Decoupled and Unified Approaches for Solving Transmission and Distribution Co-Simulations

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***Abstract*—Increased penetration of flexible loads and distributed resources on power distribution circuits will lead to possibility of aggregating such small distributed resources for several grid services both at the distribution and transmission levels. This will lead to increased interaction of transmission and distribution system operators, as well as the control actions at LV/MV grid in aggregation will have significant impact on the operations of bulk transmission systems. Therefore, lately, there has been some advancements to developing co-simulation platform for solving transmission and distribution (T&D) systems simultaneously for dynamic and power flow type of studies. However, the existing co-simulation platforms base mainly on decoupled approaches. Therefore, a unified approach of solving T&D (for solution benchmarking purpose) is largely missing in the literature. In this context, this paper shows initial results of a decoupled method applied for solving T&D power flow co-simulation and is benchmarked against a unified solution. The case studies based on augmented 14+33 bus T&D systems show that the decoupled approach is fairly accurate.**

***Index Terms*—Transmission Simulation, Distribution Simulation, Co-Simulation, Power Flow.**

I. Introduction

The penetration of distributed resources such as electric vehicles, storage, flexible loads has been rapidly increasing on power distribution systems such that it has transformed the low/medium voltage grids into large dispatchable virtual batteries. This will lead to possibility of aggregating such distributed resources for several grid services-not only at the distribution level but also at the bulk transmission level. Future distribution systems with highly variable generations, unconventional dynamics due to interactions among multiple inverters, and bidirectional flow of energy will lead to increased interaction between transmission and distribution systems. Therefore, with increased penetration of flexible loads and distributed resources on power distribution circuits, its aggregate impact on the operation of bulk transmission systems should not be overlooked [1]–[3]. To maintain secure operations of transmission and distribution systems, it is important to have a better understanding of the coupling of transmission and distribution systems. The efficient way to examine such interactions in smart grid systems is to establish simulation process that integrates grids at different voltage levels. Hence, a need may arise to study transmission and distribution systems collectively.

In conventional simulation tools, transmission and distribution circuits are treated as separate systems and are analyzed independently. In such tools, distribution systems are represented by lumped loads while solving the transmission system. On the other hand, the transmission system is represented using a constant voltage source while solving distribution circuits [4]. Although, the existing electromagnetic transient (EMT) simulators can solve integrated transmission and distribution systems, the simulation procedure may not be scalable for large-scale systems due to inherent computational burden. Moreover, EMT level simulation may not be necessary for the entire power grid if power flow analysis, stability analysis or dynamic studies are the focus. Therefore, in order to reduce computational burden of EMT simulations, efforts have made in the past to develop co-simulation platforms that can combine EMT simulation and transient stability analysis (TSA) tools together such that the detailed EMT type of studies are carried out only for high frequency switching devices [5]–[7]. Similarly, in [8], a co-simulation platform is built that can interface phasor-based and EMT simulators. Lately, efforts have made to built co-simulation platforms to solve Transmission and Distribution Systems (T&D) directly in phasor domain for several applications (power flow [1], [9], dynamic simulation [9], contingency analysis [10], etc.)

Existing T&D co-simulation methods follow decoupled approaches, where transmission and distribution systems are decoupled at interface buses. Most of the methods use off-the-shelf simulators for solving transmission and distribution systems separately, and user-built interfaces are used for data exchange between the simulators. A unified approach for simultaneously solving T&D models are discussed in [11] (for power flow) and [12] (for dynamic simulation). In [9], a global power flow analysis based on master–slave splitting of T&D system is used. Convergence of the method is demonstrated; however, it is not known if the power flow solution of T&D using decoupled approach would converge to the true solution compared to a unified approach as in [11]. In [2], an open-source framework, ‘framework for network co-simulation’ (FNCS), is developed for T&D co-simulation, where FNCS is used as the information interface, GridPACK as transmission system simulator, and GridLAB-D as distribution system simulator. In [1], a Diakoptics based simulation approach for T&D co-simulation (both power flow

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and dynamic simulation) are built. In the modeling, the transmission system is based on three-sequence modeling and distribution system is based on three-phase modeling. The authors argued that the analysis in mixed frame of reference allows use of existing mature methodology/tools in solving respective transmission and distribution models.

