# Design of Substrate Integrated Waveguides Supporting Degenerate Band-Edge Resonances

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***Abstract*—A degenerate band edge is a resonant dispersive behavior of coupled transmission lines arising from a fourth-order degeneracy due to the coalescence of two propagating and two evanescent modes. It leads to a so-called giant resonance resulting in field enhancement inside the transmission line. In this paper, we propose a SIW periodic line supporting a degenerate band edge and we study the impact of losses. Conductor and dielectric losses are analyzed in the full-wave simulations of the unit cell and of truncated structures.**

***Index Terms*—Loss analysis, degenerate band edge, substrate integrated waveguide, periodic structures.**

## I. Introduction

The degenerate band edge (DBE) is a special degeneracy condition which occurs in coupled periodic lines. It was firstly studied in the context of the slow wave propagation across anisotropic periodic layers [1]. The DBE condition provides a very flat dispersion relation near the edge of the Brillouin zone ($k_d = \pi/p$), and therefore a very low group velocity and a high quality factor near the band edge [2]. This feature can be used to enhance the field transmitted in a resonating device [1] and to design slow-wave structures [3]. The DBE condition has many applications, such as pulse compression [4], oscillators [5], filters [6] and slow-wave structures [3].

The DBE in a periodic structure occurs as a fourth-order degeneracy of the eigenmodes and its dispersion relation is described by a quartic shape [1].

$$f - f_d = -\alpha(k - k_d)^4 \tag{1}$$

where $f_d$ is the DBE frequency, $k_d = \pi/p$ and $p$ is the spatial period and $\alpha$ is a positive constant that depends on the geometry of the structure. Since the condition (1) requires four degenerate Bloch modes to meet at the same frequency $f_d$ (two propagating and two evanescent modes), the waveguide should support two different modes along each direction, and should be able to couple them in a suitable way. The DBE condition has already been implemented into some structures, such as circular waveguides [7], silicon optical waveguides [8], coupled transmission lines [6], photonic crystals [2] and coupled resonating optical waveguide [9]. All these structures implement a kind of coupling mechanism between modes (two adjacent microstrips, two degenerate modes in a circular waveguide, etc).

In this work, we propose a periodic substrate-integrated waveguide (SIW) capable to support a a DBE. In a first design we assume lossless materials for simplicity. Afterwards, we analyze different types of losses influence on the DBE resonance and show dispersion relations when considering losses. The method proposed can also be used to analyze other DBE structures and the impact of losses on their performance.

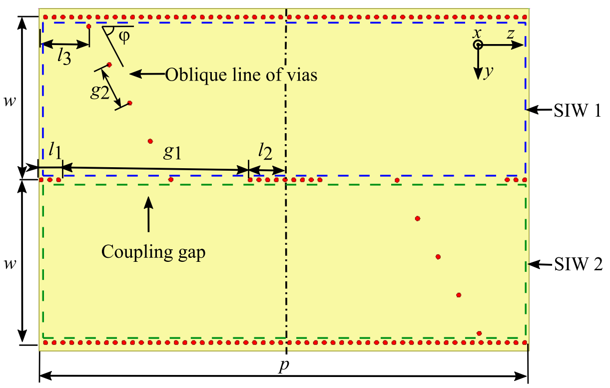
## II. SIW Design for DBE Synthesis

The unit cell in Fig. 1(a) shows two adjacent SIW coupled through the presence of two central regions without holes of length $g_1$ and through two oblique arrangements of metallic pins (the pins are depicted in red). The parameters of this structure are the following ones: the period is $p$ =90 mm, the two parallel SIWs have the same width $w = 30$ mm and the thickness of the substrate is 1 mm. The radius of the posts is 0.4 mm and the distance between adjacent posts is 0.8 mm. The two lengths of the middle posts lines are $l_1 = 1$ mm and $l_2 = 6.8$ mm, respectively. The two fundamental modes of each SIW are coupled at the gaps $g_1$, whose length is $g_1 = 34.4$ mm. The oblique line of vias determines the amount of coupling between modes and is the key element to achieve the DBE mode. The distance of the oblique line of vias from the left boundary of the cell is $l_3 = 9.1$ mm and the distance between adjacent oblique vias is $g_2 = 8$ mm. After a parametric study, an angle $\varphi$ of this oblique line necessary to develop the DBE condition has been found equal ro $\varphi = 61.7°$. The material of the substrate is Rogers RO3010, having relative permittivity $\epsilon_r$=10.2. We assume in this section a lossless structure.

