

Neutrino emission upper limits with maximum likelihood estimators for joint astrophysical neutrino searches with large sky localizations

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Since the start of the gravitational wave observation era, no joint high energy neutrino and gravitational wave event has been found. These non-detections could be used for setting an upper bound on the neutrino emission properties for gravitational wave events individually or for a set of them. Although in the previous joint high energy neutrino and gravitational wave event searches upper limits have been found, there is a lack of consistent method for the calculation. The problem addressed in this paper is finding those limits for astrophysical events which are localized poorly in the sky where the sensitivities of the neutrino detectors change significantly and can also emit neutrinos, for example the gravitational wave detections. Here we describe methods for assigning limits for expected neutrino count, emission fluence and isotropically equivalent emission based on maximum likelihood estimators. Then we apply described methods on the three GW detections from aLIGO's first observing run (O1) and find upper limits for them.

I. INTRODUCTION

With the observational discovery of gravitational waves (GW), humanity acquired another way of observing the universe [1]. It could allow us to observe phenomena which we are not able to see with other methods such as observations based on electromagnetic or neutrino emission, as well as observing cosmic events which could be observed via a multitude of messengers [2, 3]. There has been three such fundamental observations so far; the binary neutron star (BNS) merger which was observed with GWs and electromagnetic radiation in various bands [4], the blazar which was observed with high energy neutrinos (HEN) and electromagnetic radiation [5] and the SN1987a supernova which was observed with lower energy neutrinos (in MeV energy) and in various electromagnetic bands [6]. Having one or more additional messengers beyond GWs might shed a brighter light on the physical processes happened before, during, and after the astrophysical event [7, 8] which can enable having new observations that wouldn't be possible [9, 10]. Moreover absence of an additional messenger is also informative as in the case of the first multimessenger observational result with GWs that addressed the origin of a short-hard gamma-ray burst from the direction of the Andromeda galaxy [11]. Despite well over a

decade long effort [12–22], one remaining two messenger combination is a joint observation with HENs and GWs.

During aLIGO's first observing run (O1) and second observing run with aVirgo (O2) searches for joint GW and HEN events couldn't find any significant event [19–22]. Moreover during the first half of the third observing run of aLIGO and aVirgo (O3a), search on every public alert of LIGO-Virgo Collaboration is performed [23]; no significant event was found and the results for each search was reported via The Gamma-ray Coordinates Network (GCN) Notices by IceCube Collaboration [24–26]. Such searches are based on assigning a test statistic (TS) to each observed HEN, which are detected in a fixed time window before and after the detected GW [15, 27]. Based on the value of the TS for each detected event, a significance is assigned to each event by comparing it to a known background TS distribution. Unless the observed event's TS exceeds a threshold, which is determined by a fixed significance level and the background TS distribution, the event is not counted as a multi-messenger detection. One physical information that can be extracted from GW detections is an upper limit on the neutrino emission fluence, on the expected neutrino count or an isotropically equivalent neutrino emission energy from the GW event's source. In case of a non-detection these upper limits can still be used to make inferences. Although in the previous searches upper limits were calculated for these quan-

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tities in various ways, there hasn't been a consistent method used for those calculations [19, 21, 22]. For individual events, upper limit range for the isotropically equivalent emitted energy in neutrinos for every point in the sky was found; but a single limit for the each event wasn't found. Due to the need of having such a formalism, with this paper we propose a method for calculating frequentist upper limits based on maximum likelihood estimators.

As the sensitivities of GW observatories get better and number of GW observatories in the GW detection network increases, the rate of GW observations will increase. For example, in the first 6 months of the third observing run of aLIGO and aVirgo 33 GW candidate detections have been announced publicly compared to total 11 detections announced from O1 and O2 after offline analysis [28]. Thus, it is expected to have a population for the GW detections in future. Therefore a proper quantification of the upper limits for the counterparts of these GW events will let us infer about the physics involved in them.

In this paper we describe a method of finding frequentist upper limits based on maximum likelihood estimators for expected neutrino count and neutrino emission fluence for single GW events which we explain in Sections II and III respectively; and in Section IV we describe the method for finding the upper limit on the isotropically equivalent emitted energy for an ensemble of events of same kind, assuming the energy is same for all. Considering the correspondence of frequentist limit from maximum likelihood estimation and Bayesian limit for uniform prior on an ideally counted Poisson variable, we also compare frequentist and Bayesian limits for uniform prior for the neutrino count which is inferred by a TS in Sec. II C. Finally in Section V we demonstrate our method by finding the upper limits for the 3 GW events detected during the first observing run of aLIGO (O1).