In [4], a very large T&D system (100,000+ nodes) is solved using commercial ePHASORSIM solver, and a real-time performance was achieved exploiting the parallel processing. Decoupled approaches, as in [1], [4], [11], [13], exhibit benefits that the algorithms can be parallelized. However, inappropriate choice of simulation time step and inherent time delays on the decoupled approach may lead to divergence of the solution [14]. Moreover, solutions obtained from phasor-based decoupled approaches needs to benchmarked against phasor-based unified T&D co-simulation approaches [11], [12]. Since the benchmarking solutions to T&D co-simulation are not available, efforts have made in the past to compare against solutions obtained from the EMT solvers (and by using Fourier filters) [1].

Existing decoupled approaches take the advantage of using off-the-shelf simulators to solve the integrated T&D systems through proper information exchange with reduced computational burden. In all the aforementioned decoupled approaches, one major missing component is the absence of an appropriate benchmark to validate the results as there exists no standard T&D test results based on unified co-simulation approach in phasor domain. Therefore, one of the objectives of this work is to develop a unified T&D test system that can serve as a benchmark for decoupled approaches.

The rest of the paper is structured as follows. Section II discusses the mathematical modeling of the decoupled approach, Section III discusses the unified approach, Section IV presents case studies, and Section V presents main conlusions drawn from this work.

II. Decoupled Approach

Consider an $\mathbf{N}$-bus transmission system, where the first $\mathbf{m}$ buses have lumped loads, and buses $\mathbf{m+1}$ through $\mathbf{N}$ have distribution feeders connected downstream the buses. In a decoupled approach, power flow model of the transmission system for the positive sequence can be written as,

$$\mathbf{V^+_j = \sum_{k \in N} Y^+_{j,k}\, I^+_k \quad \forall j \in N} \tag{1}$$

$$\mathbf{P^+_j = Real\left(V^+_j\ I^{+*}_j\right) \quad \forall j \in 1,2,..,m} \tag{2}$$

$$\mathbf{Q^+_j = Imag\left(V^+_j\ I^{+*}_j\right) \quad \forall j \in 1,2,..,m} \tag{3}$$

$$\mathbf{\widetilde{P^+_j} = Real\left(V^+_j\ I^{+*}_j\right) \quad \forall j \in m+1,..,N} \tag{4}$$

$$\mathbf{\widetilde{Q^+_j} = Imag\left(V^+_j\ I^{+*}_j\right) \quad \forall j \in m+1,..,N} \tag{5}$$

$$\mathbf{\widetilde{V^+_j} = V^+_j \quad \forall j \in m+1,..,N}. \tag{6}$$

where $\mathbf{V^+}$, $\mathbf{I^+}$, $\mathbf{P^+}$, $\mathbf{Q^+}$, and $\mathbf{Y^+}$ represent positive sequence of bus voltage, injection current, active power injection, reactive power injection, and Y-bus matrix, respectively. $\mathbf{\widetilde{P^+_j}}$ and $\mathbf{\widetilde{Q^+_j}}$ represent equivalent positive sequence lumped load representation of distribution feeders connected at bus $\mathbf{j}$. In decoupled approaches, equivalent lumped loads ($\mathbf{\widetilde{P^+}}$ and $\mathbf{\widetilde{Q^+}}$) are obtained by separately solving the power flow of each distribution systems connected to the transmission network. $\mathbf{\widetilde{V^+}}$ represents voltage of transmission buses with distribution feeders downstream the buses.

