behavior in order to prevent such a system-wide collapse. In practice, such wide limits are rare, and as such control action would likely be needed considerably sooner.

Parameter κ weakly affects the minimum inverter capacity needed to exercise market power, given rather wide base case voltage limits, but dramatically changes the voltage profile across the entire distribution system. Figure 5 displays the voltage profiles across the distribution system for multiple cases, where strategically acting DERs are individually placed at nodes 2, 3, 6 and 33. The first three nodes are selected to simulate individually exercising market power at branching nodes of the distribution system, while last node is selected because it is the most electrically remote node in the system.

Analyzing the voltage profiles displayed in Figure 5 is important from the perspective of the distribution utility, because complying with voltage limits falls within its responsibility and, therefore, it must ensure sufficient voltage control capacity in the system. First, exercising market power never leads to violations of upper voltage limit, i.e. it only causes increased voltage sags across distribution lines. On the other hand, as the value of parameter κ decreases, i.e. the voltage limits become tighter, the number of violations of the lower voltage limits increases. However, the magnitude and frequency of such violations are not strictly proportional to the percentage increase in κ. Note that violations are also observed at the nodes that do not host strategically acting DERs due to the greater voltage sag across distribution lines. As such

These violations are also sensitive to the location of the strategically acting DER in the distribution system. For example, as seen in Figure 5a, if the DER is placed at node 2 , it causes voltage limit violations across almost the entire system at the tightest limits. Other cases with strategic DERs at more downstream nodes, relative to node 2, lead to fewer violations Figure 5.

IV. Conclusion

This paper presented an approach to measure market power of an independently-owned DER located in a radial distribution system. The proposed metric relates the ability of a strategically acting DER to exercise market power and the capacity of the inverter that connects the DER to the distribution system. The case study demonstrates that the ability to exercise market power depends on a nodal location of the DER and various externalities (e.g. uncertainty parameters and voltage limits). In future work it will be interesting to consider cases of multiple DERs, with both independently-owned strategic acting DERs, and with system-controlled DERs. Both of these cases will further study the effects of multiple distribution end injections of power into the system. Further, the presence of additional DERs, particularly units in nodes electrically close to the strategically acting agent, allows one to consider control and mitigation schemes to combat such behavior. It would also be interesting to consider multiple strategically acting DERs that may be non-cooperative agents whose injections could could affect the ability of the other to act this way.

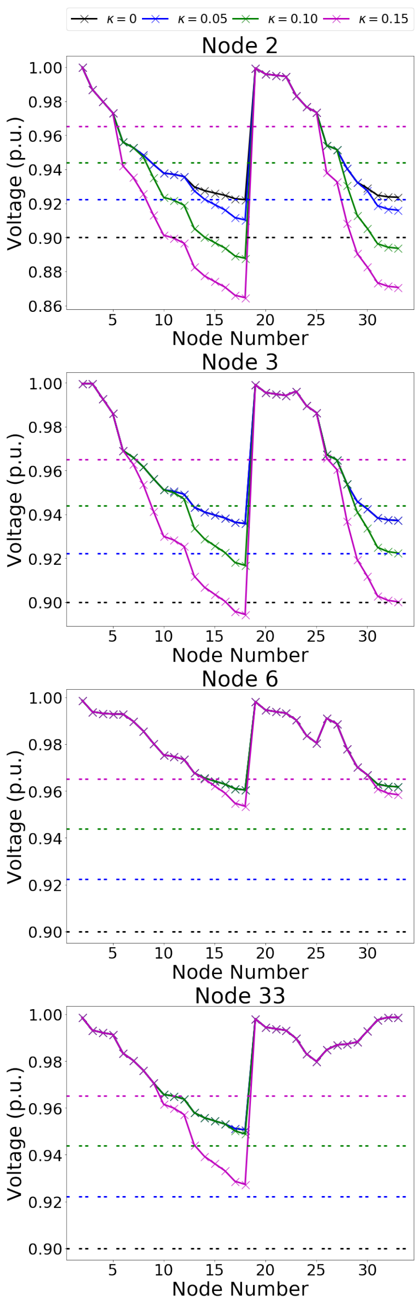

Fig. 5. The voltage profile with strategically acting DERs at given nodes. The black lines represent the base case, blue for $\kappa = 0.05$, green for $\kappa = 0.10$, and magenta for $\kappa = 0.15$. The dashed horizontal lines correspond to the minimum voltage limit for the given κ.

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