Generally a 90% upper limit is desired to be found in the joint GW-HEN event searches [19, 21, 22] although it is an arbitrary confidence interval. Without loss of generality, throughout this paper we aim to find 90% upper limits for clarity. However any kind of confidence interval at any confidence level can be found with this method, by requiring a different relationship between the estimators of the true quantity and the measurement at the very end of the calculation. Furthermore, due to inferred low chance of detecting a joint HEN with GW from absence of such an example, again without loss of generality we also assume that we are dealing with 2 or less signal neutrinos for simplicity.

II. UPPER LIMIT FOR NEUTRINO COUNT

A. Frequentist upper limit

In this section we find the 90% upper limit for the expected number of neutrinos from a single measurement via maximum likelihood estimators, where the measurement's significance is quantified by a test statistic (TS). In order to write the likelihood we need to have the TS distributions for 0, 1 and 2 detected neutrinos. First write the likelihood and denote the expected number of neutrinos with θ ;

$$\begin{aligned}\mathcal{L}(\theta; TS_m) &= P(TS_m|\theta) \\ &= \sum_{n=0}^{\infty} P(TS_m|\theta, n)P(n|\theta), \quad \theta \geq 0\end{aligned}\quad (1)$$

where n is the number of detected neutrinos from the joint source and TS_m is the measured TS. As said before we assume to have at most 2 signal neutrinos.

$$\begin{aligned}\mathcal{L}(\theta; TS_m) &= \\ &e^{-\theta} P_0(TS_m) + \theta e^{-\theta} P_1(TS_m) + \frac{\theta^2}{2} e^{-\theta} P_2(TS_m)\end{aligned}\quad (2)$$

where P_0 , P_1 , P_2 are the TS distributions corresponding to 0, 1, and 2 detected neutrinos. Now find the local maximum of the likelihood. After taking derivative with respect to θ and equating to 0, we have

$$\begin{aligned}P_1(TS_m) - P_0(TS_m) \\ + (P_2(TS_m) - P_1(TS_m))\theta - P_2(TS_m)\frac{\theta^2}{2} = 0\end{aligned}\quad (3)$$

we find the local maximum at

$$\hat{\theta} = 1 - \frac{P_1(TS_m)}{P_2(TS_m)} + \sqrt{1 + \frac{P_1(TS_m)^2}{P_2(TS_m)^2} - \frac{2P_0(TS_m)}{P_2(TS_m)}}\quad (4)$$

(other root corresponds to local minimum). For $P_2(TS_m) = 0$ it is

$$\hat{\theta} = 1 - \frac{P_0(TS_m)}{P_1(TS_m)}\quad (5)$$

For $P_2(TS_m) = P_1(TS_m) = 0$ there is no local maximum. The absolute maximum is at,

$$\hat{\theta} = 0\quad (6)$$

The hat on θ is for denoting the maximum likelihood estimator. Local maximum may not be

the absolute maximum in $\theta = [0, \infty)$. The only other candidate for absolute maximum is the border value $\hat{\theta} = 0$. So maximum likelihood estimator for expected neutrino number is either 0 or $1 - \frac{P_1(TS_m)}{P_2(TS_m)} + \sqrt{1 + \frac{P_1(TS_m)^2}{P_2(TS_m)^2} - \frac{2P_0(TS_m)}{P_2(TS_m)}}$, it can be determined by comparing the value of likelihood at these two points. Let's keep denoting it as $\hat{\theta}$.

Now the upper limit for expected neutrino number is defined as the neutrino number above which we have 90% probability to have a higher maximum likelihood estimator than the one for our measurement, in mathematical notation we find the θ which satisfies the Equation (7)

$$P(\hat{\theta} < \hat{\theta}'|\theta) = 0.9 \quad (7)$$

where $\hat{\theta}'$ is the maximum likelihood estimator for θ , and $\hat{\theta}$ is the maximum likelihood estimator for our current measurement. It is equivalent to

$$\int_{0-}^{\hat{\theta}+0+} f(\hat{\theta}'|\theta) d\hat{\theta}' = 0.1 \quad (8)$$

where $f(\hat{\theta}'|\theta)$ is the probability distribution function for the maximum likelihood estimator for true expected neutrino number θ . It consists of delta distributions at integers and a continuous distribution between $\hat{\theta}' = [0, 2]$. Delta distribution at integers ≥ 3 are identical to Poisson distribution and strengths' of delta distributions at 0, 1 and 2 are less than the corresponding Poisson strengths. The missing probability is contained in the continuous distribution.

B. Bayesian limit with uniform prior

Now we discuss the Bayesian credible limits with for a uniform prior θ . Frequentist limits with maximum likelihood estimators and Bayesian limits for uniform priors give the same limits if the quantity of interest is a location parameter [29], such as the mean of a Gaussian distribution. Moreover as shown in section IIC1, they also correspond to each other for the mean of a Poisson distribution although the mean of Poisson distribution is not a location parameter.