Consider a $\mathbf{R}$-node distribution system connected to an arbitrary $\mathbf{j}$-bus of the transmission network. Without loss of generality, we assumed the first three nodes as three-phase sub-station when analyzing the distribution system. Distribution power flow analysis can be formulated similar to the transmission system, except the distribution systems are modeled in phase frame of reference.

$$\mathbf{V^\varphi_i = \sum_{r \in R} Y^\varphi_{i,r}\, I^\varphi_r \quad \forall i \in R} \tag{7}$$

$$\mathbf{P^\varphi_i = Real\left(V^\varphi_i\ I^{\varphi*}_i\right) \quad \forall i \in R} \tag{8}$$

$$\mathbf{Q^\varphi_i = Imag\left(V^\varphi_i\ I^{\varphi*}_i\right) \quad \forall i \in R} \tag{9}$$

$$\mathbf{\widetilde{P^+_j} = -\mathcal{S}\left(P^\varphi_1, P^\varphi_2, P^\varphi_3\right)} \tag{10}$$

$$\mathbf{\widetilde{Q^+_j} = -\mathcal{S}\left(Q^\varphi_1, Q^\varphi_2, Q^\varphi_3\right)} \tag{11}$$

$$\mathbf{\left[V^\varphi_1, V^\varphi_2, V^\varphi_3\right] = \mathcal{G}\left(\widetilde{V^+_j}\right)}. \tag{12}$$

where $\mathbf{V^\varphi}$, $\mathbf{I^\varphi}$, $\mathbf{P^\varphi}$, and $\mathbf{Q^\varphi}$ represent node voltage, injection current, active power injection, and reactive power injection, respectively, in phase frame of reference. $\mathbf{Y^\varphi}$ represents Y-bus matrix of a three-phase system in phase frame of reference. Functions $\mathcal{S}(\ )$ and $\mathcal{G}(\ )$ convert powers on three phases to positive sequence power and positive sequence voltage to three-phase voltages, respectively. These conversions assume that the transmission system is balanced; hence, the three-phase voltages at the sub-station nodes ($\mathbf{V^\varphi_1}$, $\mathbf{V^\varphi_2}$, $\mathbf{V^\varphi_3}$) of the distribution system are balanced.

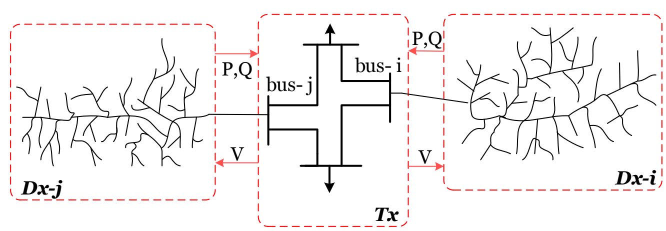

Fig. 1: A T&D system showing boundaries and information exchange for solving transmission (Tx) and distribution (Dx) co-simulation using a decoupled approach.

The solution procedure of mathematical models (1)-(12) of the decoupled approach can be explained using Fig. 1. In the Fig. 1, an integrated T&D system is shown where distribution feeders are connected at nodes $\mathbf{i}$ and $\mathbf{j}$. In the decoupled approach, the transmission and distribution systems are solved iteratively and based on some information exchange. First, assuming a nominal sub-station voltages, power flow model (7)-(11) are solved for each distribution systems in phase frame of reference. This provides total active and reactive power consumed by each distribution feeders (including losses) and are converted to positive sequence powers. Now with this information from all distribution circuits, the distribution systems are represented as lumped loads on respective transmission buses and transmission system power flow model (1)-(6) are solved. Solution of this provides positive sequence bus voltages, which are then converted to voltages in phase frame of reference using (12). The bus voltage obtained from transmission power flow in phase frame are used to update the distribution system three-phase substation voltage using (12) and distribution level power flow models (7)-(11) are solved again for each distribution systems. These transmission and distribution power flow models are solved iterative using P,Q,V updates until the convergence criterion is met.

