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Cadence Tracking for Switched FES Cycling with Unknown Input Delay

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***Abstract*—Functional electrical stimulation (FES) induced cycling provides a means of therapeutic exercise and functional restoration for people affected by neuromuscular disorders. A challenge in closed-loop FES control of coordinated motion is the presence of a potentially destabilizing input delay between the application of the electrical stimulation and the resulting muscle contraction. Moreover, switching amongst multiple actuators (e.g., between FES control of various muscle groups and a controlled electric motor) presents additional challenges for overall system stability. In this paper, a closed-loop controller is developed to yield exponential cadence tracking, despite an unknown input delay, switching between FES and motor only control, uncertain nonlinear dynamics, and additive disturbances. Lyapunov-Krasovskii functionals are used in a Lyapunov-based stability analysis to ensure exponential convergence for all time.**

***Index Terms*—Functional electrical stimulation (FES), input delay, switched systems, human-robot interaction, rehabilitation robotics.**

I. Introduction

Functional electrical stimulation (FES) induced cycling is a common rehabilitative exercise for people with lower limb movement disorders [1]–[4]. FES evokes muscle contractions by applying an external electrical stimulus across the motor neurons of a muscle. While FES has been shown to improve muscle strength [5] and range of motion [6], FES-cycling in particular has been shown to improve cardiovascular parameters [7], bone mineral density [8], and physiological motor control [9].

Challenges of closed-loop control of FES include muscle force decaying under a constant stimulation intensity due to fatigue [10], the unknown and nonlinear mapping from electrical input to generated muscle force [11], and unmodeled disturbances and uncertain parameters in the dynamic muscle model [12]. Furthermore, switching between different muscle groups and motor only control is required for more complicated functional tasks (e.g., FES cycling) [13], and the muscle response from electrical stimulation exhibits delayed contractions [14]. The delayed muscle response is modeled as an input delay due to the complex electro-physiological mechanism involved in muscle torque production. Specifically, experimental evidence has demonstrated that there is a time lag, termed electromechanical delay (EMD), between the electrical activation and the onset of muscle force [14]–[19]. The result in [18] concluded that the muscles exhibited delays ranging from 75-200 ms; however, no delay compensation was provided. Since EMD results in delayed torque generation, it can potentially destabilize human motor control tasks [16], [19]. Thus, there is a need for closed-loop switched FES control strategies that are robust to uncertain nonlinear muscle dynamics and unknown input delay.

Input delayed systems and their associated stability analysis have been extensively studied in recent years [20]–[31]. In results such as [23], exact model knowledge with a known delay is assumed. Other results such as [24] and [25] instead focus on developing non-model based controllers for an uncertain nonlinear system with a known input delay. Since it is problematic for many practical engineering applications to measure the input delay [17], results such as [26] and [27] analyze nonlinear systems with an unknown input delay. Adaptive optimal output regulation of discrete-time linear systems with unknown input delay was developed in [28]. More recently, methods to compensate for input delay in a switched system have been studied. Lyapunov-Krasovskii functions are constructed in [29] to ensure input-to-state stability of switched nonlinear systems with time-varying input delay. In [30], linear control methods were used to guarantee uniform stability of equilibrium points for a switched system with input delay. In [31], a technique is proposed to ensure global asymptotic stability for an origin of switched time-varying systems with time-varying discontinuous delays. However, the aforementioned results do not yield exponential convergence to an ultimate bound and do not account for effects that are unique to FES input delay, such as a delayed activation of the muscle in addition to a residual contraction that persists when the stimulation ceases.

Recently, FES controllers have been developed to account for the delayed response of muscle. The results in [32] assume a known delay and uncertain dynamics and ensure uniformly ultimately bounded tracking. A global asymptotic tracking controller was developed for an unknown constant delay in [33] but assumes exact model knowledge of the lower limb dynamics. In [34] and [35] an unknown time-varying input delay was assumed, and in [35], uniformly ultimately bounded tracking was achieved by assuming a bound on the time-varying rate of delay. To date, a closed-loop switched FES controller has not been developed that is robust to input delay.

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In this paper, a switched cadence tracking controller is developed for FES cycling. A controller is designed to compensate for the delay disturbance using Lyapunov-Krasovskii functionals and an auxiliary tracking error signal that is designed to inject a delay-free FES control signal in the closed-loop dynamics. Trigger conditions are developed to activate and deactivate the FES such that the muscles will be generating torque when entering kinematically efficient regions of the cycle and stop generating torque when entering inefficient regions (i.e., kinematic dead zones). A Lyapunov-based stability analysis proves exponential convergence of the cadence error system despite uncertainties in the nonlinear model, additive disturbances, and an unknown input delay.