Bayesian upper limit requires having a posterior distribution for θ as

$$P(\theta|TS_m) = \frac{P(TS_m|\theta)P(\theta)}{P(TS_m)} \quad (9)$$

where $P(TS_m)$ acts like a normalization constant. For uniform prior $P(\theta) = k$

$$P(\theta|TS_m) = \frac{kP(TS_m|\theta)}{P(TS_m)} \quad (10)$$

Again if we are sure that detected neutrino number ≤ 2

$$P(\theta|TS_m) = \frac{e^{-\theta}P_0(TS_m) + \theta e^{-\theta}P_1(TS_m) + \frac{\theta^2}{2}e^{-\theta}P_2(TS_m)}{P_0(TS_m) + P_1(TS_m) + P_2(TS_m)} \quad (11)$$

The Bayesian 90% upper limit is defined as the point where cumulative posterior probability is 0.9, or the θ' which satisfies the Equation (12)

$$P(\theta < \theta'|TS_m) = \int_0^{\theta'} P(\theta|TS_m) d\theta = 0.9 \quad (12)$$

Hence

$$e^{-\theta'} \left(1 + \frac{\theta'(P_1(TS_m) + P_2(TS_m)) + \frac{\theta'^2}{2}P_2(TS_m)}{P_0(TS_m) + P_1(TS_m) + P_2(TS_m)} \right) = 0.1 \quad (13)$$

C. Comparison of frequentist and Bayesian limits for uniform prior

Now we compare the frequentist and Bayesian limits for a uniform prior for the quantity we estimate. First we show that these two limits have correspondence for the ideal Poisson counting case and then we show that when we have a measurement with a TS they do not necessarily correspond to each other.

1. Ideal Poisson counting experiment

Before comparing frequentist and Bayesian limits for uniform prior for the case of interest, compare them for a Poisson counting experiment where we count the events and the maximum likelihood estimator and the TS are equivalent to the observed event count, in other words there is no ambiguity in the detected neutrino count. Denote it with θ_{obs} . The frequentist 90% upper limit θ_L satisfies

$$\sum_{n=\theta_{obs}+1}^{\infty} Poisson(n, \theta_L) = \sum_{n=\theta_{obs}+1}^{\infty} \frac{\theta_L^n e^{-\theta_L}}{n!} = 0.9 \quad (14)$$

The Bayesian posterior distribution with uniform prior becomes

$$P(\theta|\theta_{obs}) = \text{Poisson}(\theta_{obs}, \theta) \quad (15)$$

and the Bayesian limit θ_L satisfies

$$\int_0^{\theta_L} \frac{\theta^{\theta_{obs}} e^{-\theta}}{\theta_{obs}!} d\theta = 0.9 \quad (16)$$

Now compare $\int_0^{\theta_L} \frac{\theta^{\theta_{obs}} e^{-\theta}}{\theta_{obs}!} d\theta$ and $\sum_{n=\theta_{obs}+1}^{\infty} \frac{\theta_L^n e^{-\theta_L}}{n!}$. First differentiate them with respect to θ_L . From the sum's derivative we have

$$\sum_{n=\theta_{obs}+1}^{\infty} \frac{\theta_L^{n-1} e^{-\theta_L}}{(n-1)!} - \frac{\theta_L^n e^{-\theta_L}}{n!} = \frac{\theta_L^{\theta_{obs}} e^{-\theta_L}}{\theta_{obs}!} \quad (17)$$

From the integral's derivative we have

$$\frac{\theta_L^{\theta_{obs}} e^{-\theta_L}}{\theta_{obs}!} \quad (18)$$

the same expression. Hence $\int_0^{\theta_L} \frac{\theta^{\theta_{obs}} e^{-\theta}}{\theta_{obs}!} d\theta$ and $\sum_{n=\theta_{obs}+1}^{\infty} \frac{\theta_L^n e^{-\theta_L}}{n!}$ can only differ by a constant. However when $\theta_L = 0$ they are both 0. Therefore equations

$$\int_0^{\theta_L} \frac{\theta^{\theta_{obs}} e^{-\theta}}{\theta_{obs}!} d\theta = 0.9 \quad (19)$$

and

$$\sum_{n=\theta_{obs}+1}^{\infty} \frac{\theta_L^n e^{-\theta_L}}{n!} = 0.9 \quad (20)$$

give the same limit values. Frequentist and Bayesian upper limits with uniform prior are same. This is not specific to Poisson distribution and valid for all distributions where the estimated parameter is a location parameter [29] although for Poisson distribution mean is not a location parameter. Hence this is not a trivial result for Poisson distribution.