III. Unified Approach

A. Power Flow Model

In this section, an integrated transmission and distribution system model is developed and a unified load flow which runs for combined system is discussed. Our Unified approach considers transmission as a balanced system and distribution as unbalanced system. Therefore, we used positive sequence for transmission system and all three-sequence network for distribution system analysis. The distribution system represented using three phase parameters is converted to sequence frame and combined with transmission system parameters to develop an integrated T&D system.

Consider an **N**-bus transmission system connected to **M**-bus distribution system. In unified approach, power flow model of the combined system for the positive sequence can be written as,

$$\mathbf{V_j^+ = \sum_{k\in N+M} Y_{j,k}^+ I_k^+ \quad \forall j \in N+M} \tag{13}$$

$$\mathbf{P_j^+ = Real\left(V_j^+ \; I_j^{+*}\right) \quad \forall j \in 1,2,..,N+M} \tag{14}$$

$$\mathbf{Q_j^+ = Imag\left(V_j^+ \; I_j^{+*}\right) \quad \forall j \in 1,2,..,N+M.} \tag{15}$$

where $\mathbf{V^+}$, $\mathbf{I^+}$, $\mathbf{P^+}$, $\mathbf{Q^+}$, and $\mathbf{Y^+}$ represent positive sequence of bus voltage, injection current, active power injection, reactive power injection, and positive sequence Y-bus matrix. Transmission system parameters are readily available in sequence frame of reference, while the three-phase distribution impedance parameters need to be converted to sequence impedances. The formulation can be readily extended to include the negative and zero sequence component circuits for distribution grid. The distribution three-phase parameters are converted to three-sequence details using a stacked Y-bus methodology. Then, the sequence Y-bus of distribution system is combined with the sequence Y-bus of transmission system to obtain integrated T&D Y-bus. The load data of integrated system is per unitized to a common base which is used with integrated Y-bus to begin load flow analysis. Newton Raphson method is used to solve load flow. Since the distribution system considered in this work is a balanced system, the Newton Raphson method would converge for the combined system. A flow chart of the unified simulation of integrated T&D system is presented in Fig. 2.

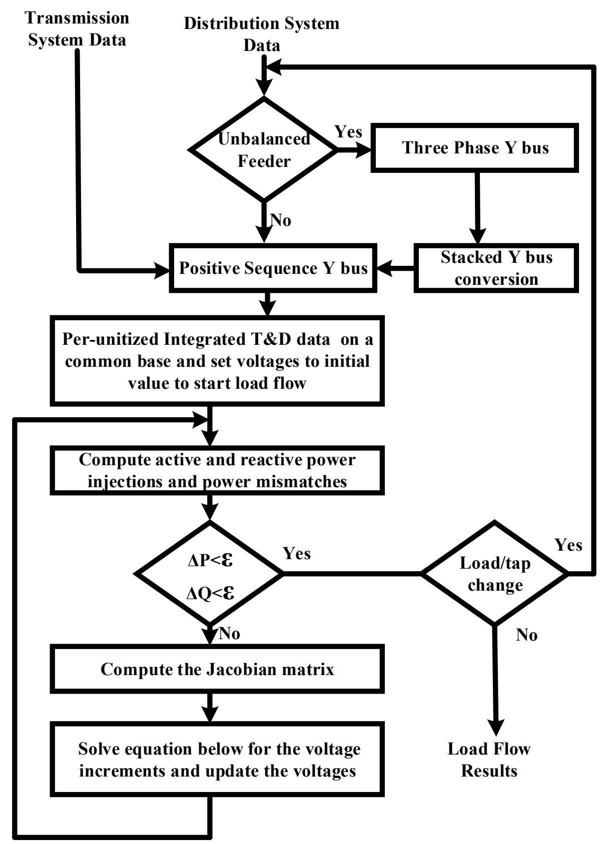

Fig. 2: Flowchart of Unified T&D Simulation.

