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**A NOVEL INTELLIGENT LEARNING CONTROL SCHEME FOR DISCRETE-TIME
NONLINEAR UNCERTAIN SYSTEMS IN MULTIPLE ENVIRONMENTS***

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ABSTRACT

In this paper, we propose a novel intelligent control scheme for a class of discrete-time nonlinear uncertain systems operating under multiple environments/control situations. First, based on the deterministic learning theory, artificial neural networks (NNs) are employed to accurately learn/identify the uncertain system dynamics under each individual environment. The learned knowledge is then utilized to: (i) achieve improved control performance by developing a family of experience-based controllers (EBCs), each of which is tailored to an individual environment; and (ii) determine real-time activation of the EBCs by developing a pattern recognition mechanism for online identifying the active control situation. In addition, a robust quasi-sliding mode controller is further designed and embedded in the overall control scheme to guarantee system stability during the transition process among multiple environments. The novelty of the proposed control scheme lies in its intelligent capabilities of knowledge acquisition and re-utilization in real-time control, enabling self-adaption to uncertain changing control environments. A simulation example is included to verify the effectiveness of the proposed results.

1 INTRODUCTION

Many aspects of modern life involve the use of intelligent automated machines, which perform tasks involving complex

dynamic interactions with their environments (e.g., [1–3]). A notable example is the intelligent humanoid robots [4], which are built to functionally operate in various environments and replace human workforce to perform sets of tasks (e.g., grasping and manipulating objects [5]). Over the past decades, considerable efforts have been devoted to the development of advanced intelligent control schemes to enable autonomy of such machines operating under multiple environments (e.g., [6–8]). However, existing methods have their own limitations, for example, they largely require the operating environments to vary slowly with time. For those cases when the environments are changing abruptly, the associated intelligent control design problem is rather challenging and has received less discussions in current literature. One promising approach to overcome this challenge is the multiple model adaptive control approach (MMAC) proposed by [9, 10]. With this approach, multiple models are first developed to approximate/represent different control situations, and multiple controllers are then designed to provide suitable control actions for each corresponding control situation. However, the learning capability of adaptive controllers developed from such a MMAC framework has not been well-explored; as a result, it lacks the real intelligent capabilities of knowledge acquisition and re-utilization. To address this issue, a so-called pattern based neural network (NN) learning control scheme was proposed in [11], which seeks to develop multiple controllers with the knowledge learned from various environments. Unfortunately, one limitation of this approach is that the changing sequence of the operating environments needs to be constrained,

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i.e., the operating environment is only allowed to change from a specific one (i.e., normal control situation) to the others, which might not be always feasible in practice.

In this paper, we focus on the intelligent control problem of a class of discrete-time nonlinear uncertain systems operating under multiple environments/control situations. Specifically, we aim at dealing with several intelligent control issues, including: (i) how to provide high control performance with tractable computational complexity under each individual environment; (ii) how to guarantee the system stability when the operating environment is abruptly changing from one to another; (iii) how to determine activation of the proposed multiple controllers under the varying environments. For the first issue, we consider that the controlled plant is operating in a specific individual environment, we propose to design an adaptive learning controller based on the deterministic learning theory [12], so as to achieve accurate learning and stable tracking control. Through this learning control process, the knowledge of the associated uncertain system dynamics can be accurately learned/identified and stored in constant radial basis function neural network (RBF NN) models. With these obtained models, a set of multiple experience-based controllers (EBCs) can be further developed with each one tailored to a particular control situation. One important feature of such EBCs is that they can guarantee improved control performance with significantly reduced computational burden that is typically demanded in traditional adaptive controls. For the second issue, it is noted that when the operating control situation is abruptly changed, the new control situation might not be able to be recognized immediately. In this case, a robust quasi-sliding mode controller (QSMC) is necessary to stabilize the overall system. Such a controller will also guarantee the switching process of multiple EBCs to be completed in a stable fashion. For the third issue, to determine online activation of the QSMC and the EBCs, we propose a pattern detection mechanism (to detect the occurrence of control situation switching) and a pattern recognition mechanism (to recognize the active control situation). Specifically, in the detection mechanism, a threshold is designed to bound the control tracking error under a steadily-operating control situation. When the operating situation is abruptly changed, the tracking error will increase and exceed the threshold, which can be used to indicate occurrence of control situation switching. Once the change of environments is detected, the operating controller will be immediately switched to the QSMC for stabilizing the overall system, and the recognition process will be activated. For recognition, we consider that the controlled plant is steadily operating in each individual control situation with the QSMC, an identification approach that is developed based on the deterministic learning theory [12] is employed to achieve locally-accurate identification of the overall uncertain system dynamics; the obtained knowledge is then used to construct a bank of estimators for online recognition of active control situations. The recognition decision making is based on

the smallest residual principle (SRP). Once the active control situation is recognized, the operating controller will be switched to the related EBC.

