Physics-Based Modeling of Arterial Hemodynamics in Humans: Tapered versus Uniform Tube-Load Models

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***Abstract*— In this paper, tapered versus uniform tube-load (TL) models were examined as alternatives for the mathematical representations for blood pressure (BP) wave propagation in human aorta. The two TL models were tested using invasively measured BP waveforms in cardiac surgery patients. Both models exhibited comparable goodness of fit. The uniform TL model performed better in terms of the Akaike Information Criterion (AIC).**

I. Introduction

Recent studies have shown that central aortic BP (CABP) measured near the heart as opposed to peripheral BP may serve as a superior determinant in cardiovascular (CV) health assessment. However, its direct measurement is often inconvenient. Using the tube-load (TL) models is an attractive approach to estimate CABP and utilize its superior clinical value while leveraging the convenience of peripheral BP measurements. Here, we comparatively assess the validity of two alternative TL models as the mathematical representations for BP wave propagation in the human aorta. The goal was to specifically examine if there is a benefit in incorporating the exponential tapering into the TL model.

II. Methods

We studied human data that we previously collected under the approval of the University of Alberta Health Research Ethics Board. The data included invasive central aortic and femoral BP collected from 13 patients undergoing cardiac surgery with cardiopulmonary bypass (CPB). We considered two TL models: (1) a uniform TL model in which the aorta is modeled as a uniform lossless tube and (2) a variant of the exponentially tapered TL model developed by Fogliardi et al [1] (Fig. 1). The relationship between the aortic inlet (P(jω,0)) and outlet (P(jω,L)) pressures in the tapered TL model can be expressed via a transfer function ($H(j\omega|\theta_1, \theta_2, \theta_3)$) which is characterized by three parameters:

$$P(j\omega,L) = H(j\omega|\theta_1, \theta_2, \theta_3)\, P(j\omega,0) \quad (1)$$

where $\theta_1 = qL$, $\theta_2 = \tau$ and $\theta_3 = R_p/Z_{c0}$. Here, q denotes the rate of tube tapering, L denotes the tube length, τ denotes the pulse transit time (PTT); r_0 and r_L denote radii at the tube inlet and outlet; and R_p and Z_{c0} denote the terminal load resistance and tube characteristic impedance. Uniform TL model is a special case of exponentially tapered TL model when q = 0 and it is characterized by 2 parameters θ_2 and θ_3.

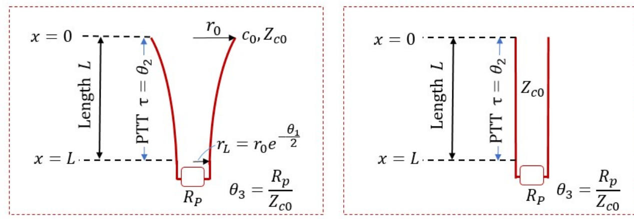

Figure 1 *Exponentially tapered (a) vs uniform (b) tube-load (TL) models*

In each subject, a 15s-long segment of data was used to fit the models to the ascending aortic and femoral arterial BP waveforms and derive the optimal parameter estimates $\boldsymbol{\theta}^*$. For model fitting, a constrained optimization problem was solved to minimize the sum of squared errors between the measured aortic outlet BP and its estimate from the models when the aortic inlet BP was inputted. The validity of the two models was assessed in terms of the goodness of fit (including the root-mean-squared error (RMSE) and correlation coefficient (r value)) and AIC.

III. Results and Conclusion

The exponentially tapered and uniform TL models showed comparable goodness of fit both in terms of RMSE and correlation coefficient (Table I).

Table I The validity metrics of the tapered vs. uniform TL models

	RMSE [mmHg]	r Value	AIC
Tapered TL	3.3+/-1.1	0.98+/-0.02	6
Uniform TL	3.4+/-1.1	0.98+/-0.01	7

Therefore, the uniform TL model was superior to the exponentially tapered TL model in terms of AIC. Also, the tapering constant was 0.6 on average which was far from the range of its anatomically anticipated value (1.7~3). In sum, in comparison with the uniform TL model, the exponentially tapered TL model may not provide valid physiological insight on the aortic tapering, and the improvement in the goodness of fit offered by the exponential aortic tapering may only be marginal.

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References

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