II. MODEL

The motorized cycle-rider system can be modeled as [13][1]

$$\tau_M(q,\dot{q},\tau,t)+\tau_e(q,t)=M(q)\ddot{q}+V(q,\dot{q})\dot{q} +G(q)+P(q,\dot{q})+b_c\dot{q}+d(t), \tag{1}$$

where $q:\mathbb{R}_{\geq 0}\to\mathcal{Q}$, $\dot{q}:\mathbb{R}_{\geq 0}\to\mathbb{R}$, and $\ddot{q}:\mathbb{R}_{\geq 0}\to\mathbb{R}$ denote the measurable crank angle and velocity, and unmeasurable acceleration, respectively. The set $\mathcal{Q}\subseteq\mathbb{R}$ denotes all possible crank angles and time is denoted by $t\in\mathbb{R}_{\geq 0}$. The electromechanical delay[2], i.e., the delay between the application/removal of the current and the onset/elimination of muscle force production is denoted by a constant[3] unknown delay denoted by $\tau\in\mathbb{R}_{>0}$. The inertial effects, centripetal-Coriolis effects, gravitational effects, and passive viscoelastic tissue forces from the legs are denoted as $M:\mathcal{Q}\to\mathbb{R}_{>0}$, $V:\mathcal{Q}\times\mathbb{R}\to\mathbb{R}$, $G:\mathcal{Q}\to\mathbb{R}$, and $P:\mathcal{Q}\times\mathbb{R}\to\mathbb{R}$, respectively. The viscous damping effects and disturbances applied about the crank axis are denoted by $b_c\in\mathbb{R}_{>0}$ and $d:\mathbb{R}_{\geq 0}\to\mathbb{R}$, respectively. The torque contributions due to the motor and FES induced muscle contractions are denoted as $\tau_e:\mathcal{Q}\times\mathbb{R}_{\geq 0}\to\mathbb{R}$ and $\tau_M:\mathcal{Q}\times\mathbb{R}\times\mathbb{R}\times\mathbb{R}_{\geq 0}\to\mathbb{R}$, respectively defined as

$$\tau_e(q,t)\triangleq B_e u_E(q,t), \tag{2}$$

$$\tau_M(q,\dot{q},\tau,t)\triangleq\sum_{m\in\mathcal{M}}B_m(q,\dot{q})\,u_m(q,\dot{q},\tau,t), \tag{3}$$

where the unknown motor control effectiveness is denoted by $B_e\in\mathbb{R}_{>0}$. The control effectiveness for the electrically stimulated muscle groups in (3) is denoted by $B_m:\mathcal{Q}\times\mathbb{R}\to\mathbb{R}_{>0}\ \forall m\in\mathcal{M}$, where $m\in\mathcal{M}\triangleq\{RG, RQ, RH, LG, LQ, LH\}$ indicates the right (R) and left (L) gluteal (G), quadriceps femoris (Q), and hamstrings (H) muscle groups. The electrical stimulation input (i.e., pulse width) delivered to the rider's muscles, denoted by $u_m:\mathcal{Q}\times\mathbb{R}\times\mathbb{R}\times\mathbb{R}_{\geq 0}\to\mathbb{R}\ \forall m\in\mathcal{M}$, and the control current to the electric motor denoted by $u_E:\mathcal{Q}\times\mathbb{R}_{\geq 0}\to\mathbb{R}$, are defined as

$$u_m(q,\dot{q},\tau,t)\triangleq k_m\sigma_m(q,\dot{q},\overline{\tau},\underline{\tau})\,u(t-\tau), \tag{4}$$

$$u_E(q,t)\triangleq k_e u_e(t), \tag{5}$$

where $k_m,k_e\in\mathbb{R}_{>0}\ \forall m\in\mathcal{M}$ are selectable constants and $\overline{\tau},\underline{\tau}\in\mathbb{R}_{>0}$ are known constants that represent the upper and lower bound of the delay, respectively (e.g., determined from experimental results such as [17]). The subsequently designed FES control in (4) and motor input in (5) are denoted by $u:\mathbb{R}\times\mathbb{R}_{\geq 0}\to\mathbb{R}$ and $u_e:\mathbb{R}_{\geq 0}\to\mathbb{R}$, respectively. In (4), $\sigma_m:\mathcal{Q}\times\mathbb{R}\to\{0,1\}$ denotes a piecewise left-continuous switching signal for each muscle group and is defined as

$$\sigma_m(q,\dot{q},\overline{\tau},\underline{\tau})\triangleq\begin{cases}1, & q_\alpha\in\mathcal{Q}_m, q\in\mathcal{Q}_e\\ 1, & q_\beta\in\mathcal{Q}_m\\ 0, & \text{otherwise}\end{cases}, \tag{6}$$

$\forall m\in\mathcal{M}$, where trigger conditions $q_\alpha,q_\beta:\mathcal{Q}\times\mathbb{R}\to\mathbb{R}$ are defined as $q_\alpha\triangleq f(q,\dot{q},\overline{\tau})$ and $q_\beta\triangleq f(q,\dot{q},\underline{\tau})$, where f is designed to stimulate the rider's muscles sufficiently prior to the crank entering the FES region and for stimulation to cease sufficiently prior to the crank leaving the FES region.