B. Y-bus in Sequence Frame

The stacked Y-bus methodology builds from the admittance matrices of individual elements. We briefly discuss the formation of admittance matrix of individual series components next. Note that even though the formulation is shown only for a generic conductors/cables and transformer models, we consider a comprehensive set of series elements (e.g., 1-phase feeders, 2-phase feeder, 3-phase feeders, 1-phase and 3-phase load tap changers, delta–wye/wye–delta/ wye–wye/ delta–delta transformers, 1-phase transformers, etc.).

For typical multi-phase cables and conductors connected between node n and node m, component of 3-phase Y-bus can be obtained as,

$$\mathbf{Y}_{nn} = \mathbf{Y}_{mm} = \frac{1}{2}\mathbf{B}_{nm} + \mathbf{Z}_{nm}^{-1} \tag{16}$$

$$\mathbf{Y}_{nm} = \mathbf{Y}_{mn} = \mathbf{Z}_{nm}^{-1}. \tag{17}$$

Similarly, a three-phase transformer can be represented by a series block representing the per unit leakage admittance, and a shunt block modeling transformer core losses. The nodal admittances of different connections types of transformer are shown in Table I.

TABLE I: Y-bus for Various Transformer Connections.

Node n	Node m	Ynn	Ynm	Ymn	Ymm
Wye-G	Wye-G	Y1	Y1	Y1	Y1
Wye	Wye	Y2	Y2	Y2	Y2
Wye	Delta	Y2	-Y3	$-Y3^T$	Y2
Delta	Delta	Y2	Y2	Y2	Y2

$$\mathbf{Y}_1 = \begin{bmatrix} y_t & 0 & 0 \\ 0 & y_t & 0 \\ 0 & 0 & y_t \end{bmatrix}, \mathbf{Y}_2 = \frac{1}{3}\begin{bmatrix} 2y_t & -y_t & -y_t \\ -y_t & 2y_t & -y_t \\ -y_t & -y_t & 2y_t \end{bmatrix} \quad (18)$$

$$\mathbf{Y}_3 = \frac{1}{\sqrt{3}}\begin{bmatrix} -y_t & y_t & 0 \\ 0 & -y_t & y_t \\ y_t & 0 & -y_t \end{bmatrix}, \mathbf{Y}_4 = \frac{1}{3}\begin{bmatrix} y_t & -y_t & 0 \\ -y_t & 2y_t & -y_t \\ 0 & -y_t & y_t \end{bmatrix} \quad (19)$$

$$\mathbf{Y}_5 = \begin{bmatrix} y_t & 0 \\ 0 & y_t \end{bmatrix}, \mathbf{Y}_6 = \frac{1}{\sqrt{3}}\begin{bmatrix} -y_t & y_t & 0 \\ 0 & -y_t & y_t \end{bmatrix}. \quad (20)$$

where y_t is the per unit leakage admittance.

After the admittance matrices for individual series components are computed, stacked Y-bus method can be used as following,

- Using 3-phase primitive Y-bus for individual component, built 3-phase Y-bus of distribution system .
- Take each 3x3 matrix from the 3-phase Y-bus.
- Find inverse to obtain Z.
- Convert Z to sequence components.
- Inverse the sequence impedance components to obtain sequence admittance component.
- Append this distribution system sequence Y-bus to positive sequence Y-bus of transmission system to obtain Y-bus of integrated T&D system.

IV. Case Studies

A. Validation of Power Flow Models

A current injection method similar to [15] is used to solve the power flow models, where the current injection equations are written in rectangular coordinates. In the current injection method, the Jacobian matrix has the same structure as the nodal admittance matrix except for PV buses. Off-diagonal blocks of the Jacobian matrix are equal to those of the nodal admittance matrix. The diagonal blocks are updated according to type of load model considered for that bus. For each PV bus, a new dependent variable is introduced with an additional equation imposing zero bus voltage deviation.