2. Measurement with a TS

The ideal Poisson counting experiment is a special case of measurement with a TS where the TS distributions of each detection count are separate. When we check the relationship between frequentist and Bayesian with uniform prior limits we see that there is not such a coincidence like the ideal counting experiment. This can be demonstrated with a simple counter example. Consider having uniform TS distribution for 0 detections in between $[a, b]$ and for 1 detection in between $[c, d]$ such that $c < b$ and $b - a > d - c$. TS distributions for other detection counts are separate such that there is no ambiguity there. For a TS measurement which may correspond to 0 or 1 count ($c < TS_m < b$) one needs to solve the Equation (21) in order to find the Bayesian limit for uniform prior

$$\begin{aligned} e^{-\theta} \left(1 + \frac{\theta(d-c)^{-1}}{(b-a)^{-1} + (d-c)^{-1}} \right) \\ = e^{-\theta} \left(1 + \frac{\theta(b-a)}{b-a+d-c} \right) = 0.1 \end{aligned} \quad (21)$$

where we plugged in $P_1(TS_m) = (d-c)^{-1}$ and $P_0(TS_m) = (b-a)^{-1}$ to Equation (13).

However, in the frequentist approach we solve Equation (22)

$$e^{-\theta} \left(1 + \frac{\theta(b-c)}{d-c} \right) = 0.1 \quad (22)$$

which follows from Equation (8). In frequentist approach, the length of the intersection of $P_0(TS)$ and $P_1(TS)$ affects the limit whereas in the Bayesian interpretation it has no such direct role. When the intersections between the distributions vanish, we get the ideal counting example.

III. UPPER LIMIT FOR FLUENCE WITH A GRAVITATIONAL WAVE SKYMAP

Now instead of the neutrino count we want to estimate the fluence. Neutrino detectors, have a sky position dependent sensitivity due to the interaction of neutrinos and cosmic rays with the Earth and atmosphere; hence detected number of neutrinos depend on their position on the sky for a constant fluence. Again start by writing the likelihood, denote the fluence by ϕ ,

$$\begin{aligned}
\mathcal{L}(\phi; TS_m, \mathcal{P}_{GW}) &= P(TS_m, \mathcal{P}_{GW}|\phi) = P(\mathcal{P}_{GW}|\phi)P(TS_m|\mathcal{P}_{GW}, \phi) \\
&= \alpha P(TS_m|\mathcal{P}_{GW}, \phi) = \alpha \sum_{n=0}^{\infty} P(TS_m|\phi, n, \mathcal{P}_{GW})P(n|\phi, \mathcal{P}_{GW}) \\
&= \alpha \sum_{n=0}^{\infty} P(TS_m|\phi, n, \mathcal{P}_{GW}) \int P(n|\phi, \Omega, \mathcal{P}_{GW})P(\Omega|\phi, \mathcal{P}_{GW})d\Omega, \quad \phi \geq 0 \quad (23)
\end{aligned}$$

where n is the detected neutrino number from the joint source, Ω is the sky position of source and $\mathcal{P}_{GW}$ is the probability distribution of sky position acquired from the gravitational wave detection, namely the skymap. $P(\Omega|\phi, \mathcal{P}_{GW})$ has no fluence dependency and is $\mathcal{P}_{GW}(\Omega)$. $P(\mathcal{P}_{GW}|\phi)$ doesn't have a fluence dependency and is denoted with α . Since it

doesn't affect the maximum likelihood estimator it will be dropped for the rest of analysis. Also denote the position dependent coefficient which relates fluence to the expected neutrino number as $c(\Omega)$ which is proportional to the effective area of the neutrino detector [30]. Again assume we are sure that we only have at most 2 neutrinos, then we have

$$\begin{aligned}
\mathcal{L}(\phi; TS_m, \mathcal{P}_{GW}) &= P_0(TS_m) \int e^{-c(\Omega)\phi} \mathcal{P}_{GW}(\Omega) d\Omega + P_1(TS_m) \int c(\Omega)\phi e^{-c(\Omega)\phi} \mathcal{P}_{GW}(\Omega) d\Omega \\
&\quad + P_2(TS_m) \int \frac{(c(\Omega)\phi)^2}{2} e^{-c(\Omega)\phi} \mathcal{P}_{GW}(\Omega) d\Omega \quad (24)
\end{aligned}$$

After taking the derivative with respect to ϕ and

equating to 0 for finding the local maximum, we have the condition for ϕ

$$\int \mathcal{P}_{GW}(\Omega) e^{-c(\Omega)\phi} c(\Omega) [P_1(TS_m) - P_0(TS_m) + (P_2(TS_m) - P_1(TS_m))c(\Omega)\phi - P_2(TS_m)\frac{c(\Omega)^2\phi^2}{2}] d\Omega = 0 \quad (25)$$