Definitions for the subsequent FES regions, denoted by $\mathcal{Q}_m\subset\mathcal{Q}$, and switching laws are based on [13], where each muscle group is stimulated in specific regions of the crank cycle (i.e., when kinematically efficient). In this manner, $\mathcal{Q}_m$ is defined for each muscle group as

$$\mathcal{Q}_m\ \triangleq\ \{q\in\mathcal{Q}\,|\,T_m(q)>\varepsilon_m\}, \tag{7}$$

$\forall m\in\mathcal{M}$, where $\varepsilon_m\in(0,\ \max(T_m)]$ is the lower threshold for each torque transfer ratio denoted by $T_m:\mathcal{Q}\to\mathbb{R}$, which limits the FES regions for each muscle so that each muscle group can only contribute to forward pedaling (i.e., positive crank motion). The union of all muscle regions defined in (7) represents the entire FES region, denoted by $\mathcal{Q}_{FES}$, defined as $\mathcal{Q}_{FES}\triangleq\underset{m\in\mathcal{M}}{\cup}\{\mathcal{Q}_m\}$. The kinematic dead zones are defined as $Q_e\triangleq Q\backslash\mathcal{Q}_{FES}$. Substituting (3)-(5) into (1) yields[4]

$$B_M^\tau u_\tau+B_E u_e=M\ddot{q}+V\dot{q}+G+P+b_c\dot{q}+d, \tag{8}$$

where $B_M^\tau\triangleq\sum_{m\in\mathcal{M}}B_m(q,\dot{q})\,k_m\sigma_m(q,\dot{q},\overline{\tau},\underline{\tau})$, $B_E\triangleq B_e k_e$, and $u_\tau\triangleq u(t-\tau)$. Throughout the paper, delayed functions are defined as

$$h_\tau\triangleq\begin{cases}h(t-\tau) & t-\tau\geq t_0\\ 0 & t-\tau<t_0\end{cases},$$

where $t_0\in\mathbb{R}_{\geq 0}$ is the initial time.

The switched system in (8) has the following properties and assumptions [13]. **Property: 1** $c_m\leq M\leq c_M$, where

[1]For notational brevity, all explicit dependence on time, t, within the terms $q(t)$, $\dot{q}(t)$, $\ddot{q}(t)$ is suppressed.

[2]For simplicity, and without loss of generality, the delay for the onset of torque and the delay resulting in residual torque are set to the same value.

[3]Time-varying delay effects due to muscle fatigue are the focus of future efforts.

[4]For notational brevity, all functional dependencies are hereafter suppressed unless required for clarity of exposition.

$c_m,\, c_M \in \mathbb{R}_{>0}$ are known constants. **Property: 2** $|V| \leq c_V|\dot{q}|$, where $c_V \in \mathbb{R}_{>0}$ is a known constant and $|\cdot|$ denotes the absolute value. **Property: 3** $|G| \leq c_G$, where $c_G \in \mathbb{R}_{>0}$ is a known constant. **Property: 4** $|P| \leq c_{P1} + c_{P2}|\dot{q}|$, where $c_{P1},\, c_{P2} \in \mathbb{R}_{>0}$ are known constants. **Property: 5** $b_c\dot{q} \leq c_c|\dot{q}|$, where $c_c \in \mathbb{R}_{>0}$ is a known constant. **Property: 6** $|d| \leq c_d$, where $c_d \in \mathbb{R}_{>0}$ is a known constant. **Property: 7** $\frac{1}{2}\dot{M} = V$. **Property: 8** The muscle control effectiveness B_m is lower and upper bounded $\forall m \in \mathcal{M}$, and thus, when $\sum\limits_{m\in\mathcal{M}} \sigma_m > 0$, $c_b \leq B_M^{\tau} \leq c_B$, where $c_b, c_B \in \mathbb{R}_{>0}$ are known constants. **Property: 9** $c_e \leq B_E \leq c_E$, where $c_e,\, c_E \in \mathbb{R}_{>0}$ are known constants.