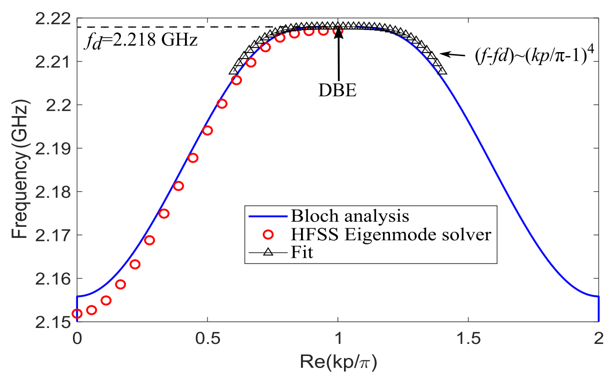
A full-wave simulation (Ansys HFSS) of the unit cell leads to the calculation of its scattering matrix, which is converted into a transfer matrix. Then, a Bloch analysis is performed to calculate the dispersion relation of the line. The dispersion diagram is shown in Fig. 1(b). A validation has been performed also with the eigensolver tool of HFSS has been performed, showing a perfect agreement with the Bloch analysis. The dispersion diagram of the optimized structure confirmes the presence of a DBE frequency at 2.218 GHz. For validation, the dispersion curve is superposed on a fourth-order polynomial fitting well the resonance near the DBE point.

## III. Loss Impact on DBE

A rigorous DBE condition is possible only in lossless structures; due to the presence of losses, the modes at the resonance




(a)




(b)

Fig. 1. (a) The unit cell of the periodic substrate integrated waveguides with DBE. The two parallel SIWs provide four coupling Bloch modes to form DBE mode. The coupling gaps and oblique lines of posts are used to provide a suitable coupling between the modes. (b) Dispersion diagram for the optimized unit cell supporting a DBE.

are not completely degenerate [10]. This will affect the DBE resonance, so that it is interesting to analyze the influences of losses on the dispersion diagram. Losses in the SIW structures can be of three types: conductor losses, dielectric losses and radiation losses [11]. The radiation losses are due to the leakage of fields through the gaps between lateral metallic posts and it is negligible when the diameter and the separation of the posts are small enough compared to the wavelength of the propagating wave in the SIW. In our structure, the diameter and separation of the post meet the rules described in [11], so that radiation losses will not be considered among the sources of losses in the following discussion. Note, however, that the numerical analyses are performed with a commercial simulator (HFSS) having absorbing boundary conditions on the side of the cell, so that no assumptions are done on the absence of radiation losses. If present, they would be visible in the results.

To investigate the contribution of conductor and dielectric losses to the DBE resonance in SIWs, we replace the PEC of the top and bottom grounds and of posts with copper whose conductivity is $5.8{\times}10^7$ S/m. The thickness of the ground layer is 0.07 mm. The dielectric substrate is Rogers RO3010 whose loss tangent is $\tan\delta = 0.0035$. The dispersion diagram near DBE frequency is shown in Fig. 2 (a). As expected, dielectric losses are the main contributions to the total losses, whereas the conductor losses only affect the dispersion relation strongly when the real part of the normalized wavenumber is larger than 0.77. In order to decrease the loss impact on DBE condition, we try now a substrate material exhibiting lower losses, the Polyflon CuFlon whose loss tangent is $\tan\delta = 0.00045$ and relative permittivity is $\epsilon_r = 2.1$. Due to the change in the relative permittivity, a DBE is found now at 4.887 GHz, and the full dispersion diagram close to the DBE is shown in Fig. 2 (b).

In all these cases the effect of losses is clearly visible. However, on the one hand the lossless and high-loss configurations of Fig. 2 (a) show significant differences already when $\mathrm{Re}(kp/\pi) > 0.6$. On the other hand the lossless and low-loss configurations of Fig. 2 (b) discrepancies are observed only in the region at $\mathrm{Re}(kp/\pi) > 0.8$.