Although for neutrino detectors, $c(\Omega)$ can be well approximated; since $\mathcal{P}_{GW}(\Omega)$ doesn't have an a-priori estimated form we cannot go further in solving the equation analytically. Therefore in order to find the maximum likelihood estimator $\hat{\phi}$ for fluence, one needs to find it numerically with known $\mathcal{P}_{GW}(\Omega)$. Similar to the neutrino count case, the upper limit for fluence is found by finding the ϕ which satisfies Equation (26)

$$P(\hat{\phi} < \hat{\phi}'|\phi) = 0.9 \quad (26)$$

where $\hat{\phi}'$ is the maximum likelihood estimator for ϕ , and $\hat{\phi}$ is the maximum likelihood estimator for our current measurement.

IV. UPPER LIMIT ON ISOTROPICALLY EQUIVALENT EMISSION ENERGIES OF AN ENSEMBLE OF EVENTS WITH VOLUME LOCALIZATION

In this section we consider finding an upper limit for an ensemble of similar GW events, for example same kind of events like binary black hole mergers (BBH) or binary neutron star (BNS) mergers. Due to expected different distances of these events, it can't be expected to have a similar neutrino count or fluence from each event due to the suppression with distance squared. Instead one quantity which can be similar for them is the isotropically equivalent

lent emission energy (E_{iso}) for neutrinos. Here by assuming all the events having the same E_{iso} , we describe the procedure of finding an upper limit on E_{iso} with maximum likelihood estimation. Although it is clear that true E_{iso} will be different for each event, this assumption enables us to infer more stringent information about the physics involved in same kind of processes. In order this assumption to be mean-

ingful, the set of events should be downselected for having same kind of events. For example using BBH and BNS events together doesn't make sense as the physics involved in those are different.

We consider having N events with volume localizations $\mathcal{V}_{GW,i}$ and measured TS values $TS_{m,i}$ for i^{th} event. We write the likelihood for E_{iso} whose value is denoted as E_{iso} .

$$\begin{aligned}\mathcal{L}(E_{\text{iso}}; TS_{m,1\dots N}, \mathcal{V}_{GW,1\dots N}) &= \prod_{i=1}^N P(TS_{m,i}, \mathcal{V}_{GW,i} | E_{\text{iso}}) = \prod_{i=1}^N P(TS_{m,i} | E_{\text{iso}}, \mathcal{V}_{GW,i}) P(\mathcal{V}_{GW,i} | E_{\text{iso}}) \\ &= \beta \prod_{i=1}^N P(TS_{m,i} | E_{\text{iso}}, \mathcal{V}_{GW,i}) = \beta \prod_{i=1}^N \sum_{n=0}^{\infty} P(TS_{m,i} | E_{\text{iso}}, \mathcal{V}_{GW,i}, n) P(n, | E_{\text{iso}}, \mathcal{V}_{GW,i}) \\ &= \beta \prod_{i=1}^N \sum_{n=0}^{\infty} P(TS_{m,i} | E_{\text{iso}}, \mathcal{V}_{GW,i}, n) \int P(n | E_{\text{iso}}, \mathcal{V}_{GW,i}, r, \Omega) P(r, \Omega | E_{\text{iso}}, \mathcal{V}_{GW,i}) r^2 dr d\Omega, E_{\text{iso}} \geq 0\end{aligned}\quad (27)$$

where n is the number of detected neutrinos from the joint source, r is the distance of the source, Ω is the sky position of the source. We assume the GW volume localizations are not affected by E_{iso}

and hence are effectively a constant for the likelihood which is denoted by β which will be dropped. For having at most 2 neutrinos from the joint source the likelihood becomes

$$\begin{aligned}\mathcal{L}(E_{\text{iso}}; TS_{m,1\dots N}, \mathcal{V}_{GW,1\dots N}) &= \prod_{i=1}^N \int (P_{0,i}(TS_{m,i}) + c'(\Omega) \frac{E_{\text{iso}}}{4\pi r^2} P_{1,i}(TS_{m,i}) + \frac{1}{2} (c'(\Omega) \frac{E_{\text{iso}}}{4\pi r^2})^2 P_{2,i}(TS_{m,i})) e^{-c'(\Omega) \frac{E_{\text{iso}}}{4\pi r^2}} \mathcal{V}_{GW,i}(r, \Omega) r^2 dr d\Omega\end{aligned}\quad (28)$$

where $P_{0,i}$, $P_{1,i}$, $P_{2,i}$ are the TS distributions corresponding to 0, 1, and 2 detected neutrinos for the i^{th} event and

$$c'(\Omega) = \frac{c(\Omega)}{\int E_{\nu} \frac{dN}{dE_{\nu}} dE_{\nu}} \quad (29)$$