III. Control Development

The control objective is for the crank to track a desired cadence $\dot{q}_d : \mathbb{R}_{\geq 0} \to \mathbb{R}$ despite the unknown input delay and uncertainties in the dynamic model. To facilitate the subsequent analysis, measurable auxiliary tracking errors, denoted by $e_1 : \mathbb{R}_{\geq 0} \to \mathbb{R}$ and $r : \mathbb{R}_{\geq 0} \to \mathbb{R}$, are defined as[5]

$$e_1 \triangleq q_d - q, \tag{9}$$

$$r \triangleq \dot{e}_1 + \alpha e_1 + \eta e_u, \tag{10}$$

where $\alpha, \eta \in \mathbb{R}_{\geq 0}$, are selectable constants. To incorporate a delay-free input term in the closed-loop error system, an auxiliary error signal, denoted by $e_u : \mathbb{R}_{\geq 0} \to \mathbb{R}$, is defined as

$$e_u \triangleq -\int_{t-\hat{\tau}}^{t} u(\theta)\, d\theta, \tag{11}$$

where $\hat{\tau} \in \mathbb{R}_{>0}$ is the delay estimate. Taking the time derivative of (10), multiplying by M, and using (8), (9), and (11), the open-loop error system can be obtained as

$$\begin{aligned} M\dot{r} \;&=\; -Vr - e_1 + \chi - B_E u_e + B_M^{\tau}\left(u_{\hat{\tau}} - u_{\tau}\right) \\ &\quad + \left(M\eta - B_M^{\tau}\right) u_{\hat{\tau}} - M\eta u, \end{aligned} \tag{12}$$

where the auxiliary term $\chi : \mathcal{Q} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ is defined as

$$\begin{aligned} \chi \;&\triangleq\; M\ddot{q}_d + V\left(\dot{q}_d + \alpha e_1 + \eta e_u\right) + G \\ &\quad + P + b_c\dot{q} + d + M\alpha\dot{e}_1 + e_1. \end{aligned}$$

From Properties 1-6, χ can be bounded as

$$|\chi| \leq \Phi + \rho\left(\|z\|\right)\|z\|, \tag{13}$$

where $\Phi \in \mathbb{R}_{>0}$ is a known constant, $\rho(\cdot)$ is a positive, radially unbounded, and strictly increasing function, and $z \in \mathbb{R}^3$ is a vector of error signals defined as

$$z \triangleq \left[\, e_1 \;\; r \;\; e_u \,\right]^T. \tag{14}$$

Based on (12) and (13), and the subsequent stability analysis, the FES control input is designed as

$$u = k_s r, \tag{15}$$

[5]The control objective can be quantified in terms of the first time derivative of e_1, (i.e., $\dot{e}_1$).

where $k_s \in \mathbb{R}_{>0}$ is a selectable constant. The motor control input is designed as

$$u_e = k_{1e}\mathrm{sgn}\left(r\right) + k_{2e}r, \tag{16}$$

where $\mathrm{sgn}\left(\cdot\right)$ denotes the signum function, and $k_{1e},\, k_{2e} \in \mathbb{R}_{>0}$ are selectable constants. Substituting (15) and (16) into (12) yields the closed-loop error system

$$\begin{aligned} M\dot{r} \;&=\; -Vr - e_1 + \chi - B_E\left(k_{1e}\mathrm{sgn}\left(r\right) + k_{2e}r\right) \\ &\quad + k_s B_M^{\tau}\left(r_{\hat{\tau}} - r_{\tau}\right) + \left(M\eta - B_M^{\tau}\right)k_s r_{\hat{\tau}} \\ &\quad - M\eta k_s r. \end{aligned} \tag{17}$$

Based on (17) and the subsequent stability analysis, let the Lyapunov-Krasovskii functionals $Q_1,\, Q_2,\, Q_3,\, Q_4 : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ be defined as

$$Q_1 \triangleq \left(\varepsilon_1\omega_1 k_s + \frac{k_s c_B}{2}\right)\int_{t-\hat{\tau}}^{t} r\left(\theta\right)^2 d\theta, \tag{18}$$

$$Q_2 \triangleq \frac{\omega_2 k_s}{\hat{\tau}}\int_{t-\hat{\tau}}^{t}\int_{s}^{t} r\left(\theta\right)^2 d\theta ds, \tag{19}$$

$$Q_3 \triangleq \frac{k_s c_B}{2}\int_{t-\tau}^{t} r\left(\theta\right)^2 d\theta, \tag{20}$$

$$Q_4 \triangleq \frac{\omega_3 k_s}{\tau}\int_{t-\tau}^{t}\int_{s}^{t} r\left(\theta\right)^2 d\theta ds, \tag{21}$$

where $\omega_1, \omega_2, \omega_3, \varepsilon_1, \varepsilon_2 \in \mathbb{R}_{>0}$ are selectable constants. To facilitate the subsequent stability analysis, auxiliary bounding constants $\beta_1, \delta_1, \beta_2, \delta_2 \in \mathbb{R}_{>0}$ are defined as

$$\begin{aligned} \beta_1 \triangleq \min\Bigg(&\alpha - \frac{\varepsilon_2\eta^2}{2},\; \frac{3}{4}c_m\eta k_s - 2c_B k_s - 2\varepsilon_1\omega_1 k_s \\ &- \omega_2 k_s - \omega_3 k_s + k_{2e}c_e,\; \frac{\omega_2}{3k_s\hat{\tau}^2} - \frac{1}{2\varepsilon_2} - \frac{\omega_1 k_s}{\varepsilon_1}\Bigg), \end{aligned} \tag{22}$$