An integrated T&D system is formed by combining a 14-bus transmission system and a 33-node balanced distribution system connected to as shown in Fig. 3. The 14-bus transmission system consists of 5 generators and 11 loads with net load of 260MW (75MVAr). The 33-node distribution system has net connected load of 3.7MW (2.3MVAr). First, the transmission system model and distribution system model are separately verified for the distributed approach. The transmission system model is also solved with spot load approach, where all the loads on distribution feeders are lumped at a corresponding transmission bus. Then, the combined T&D model is solved using distributed and unified approach, and the results are shown in Fig.7. The error plot shows that the decoupled approach results is the same solution as the unified approach.

B. 47-bus System

The augmented 47-bus system (see Fig. 3) is further used to compare the performance of voltage and power angle solution on the transmission as well as distribution part of the circuit. The load flow voltage and angle obtained from the unified approach and decoupled approach are shown in Fig. 5 and Fig. 6. The plots clearly show that the voltage and angle solutions from both approaches are very close with error less than 6×10^{-6} on voltages and 2.5×10^{-3} on angles.

C. 113-bus System

Two augmented 113-bus systems are created by using three sections of 33-node distribution feeder (see Fig. 7). The load flow voltage and angle obtained from the unified approach and decoupled approach for the first case of the circuit configuration (Case a) are shown in Fig. 8 and Fig. 9. The plots clearly show that the voltage and angle solutions from both approaches are very close with error less than 3×10^{-5} on voltages and 2.5×10^{-3} on angles. For the second case of the circuit configuration (Case b), the voltage and angle solution obtained are shown in Fig. 10 and Fig. 11. The plots clearly show that the voltage and angle solutions from both approaches are very close with error less than 3×10^{-5} on voltages and 2.5×10^{-3} on angles. The case studies demonstrate that the distributed and unified approaches for solving T&D model yield the same solutions.

V. Conclusion and Future Work

In the proposed work, we have developed a unified approach for solving T&D co-simulation for benchmarking purpose, and accuracy of a decoupled approach is compared to the unified solution. The unified T&D power flow model bases on using Y-bus of individual components and a stack Y-bus building methodology. Since in the literature a phasor-based benchmark to validate decoupled approach was lacking, we present some

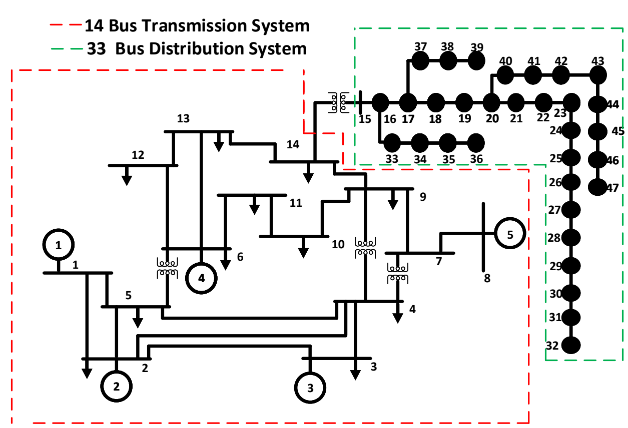

Fig. 3: One line diagram of 47-bus T&D system.

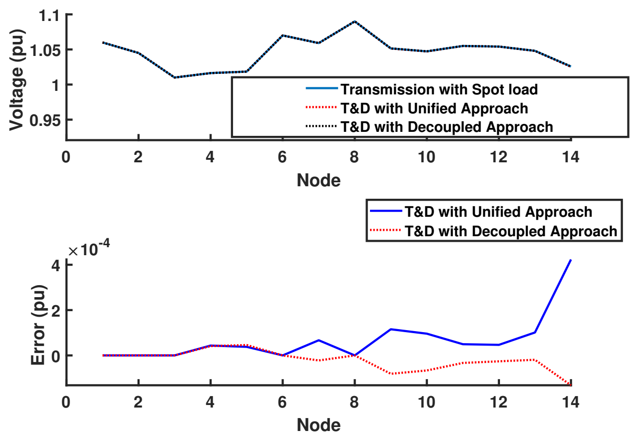

Fig. 4: Voltage solution and error on transmission circuits.