since ϕ and E_{iso} are connected as

$$\int E_{\nu} \phi \frac{dN}{dE_{\nu}} dE_{\nu} = \frac{E_{\text{iso}}}{4\pi r^2} \quad (30)$$

where E_{ν} is the neutrino energy and $\frac{dN}{dE_{\nu}}$ is the energy dependency of the differential neutrino fluence; i.e. $= E_{\nu}^{-2}$ spectrum over a range of energies which is

expected from Fermi acceleration [31]. Similarly to the fluence case, we can't go further in analytically maximizing the likelihood. The maximum likelihood estimators should be found by numerically evaluating the likelihood with known $\mathcal{V}_{GW,i}(r, \Omega)$. Then the 90% upper limit is found for E_{iso} by finding E_{iso} which satisfies

$$P(\hat{E}_{\text{iso}} < \hat{E}_{\text{iso}}' | E_{\text{iso}}) = 0.9 \quad (31)$$

where $\hat{E}_{\text{iso}}'$ is the maximum likelihood estimator for E_{iso} and $\hat{E}_{\text{iso}}$ is the maximum likelihood estimator for the measurements of the ensemble of events.

V. LIMITS FOR THE GW EVENTS IN ALIGO'S FIRST OBSERVING RUN

Now, by using publicly available data, we illustrate our method and find the neutrino emission limits on the 3 GW events from aLIGO's first observing run O1; GW150914, GW151012 and GW151226 [1, 28, 32, 33]. These events were analyzed before and the temporally coincident neutrinos in $\pm 500s$ window are reported [19, 21]. The list of neutrinos can be found in Table I and the GW localizations overlaid with the neutrinos can be found in Fig. 1. Volume localization of the GW events are also available. We use the significance calculation method of the Low-Latency Algorithm for Multi-messenger Astrophysics (LLAMA) search which has been used in the third observing run of aLIGO and aVirgo (O3) for joint GW-HEN event search [27, 34]. 9 of the 10 reported neutrinos were detected by the IceCube Observatory and the other one was detected by ANTARES Observatory which was coincident with GW151226 [21]. Here we only use the neutrinos detected by IceCube for simplicity as the method for calculating the significance assumes a single neutrino detector at the moment. We assume these 3 GW events are certain detections without any significance ambiguity. The significance calculation uses detector specific background distributions. In the case of certain GW events, only a background neutrino sample is needed. We use the most recent publicly available all-sky point source sample of IceCube from year 2012 in its final configuration with 86 strings [35]. We assume the sensitivity of IceCube hasn't changed significantly from 2012 to 2015 as it has reached to its final configuration with 86 strings in 2011 [36].

A. Event generation

The neutrino detection rate in the sample is about 3.5 in 1000s. Consequently we have the average background neutrino rate of 3.5 per GW, as we consider neutrinos in $\pm 500s$ window around the GW event. In order to construct the significance distribution for zero signal neutrinos, we drew average of 3.5 neutrinos from the public events list according to a Poisson distribution and distributed these events uniformly on a $\pm 500s$ window around the GW event. We calculated significance for 1000 such events for obtaining P_0 for each event. For obtaining P_1 , we generated signal neutrinos by using the effective area distribution of IceCube for the same configuration

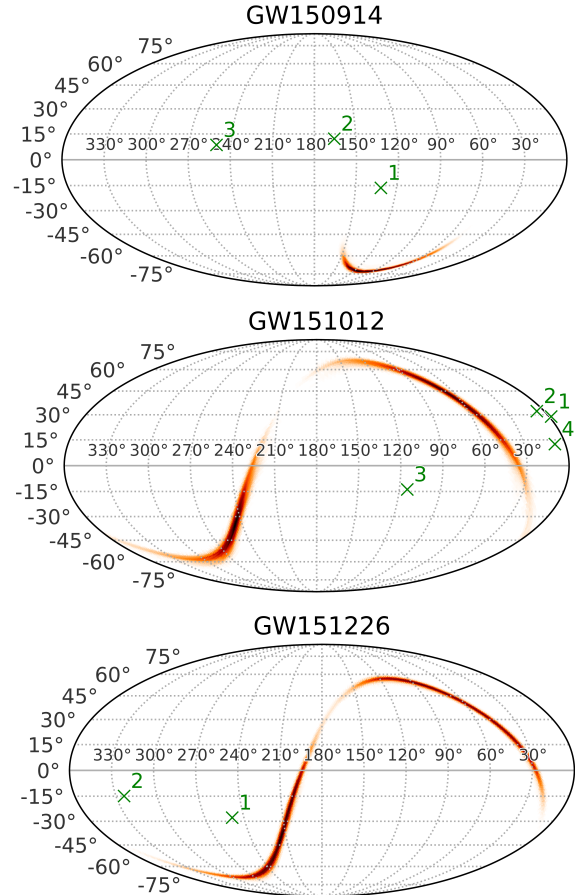


FIG. 1. Sky localization of the three GW events overlaid with neutrinos according to the labelling in Table I. Darker color represents higher probability density for the GW source location. Green crosses shows the location of the neutrinos.