$$\delta_1 \triangleq \min\left(\frac{\beta_1}{2},\; \frac{\omega_2}{3\hat{\tau}\left(\varepsilon_1\omega_1 + \frac{1}{2}c_B\right)},\; \frac{1}{3\hat{\tau}},\; \frac{\omega_3}{c_B\overline{\tau}},\; \frac{1}{2\overline{\tau}}\right), \tag{23}$$

$$\begin{aligned} \beta_2 \triangleq \min\Bigg(&\alpha - \frac{\varepsilon_2\eta^2}{2},\; \frac{1}{2}c_e k_{2e} + c_m\eta k_s - k_s c_B - \omega_3 k_s \\ &- 2\varepsilon_1\omega_1 k_s - \omega_2 k_s,\; \frac{\omega_2}{3k_s\hat{\tau}^2} - \frac{1}{2\varepsilon_2} - \frac{\omega_1 k_s}{\varepsilon_1}\Bigg), \end{aligned} \tag{24}$$

$$\delta_2 \triangleq \min\left(\frac{\beta_2}{2},\; \frac{\omega_2}{3\hat{\tau}\left(\varepsilon_1\omega_1 + \frac{1}{2}c_B\right)},\; \frac{1}{3\hat{\tau}},\; \frac{\omega_3}{c_B\overline{\tau}},\; \frac{1}{2\overline{\tau}}\right). \tag{25}$$

IV. Stability Analysis

To facilitate the analysis, switching times are denoted by $\{t_n^i\}$, $i \in \{u, e\}$, $n \in \{0, 1, 2, ...\}$, which represent the time instances when B_M^τ becomes nonzero $(i = u)$, or the time instances when B_M^τ becomes zero $(i = e)$. Let $V_L : \mathbb{R}^7 \to \mathbb{R}$ denote a continuously differentiable, positive definite, common Lyapunov function candidate defined as

$$V_L \triangleq \frac{1}{2}e_1^2 + \frac{1}{2}Mr^2 + \frac{1}{2}\omega_1 e_u^2 + Q_1 + Q_2 + Q_3 + Q_4. \quad (26)$$

The common Lyapunov function candidate V_L satisfies the following inequalities:

$$\lambda_1 \|y\|^2 \leq V_L \leq \lambda_2 \|y\|^2, \quad (27)$$

where $y \in \mathbb{R}^7$ is defined as

$$y \triangleq \begin{bmatrix} z & \sqrt{Q_1} & \sqrt{Q_2} & \sqrt{Q_3} & \sqrt{Q_4} \end{bmatrix}^T, \quad (28)$$

and $\lambda_1, \lambda_2 \in \mathbb{R}_{>0}$ are known constants defined as

$$\lambda_1 \triangleq \frac{1}{2}\min(1, c_m, \omega_1), \quad \lambda_2 \triangleq \max\left(1, \frac{c_M}{2}, \frac{\omega_1}{2}\right).$$

To facilitate the following analysis, let $\mathcal{D}$ be an open and connected set, and let the set of initial conditions $S_\mathcal{D} \subset \mathcal{D}$ be defined as[6]

$$S_\mathcal{D} \triangleq \left\{y\left(t_n^i\right) \in \mathbb{R}^7 \mid \|y\| < \inf\left\{\rho^{-1}\left(\left(\sqrt{\kappa}, \infty\right)\right)\right\}\right\}, \quad (29)$$

where $\kappa \triangleq \min\left(\frac{1}{2}\beta_1 c_m \eta k_s, \beta_2 c_e k_{2e}\right)$.

Theorem 1. *The closed-loop error system in (17) is exponentially stable in the sense that*

$$\|y(t)\| \overset{a.e.}{\leq} \sqrt{\frac{\lambda_2}{\lambda_1}\|y(t_0)\|}\exp\left(-\frac{\lambda_3}{2}(t - t_0)\right), \quad (30)$$

where $\lambda_3 \triangleq \lambda_2^{-1}\min(\delta_1, \delta_2)$ $\forall t \in [t_0, \infty)$, *provided* $\|y(t_n^u)\| \in S_\mathcal{D}$, $\|y(t_n^e)\| \in S_\mathcal{D}$, $\forall n$, *and the following gain conditions are satisfied:*

$$\alpha > \frac{\varepsilon_2\eta^2}{2}, \quad \omega_2 > \frac{3}{2}k_s\hat{\tau}^2\left(\frac{1}{\varepsilon_2} + \frac{2\omega_1 k_s}{\varepsilon_1}\right), \quad k_{1e} > \frac{\Phi}{c_e},$$
$$\varepsilon_1\omega_1 \geq \max\left(|c_M\eta - c_b|, |c_m\eta - c_B|, c_M\eta\right),$$
$$\eta \geq \frac{4}{3c_m}\left(2c_B + 2\varepsilon_1\omega_1 + \omega_2 + \omega_3 - \frac{k_{2e}c_e}{k_s}\right),$$
$$k_{2e} \geq \frac{k_s}{c_e}\left(4\varepsilon_1\omega_1 + 2\omega_2 + 2\omega_3 + 2c_B - 2c_m\eta\right). \quad (31)$$