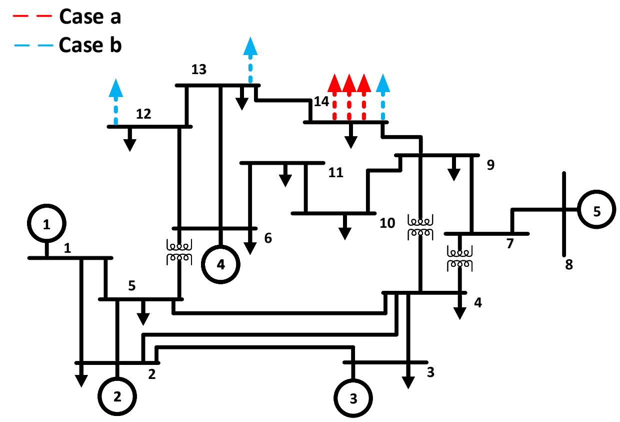

Fig. 7: Two configuration of 113-bus T&D system.

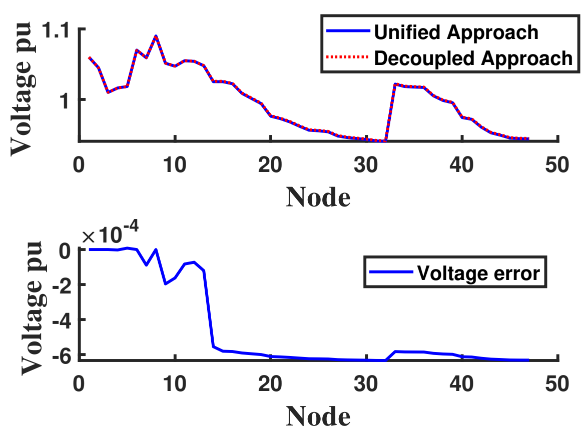

Fig. 5: Voltage solution (and error) of 47-bus T&D system obtained from decoupled and unified approaches.

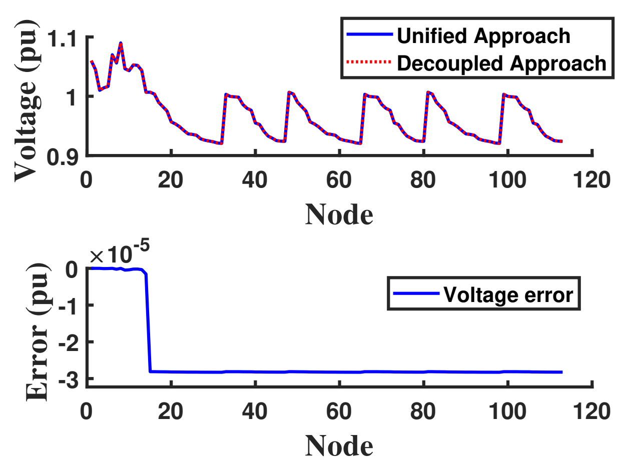

Fig. 8: Voltage solution (and error) of 113-bus T&D system (Case a) obtained from decoupled and unified approaches.

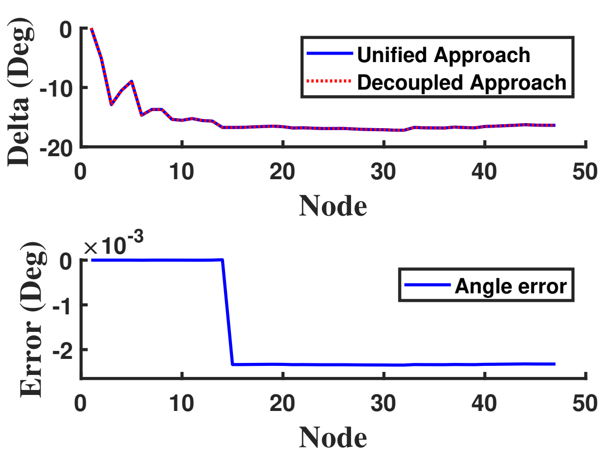

Fig. 6: Angle solution (and error) of 47-bus T&D system obtained from decoupled and unified approaches.