which is also publicly available. First we generate the sky coordinates of these neutrinos by sampling coordinates from the sky according to the probability distribution of GW localization from the parameter estimation sample. We also keep the distance information of the chosen points from the as distance distribution is needed for E_{iso} calculation and distance and sky position are not independent. We assume the time of the GW event is not a random variable and therefore all the events with the same GW data create the same localization. The effective area depends on the neutrino energy and the declination of neutrinos [30]. By assuming an E_ν^{-2} spectrum [31] and using the effective area, we assign these neutrinos energies. One remaining property of

Event	Neutrino number	Time difference [s]	Right ascension [°]	Declination [°]	Angular uncertainty [°]	Energy [TeV]
GW150914	1	37.2	132.6	-16.6	0.35	175
GW150914	2	163.2	167.0	12.0	1.95	1.22
GW150914	3	311.4	-108.5	8.4	0.47	0.33
GW151012	1	-423.3	360.0	28.7	3.5	0.38
GW151012	2	-410.0	7.5	32.0	1.1	0.45
GW151012	3	-89.8	115.5	-14.0	0.6	13.7
GW151012	4	147.0	9.0	12.3	0.3	0.35
GW151226	1	-290.9	325.5	-15.1	0.1	158
GW151226	2	-22.5	88.5	14.9	0.7	6.3

TABLE I. List of neutrinos which were in the $\pm 500s$ window around the three GW events. Data taken from [19, 21]

the generated neutrinos is their angular uncertainty. In order not to lose the dependency of angular uncertainty to energy and declination, for each generated neutrino we narrowed the list of real events via their energies and declinations. We required declination difference with the generated neutrino to be less than 10° and differences of their base 10 logarithm of energies to be less than 1.5. From the remaining real neutrinos we picked one of their angular uncertainty uniformly and assigned it to the generated neutrino. We set the energy and declination difference constraints by considering the number of neutrinos remained after cuts. Having more stringent cuts causes some of the generated neutrinos to have no similar real neutrino for taking its angular uncertainty. With a larger sample, more stringent cuts could be imposed. Due to the axial symmetry of IceCube we did not put a constraint on the right ascension of neutrinos. One caveat worth noticing here is that although the energy proxy given in the neutrino list is an estimate of the actual neutrino energy, it has expected differences with the actual neutrino energy [35]. However, we consider it as the neutrino energy in our generation. Next, we shift the position of each generated neutrino, by randomly choosing a offset distance according a normal distribution with zero mean and with variance angular error squared. Then a uniformly random angle is chosen in $0-2\pi$ and neutrinos' positions on the sky is shifted along that direction by the chosen distances. After the neutrinos are generated we sampled the time of each signal neutrino from a symmetric triangular distribution whose mode is the GW event time and extend is $\pm 500s$. This distribution is obtained if one assumes the GW and neutrino emission to be uniformly in $\pm 250s$ window around the same astrophysical event [37]. Convolution of two uniform distributions give a triangular distribution which implies that temporally closer neutrinos to the GW are more likely to be associated than background neutrinos

which are distributed uniformly around the GW event. Finally we choose background neutrinos to accompany each signal neutrino. These background neutrinos are chosen identically as the ones chosen for obtaining P_0 . Similarly we generate 1000 of such events. We do not consider any of these neutrinos to be coming from the same source as for each combination of two, the positional difference is larger than the sum of angular errors. Hence we take P_2 to be zero for all of the events.

B. Calculation and results

We find the maximum likelihood estimators for the events with actual coincident neutrinos, as well as the generated background and signal neutrinos. Then we find the value of the true quantity which satisfies Eqs. (7), (26) or (31). In order to do it, and also for finding the estimators for fluence and E_{iso} , we need to find the expected number of neutrinos for unit fluence at a given declination which has been denoted as $c(\Omega)$ in this paper. The requirement when calculating the value of the true quantity arises due to the fact that we directly sampled the signal neutrino positions from the GW skymap; but didn't account for the declination dependency of the IceCube's sensitivity or the effective area. We only used the effective area for the energies of the neutrinos after we had chosen the positions. By using $c(\Omega)$ we get the expected detected signal neutrino count distribution for a given fluence. With that distribution we weight the signal neutrino events with the Poisson probabilities whose mean is determined by the true value of the estimated quantity, declination and the distance of the simulated emission. For events with 2 or more signal neutrinos we take the corresponding estimator to be higher than the estimator of the actual event. $c(\Omega)$ is obtained by