Proof: When $B_M^\tau > 0$, the delay effect is present in the system because the rider's muscles are stimulated (i.e., $t \in [t_n^u, t_{n+1}^e)$). Furthermore, since B_M^τ is discontinuous, the time derivative of (26) exists almost everywhere (a.e.) within $t \in [t_0, \infty)$. After using (9)-(11), (17), and applying the Leibniz Rule for (18)-(21), the time derivative of (26) is

$$\begin{aligned}\dot{V}_L \overset{\text{a.e.}}{=} \; & e_1\left(r - \alpha e_1 - \eta e_u\right) + r\left(-Vr - e_1 + \chi\right. \\ & + k_s B_M^\tau\left(r_{\hat{\tau}} - r_\tau\right) + \left(M\eta - B_M^\tau\right)k_s r_{\hat{\tau}} \\ & \left. - M\eta k_s r - B_E\left(k_{1e}\text{sgn}(r) + k_{2e}r\right)\right) \\ & + \omega_1 e_u k_s\left(r_{\hat{\tau}} - r\right) + \tfrac{1}{2}\dot{M}r^2 + \tfrac{k_s c_B}{2}\left(r^2\right. \\ & \left. - r_\tau^2\right) + \left(\varepsilon_1\omega_1 k_s + \tfrac{k_s c_B}{2}\right)\left(r^2 - r_{\hat{\tau}}^2\right) \\ & + \tfrac{\omega_2 k_s}{\hat{\tau}}\left(\hat{\tau}r^2 - \int_{t-\hat{\tau}}^t r(\theta)^2 d\theta\right) \\ & + \tfrac{\omega_3 k_s}{\tau}\left(\tau r^2 - \int_{t-\tau}^t r(\theta)^2 d\theta\right).\end{aligned} \quad (32)$$

Using Properties 1, 7-9, canceling common terms, and setting ε_1 and ω_1 such that $\max\left(|c_M\eta - c_b|, |c_m\eta - c_B|\right) \leq \varepsilon_1\omega_1$, and setting $B_E = 0$ in (32) yields

$$\begin{aligned}\dot{V}_L \overset{\text{a.e.}}{\leq} \; & -\alpha e_1^2 + \eta|e_1 e_u| + |r||\chi| + k_s c_B|rr_{\hat{\tau}}| \\ & + k_s c_B|rr_\tau| + \varepsilon_1\omega_1 k_s|rr_{\hat{\tau}}| - c_m\eta k_s r^2 \\ & + \omega_1 k_s\left(|e_u r_{\hat{\tau}}| + |e_u r|\right) + \tfrac{k_s c_B}{2}\left(r^2 - r_\tau^2\right) \\ & + \left(\varepsilon_1\omega_1 k_s + \tfrac{k_s c_B}{2}\right)\left(r^2 - r_{\hat{\tau}}^2\right) - k_{1e}c_e|r| \\ & + \tfrac{\omega_2 k_s}{\hat{\tau}}\left(\hat{\tau}r^2 - \int_{t-\hat{\tau}}^t r(\theta)^2 d\theta\right) - k_{2e}c_e r^2 \\ & + \tfrac{\omega_3 k_s}{\tau}\left(\tau r^2 - \int_{t-\tau}^t r(\theta)^2 d\theta\right).\end{aligned} \quad (33)$$

To facilitate the analysis, Young's Inequality is used to obtain the following inequalities:

$$|e_1 e_u| \leq \frac{1}{2\varepsilon_2\eta}e_u^2 + \frac{\varepsilon_2\eta}{2}e_1^2, \quad (34)$$

$$|rr_{\hat{\tau}}| \leq \frac{1}{2}r^2 + \frac{1}{2}r_{\hat{\tau}}^2, \quad (35)$$

$$|rr_\tau| \leq \frac{1}{2}r^2 + \frac{1}{2}r_\tau^2, \quad (36)$$

$$|e_u r_{\hat{\tau}}| \leq \frac{1}{2\varepsilon_1}e_u^2 + \frac{\varepsilon_1}{2}r_{\hat{\tau}}^2, \quad (37)$$

$$|e_u r| \leq \frac{1}{2\varepsilon_1}e_u^2 + \frac{\varepsilon_1}{2}r^2. \quad (38)$$