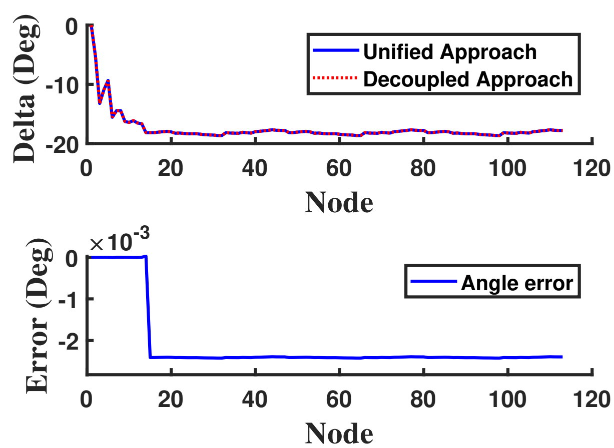

Fig. 9: Angle solution (and error) of 113-bus T&D system (Case a) obtained from decoupled and unified approaches.

of the initial results obtained towards building a benchmark unified T&D model. Our case studies on 47-bus and 113-bus T&D systems show that the decoupled approach are fairly accurate compared to the unified approches for solving T&D co-simulation. However, our observation is based on synthetic small-scale system; thus, extensive simulations on large scale systems with three-phase unbalanced distribution grids must

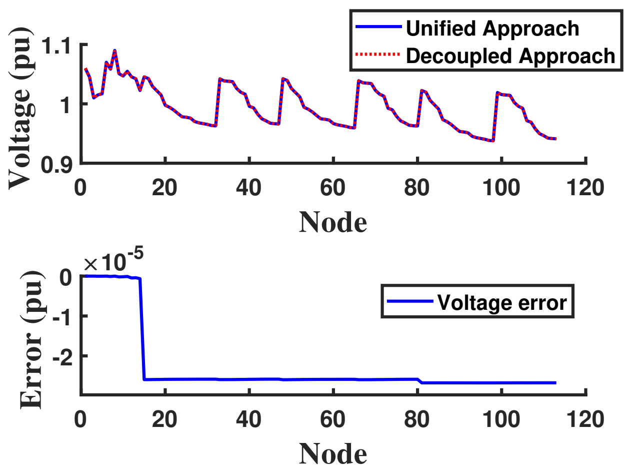

Fig. 10: Voltage solution (and error) of 113-bus T&D system (Case b) obtained from decoupled and unified approaches.

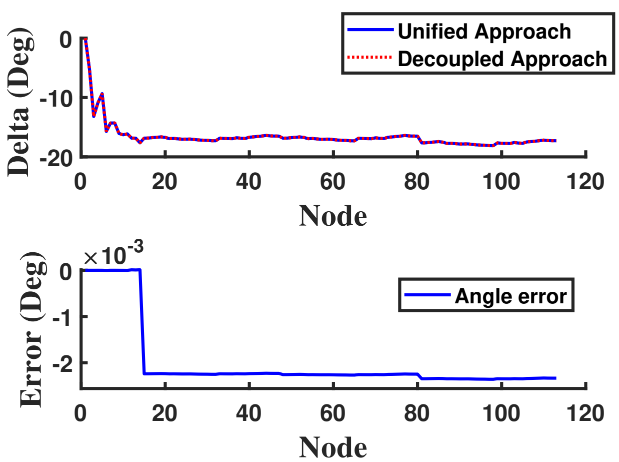

Fig. 11: Angle solution (and error) of 113-bus T&D system (Case b) obtained from decoupled and unified approaches.

be considered before making any generic conlusion regarding the accuracy of decoupled approaches.

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