Furthermore, in order to have also a comparison between different loss levels at the same frequency, we have also used a fictitious substrate with the same real part of $\epsilon_r$ of the CuFlon but higher tangent loss. In this third case the DBE mode is still at 4.887 GHz. In Fig. 2 (c) the lossless CuFlon structure is compared with the lossy Cuflon ($\tan\delta = 0.00045$) and with the fictitious Cuflon having higher losses $\tan\delta = 0.0035$. We find here the confirmation that a low-loss material does help to achieve a good approximation of a perfect DBE in the neighborhood of the resonant frequency.

These results help to quantify SIW losses on DBE, and are preliminary to the study of losses in SIW DBE with a finite number of cell. It is expected that the level of losses will limit the Q factor of finite-size resonators. In future works dielectric-less configurations of the unit cell in Fig. 1(a) will be also explored. The results of the different contributions of losses prove that limiting losses to conductors could help achieving a very good approximation of DBE condition.

## IV. Conclusion

In this work, we have proposed for the first time a periodic SIW capable to support a DBE. This kind of structure is expected to find applications in integrated filters and resonators due to the ease of fabrication. The dispersion diagram of the periodic SIW has been derived by simulating the unit cell with commercial software and performing a periodic Bloch analysis. The tuning of geometric parameters defining the coupling between the adjacent waveguides allowed to achieve a DBE condition. The effect of losses has been discussed separately for different sources of losses (radiation, conductor, dielectric) and different dielectrics have been tested to assess their impact on the resonance shape.

Future work will focus on the study and realization of finite-length SIWs. Different configurations capable to minimize losses and/or to reduce the dimensions of the cell will be privileged.

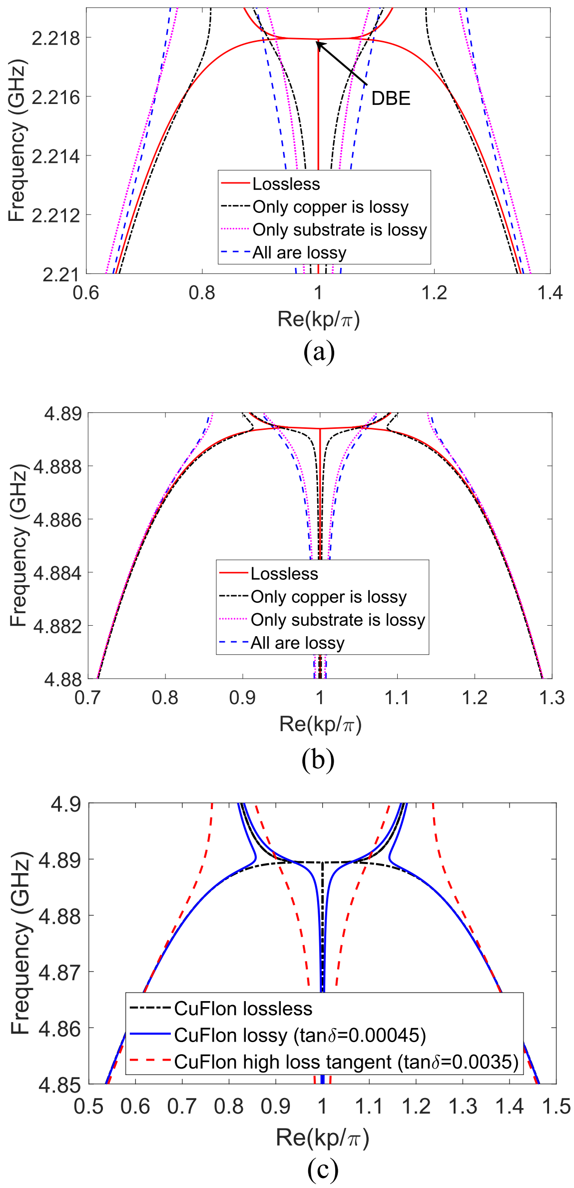



Fig. 2. The dispersion diagram when considering losses. The conductor losses and the dielectric losses are considered separately. (a) The substrate's material is Rogers RO3010 with loss tangent ($\tan\delta = 0.0035$). (b) The substrate material is Polyflon CuFlon with a lower loss tangent ($\tan\delta = 0.00045$). (c) A comparison between lossless CuFlon, lossy CuFlon, and a fictitious substrate with the same permittivity of CuFlon but a higher loss tangent ($\tan\delta = 0.0035$).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