Substituting (13) and (34)-(38) into (33), choosing $k_{1e} > \frac{\Phi}{c_e}$, and completing the squares on $|r|\rho(\|z\|)\|z\|$, yields

$$\begin{aligned}\dot{V}_L \overset{\text{a.e.}}{\leq} \; & -\left(\alpha - \tfrac{\varepsilon_2\eta^2}{2}\right)e_1^2 + \left(\tfrac{1}{2\varepsilon_2} + \tfrac{\omega_1 k_s}{\varepsilon_1}\right)e_u^2 \\ & - k_s\left(\tfrac{3}{4}c_m\eta + \tfrac{k_{2e}c_e}{k_s} - 2c_B - 2\varepsilon_1\omega_1 - \omega_3\right)r^2 \\ & + \tfrac{1}{c_m\eta k_s}\rho^2(\|z\|)\|z\|^2 - \tfrac{\omega_2 k_s}{\hat{\tau}}\int_{t-\hat{\tau}}^t r(\theta)^2 d\theta \\ & + \omega_2 k_s r^2 - \tfrac{\omega_3 k_s}{\tau}\int_{t-\tau}^t r(\theta)^2 d\theta.\end{aligned} \quad (39)$$

Using (15) and the Cauchy-Schwarz inequality, upper bounds for e_u^2, Q_2, and Q_4 can be obtained as

$$e_u^2 \leq \hat{\tau}k_s\int_{t-\hat{\tau}}^t r(\theta)^2 d\theta, \quad (40)$$

$$Q_2 \leq \omega_2 k_s\int_{t-\hat{\tau}}^t r(\theta)^2 d\theta, \quad (41)$$

[6]For a set A, the inverse image is defined as $\rho^{-1}(A) \triangleq \{a \mid \rho(a) \in A\}$.

$$Q_4 \leq \omega_3 k_s \int_{t-\tau}^{t} r\left(\theta\right)^2 d\theta. \tag{42}$$

Using (18), (20), and (40)-(42) the following upper bound can be developed

$$\begin{aligned}\dot{V}_L \overset{\text{a.e.}}{\leq} & -\left(\alpha - \tfrac{\varepsilon_2 \eta^2}{2}\right) e_1^2 - \tfrac{\omega_3}{c_B \tau} Q_3 - \tfrac{1}{2\tau} Q_4 \\ & -k_s \left(\tfrac{3}{4} c_m \eta + \tfrac{k_{2e} c_e}{k_s} - 2c_B - 2\varepsilon_1 \omega_1 - \omega_2 - \omega_3\right) r^2 \\ & -\left(\tfrac{\omega_2}{3k_s \hat{\tau}^2} - \tfrac{1}{2\varepsilon_2} - \tfrac{\omega_1 k_s}{\varepsilon_1}\right) e_u^2 \\ & -\tfrac{\omega_2}{3\hat{\tau}\left(\varepsilon_1 \omega_1 + \frac{1}{2} c_B\right)} Q_1 - \tfrac{1}{3\hat{\tau}} Q_2 \\ & +\tfrac{1}{c_m \eta k_s} \rho^2 \left(\|z\|\right) \|z\|^2 .\end{aligned} \tag{43}$$

Based on the definition of β_1 in (22), the fact that $\|y\| \geq \|z\|$ and $\bar{\tau} \geq \tau$, and imposing the aforementioned gain conditions in (31), the following upper bound is obtained

$$\begin{aligned}\dot{V}_L \overset{\text{a.e.}}{\leq} & -\left(\tfrac{\beta_1}{2} - \tfrac{1}{c_m \eta k_s} \rho^2 \left(\|y\|\right)\right) \|z\|^2 \\ & -\tfrac{\beta_1}{2} \|z\|^2 - \tfrac{\omega_2}{3\bar{\tau}\left(\varepsilon_1 \omega_1 + \frac{1}{2} c_B\right)} Q_1 - \tfrac{1}{3\bar{\tau}} Q_2 \\ & -\tfrac{\omega_3}{c_B \bar{\tau}} Q_3 - \tfrac{1}{2\bar{\tau}} Q_4 .\end{aligned} \tag{44}$$

Provided $\|y\left(t_n^u\right)\| \in S_{\mathcal{D}}$ and using (23) with (44) the following upper bound is obtained

$$\dot{V}_L \overset{\text{a.e.}}{\leq} -\delta_1 \|y\|^2 . \tag{45}$$

From (27), the bound in (45) can be further bounded as

$$\dot{V}_L \leq -\frac{\delta_1}{\lambda_2} V_L, \tag{46}$$

$\forall t \in \left[t_n^u, t_{n+1}^e\right)$.