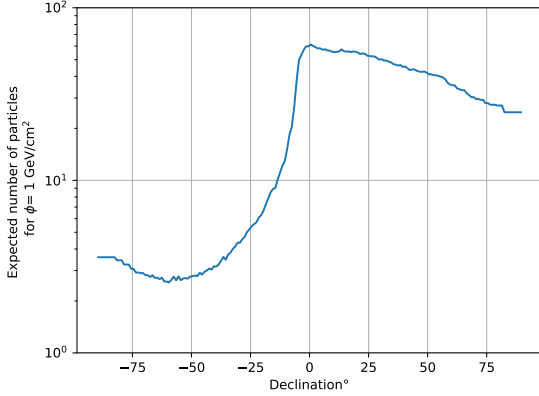


FIG. 2. Expected number of particles for the differential fluence of $1 \text{ GeV}/\text{cm}^2 E_\nu^{-2}$ vs. declination, obtained from the effective area distribution

Event	Detected signal neutrino count	Fluence [GeV/cm^2]	E_{iso} [ergs]
GW150914	2.3	0.82	5.0×10^{53}
GW151012	2.1	0.36	1.1×10^{54}
GW151226	2.3	0.45	2.5×10^{53}
Combined	-	-	5.7×10^{52}

TABLE II. 90% upper limits for the detected signal neutrino count, fluence and E_{iso} for three events and E_{iso} for the population of three events.

integrating the effective area of each declination in energy after scaling with E_ν^{-2} . For E_ν^{-2} spectrum the obtained $c(\Omega)$ is shown in Fig 2.

The obtained 90% frequentist upper limits with maximum likelihood estimators are shown in Table II. The Bayesian upper limits for the neutrino count with uniform prior are found to be 2.3 for all events.

C. Discussion

The 90% frequentist upper limit for the mean count of a background free ideal Poisson process is 2.3 and as shown in Sec. II C it also corresponds to a 90% Bayesian upper limit with uniform prior for the count. When we look at the neutrino count limits we have we see that except the frequentist limit for the event GW151012 we have the upper limit as 2.3 neutrinos. It shows that for GW150914 and GW151226 the background and signal distributions are separated sufficiently for behaving as ideal counting processes. The fact that for GW151012, the frequentist limit is less than 2.3 points a substan-

tial intersection of background (P_0) and signal (P_1) distributions in a region of higher significance than the event's significance. The fact that its Bayesian limit is still 2.3 implies that for the values around event's significance the intersection of background and signal distributions is negligible. When we look at the fluence upper limits, we see that the upper limit for GW150914 is about twice of other events' limits. This can be explained by the localization of the GWs and IceCube's declination dependent sensitivity, i.e. Fig. 2. Required fluence for the same number of neutrinos is about an order larger in the south hemisphere compared to the northern hemisphere, therefore fluence requirement from southern regions dominates the northern regions for comparable probabilities. As GW150914 is completely localized in the southern hemisphere and the other two events are more or less equally localized in both hemispheres, it is expected to have twice the limits of GW151012 and GW151226 for GW150914. When we look at the upper limits for E_{iso} , we see that the limit of GW150914 is twice of GW151226. Both events have similar expected distances, 440 Mpc and 450 Mpc for GW150914 and GW151226 respectively [28]. Therefore we expect their E_{iso} upper limit ratio to be similar to their fluence upper limits. The expected distance of GW151012 is 1080 Mpc [28], about twice of GW150914 and GW151226. Therefore we expect a factor of 4 difference between the fluence upper limits and E_{iso} upper limits, which is present. In [19, 21] E_{iso} upper limits were found for every point in the sky. Our method allows one to have a single upper limit value for the whole event. Our upper limits fall in the previously reported upper limit range in the whole sky. Finally we comment on the E_{iso} upper limits for combination of the three events. We see that when three GW events which don't have sufficient significance for having a counterpart in neutrino emission are combined for an upper limit, we get an order of magnitude more stringent upper limit. This illustrates the importance of having collection of events when constraining astrophysical parameters.

VI. CONCLUSION

We described the methods of finding frequentist upper limits with maximum likelihood estimators for expected neutrino count, neutrino emission fluence and isotropically equivalent neutrino emission energy for an ensemble of events which are specifically aimed for joint GW and HEN events; but could be used for any similar search which uses TS for count-

ing discrete events. Then we applied this method on the GW events in aLIGO’s first observing run (O1) and found upper limits for them. Through the paper we considered 90% upper limits although different confidence intervals at arbitrary confidence levels can also be found with this method instead of 90% upper limits; by requiring different relationships between the estimators of the true quantity and the measurement instead of Equations (7), (26) or (31).

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