When $B_M^\tau = 0$, the delay effect is absent from the system (i.e., $t \in \left[t_n^e, t_{n+1}^u\right)$). According to the switching laws in (6), when $B_M^\tau = 0$, the system is controlled by the motor only. Using Properties 1 and 7-9, canceling common terms, choosing ε_1 and ω_1 such that $\varepsilon_1 \omega_1 \geq c_M \eta$, and setting $B_M^\tau = 0$ in (32), an upper bound for (32) can be obtained as

$$\begin{aligned}\dot{V}_L \overset{\text{a.e.}}{\leq} & -\alpha e_1^2 + \eta \left|e_1 e_u\right| + |r| \left|\chi\right| - c_e k_{1e} |r| \\ & -\left(c_e k_{2e} + c_m \eta k_s\right) r^2 + \varepsilon_1 \omega_1 k_s \left|r r_{\hat{\tau}}\right| \\ & +\omega_1 k_s \left(\left|e_u r_{\hat{\tau}}\right| + \left|e_u r\right|\right) + \tfrac{k_s c_B}{2} \left(r^2 - r_\tau^2\right) \\ & +\left(\varepsilon_1 \omega_1 k_s + \tfrac{k_s c_B}{2}\right) \left(r^2 - r_{\hat{\tau}}^2\right) \\ & +\tfrac{\omega_2 k_s}{\hat{\tau}} \left(\hat{\tau} r^2 - \int_{t-\hat{\tau}}^{t} r\left(\theta\right)^2 d\theta\right) \\ & \tfrac{\omega_3 k_s}{\tau} \left(\tau r^2 - \int_{t-\tau}^{t} r\left(\theta\right)^2 d\theta\right) .\end{aligned} \tag{47}$$

After substituting (13) and (34)-(37) into (47), selecting the gain conditions according to (31), and completing the squares on $|r|\, \rho\left(\|z\|\right) \|z\|$, the following upper bound is obtained

$$\begin{aligned}\dot{V}_L \overset{\text{a.e.}}{\leq} & -\left(\alpha - \tfrac{\varepsilon_2 \eta^2}{2}\right) e_1^2 + \left(\tfrac{1}{2\varepsilon_2} + \tfrac{\omega_1 k_s}{\varepsilon_1}\right) e_u^2 \\ & -\left(\tfrac{1}{2} c_e k_{2e} + c_m \eta k_s - 2\varepsilon_1 \omega_1 k_s - \omega_2 k_s\right. \\ & \left.-k_s c_B - \omega_3 k_s\right) r^2 - \tfrac{\omega_3 k_s}{\tau} \int_{t-\tau}^{t} r\left(\theta\right)^2 d\theta \\ & +\tfrac{1}{2 c_e k_{2e}} \rho^2 \left(\|z\|\right) \|z\|^2 - \tfrac{\omega_2 k_s}{\hat{\tau}} \int_{t-\hat{\tau}}^{t} r\left(\theta\right)^2 d\theta .\end{aligned} \tag{48}$$

After following a similar development as the case when $B_M^\tau > 0$, (48) can be upper bounded as

$$\dot{V}_L \overset{\text{a.e.}}{\leq} -\frac{\delta_2}{\lambda_2} V_L, \tag{49}$$

$\forall t \in \left[t_n^e, t_{n+1}^u\right)$ provided that $\|y\left(t_n^e\right)\| \in S_{\mathcal{D}}$. The result in (46) and (49) can be further upper bounded by substituting the decay rate $\lambda_3 \triangleq \lambda_2^{-1} \min\left(\delta_1, \delta_2\right)$ to yield

$$\dot{V}_L \overset{\text{a.e.}}{\leq} -\lambda_3 V_L . \tag{50}$$

Hence, (46) and (49) can be used with (50) to verify (26) as a common Lyapunov function across all regions of the crank cycle. Furthermore, the decay rate in (50) represents the most conservative decay rate across all regions (i.e., $\forall t \in [t_0, \infty)$). Solving the differential inequality in (50) yields the following bound

$$V_L\left(t\right) \overset{\text{a.e.}}{\leq} \left(V_L(t_0)\right) \exp\left(-\lambda_3 (t - t_0)\right) . \tag{51}$$

Provided $\|y\left(t_n^u\right)\| \in S_{\mathcal{D}}$ and $\|y\left(t_n^e\right)\| \in S_{\mathcal{D}}$ and the aforementioned gain conditions are met, (26) can be used with (51) to yield the exponential bound in (30). From (26) and (50), $e_1, r, e_u \in \mathcal{L}_\infty$. By (15) and (16), $u, u_e \in \mathcal{L}_\infty$ and the remaining signals are bounded. ■

V. Conclusion

Robust muscle delay-compensating controllers for switched FES and motor only control were designed to provide cadence tracking despite uncertain nonlinear lower limb dynamics subject to bounded unknown additive disturbances and an unknown FES input delay. An auxiliary tracking error signal was designed to inject a delay-free FES control signal in the closed-loop dynamics without measuring the FES input delay. Lyapunov-Krasovskii functionals are used in the Lyapunov stability analysis to ensure exponentially converging cadence tracking. Future efforts will focus on applying the developed control system to people with neurological disorders and will consider disorder-specific challenges to implementation. Further work can also allow for the FES input delay to be time-varying and implement an adaptive time-varying estimate of the delay in the control design instead of a constant estimate to achieve more precise cadence tracking.

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