The main contribution of this paper can be summarized as follows. A novel intelligent control scheme is proposed for a class of discrete-time nonlinear uncertain systems operating under multiple control environments, which is capable of mimicking the human's intelligence capabilities of (i) knowledge acquisition through the process of control and recognition, and (ii) knowledge re-utilization to improve the performance of recognition and control for rapidly adapting to multiple environments.

The rest of the paper is organized as follows. Section 2 provides some preliminaries and states the problem to be studied. Section 3 presents the design of EBCs, and Section 4 proposes the robust QSMC. The detection and recognition mechanisms are presented in Section 5. Section 6 presents the overall intelligent controller structure for systems operating under multiple environments. The simulation results are presented in Section 7, and the conclusion is given in Section 8.

Notations. \mathbb{R} , \mathbb{R}_+ and \mathbb{Z}_+ denote, respectively, the set of real numbers, the set of positive real numbers and the set of positive integers; $\mathbb{R}^{m \times n}$ denotes the set of $m \times n$ real matrices; \mathbb{R}^n denotes the set of $n \times 1$ real column vectors; I_n denotes the $n \times n$ identity matrix; the open ball $B_r = \{x \in \mathbb{R}^n : \|x\| < r\}$ with r being an arbitrary positive constant; $|\cdot|$ is the absolute value of a real number; $\|\cdot\|$ is the 2-norm of a vector or a matrix, i.e. $\|x\| = (x^T x)^{\frac{1}{2}}$.

2 Preliminaries and Problem Formulation

2.1 RBF NNs

RBF NN can be described by $f_{nn}(Z) = \sum_{i=1}^{N_n} w_i s_i(Z) = W^T S(Z)$ [13], where $Z \in \Omega_Z \subset \mathbb{R}^q$ is the input vector, $W = [w_1, \dots, w_{N_n}]^T \in \mathbb{R}^{N_n}$ is the weight vector, N_n is the NN node number, and $S(Z) = [s_1(\|Z - \zeta_1\|), \dots, s_{N_n}(\|Z - \zeta_{N_n}\|)]^T$, with $s_i(\cdot)$ being a radial basis function, and ζ_i ($i = 1, 2, \dots, N_n$) being distinct points in state space. The Gaussian function $s_i(\|Z - \zeta_i\|) = \exp[-\frac{(Z - \zeta_i)^T (Z - \zeta_i)}{\eta_i^2}]$ is one of the most commonly used radial basis functions, where $\zeta_i = [\zeta_{i1}, \zeta_{i2}, \dots, \zeta_{iq}]^T$ is the center of the receptive field, and η_i is the width of the receptive field. The Gaussian function belongs to the class of localized RBFs in the sense that $s_i(\|Z - \zeta_i\|) \rightarrow 0$ as $\|Z\| \rightarrow \infty$. It is easily seen that $S(Z)$ is bounded and there exists a real constant $S_M \in \mathbb{R}_+$ such that $\|S(Z)\| \leq S_M$ [12].

It has been shown in [13, 14] that for any continuous function $f(Z) : \Omega_Z \rightarrow \mathbb{R}$ where $\Omega_Z \subset \mathbb{R}^q$ is a compact set, and for the NN approximator, where the node number N_n is sufficiently large, there exists an ideal constant weight vector W^* , such that for any $\varepsilon^* > 0$, $f(Z) = W^{*T} S(Z) + \varepsilon$, $\forall Z \in \Omega_Z$, where $|\varepsilon| < \varepsilon^*$ is the ideal approximation error. The ideal weight vector W^* is an "artificial" quantity required for analysis, and is defined as the value of W that minimizes $|\varepsilon|$ for all $Z \in \Omega_Z \subset \mathbb{R}^q$, i.e. $W^* \triangleq \operatorname{argmin}_{W \in \mathbb{R}^{N_n}} \{\sup_{Z \in \Omega_Z} |f(Z) - W^T S(Z)|\}$. Moreover,

based on the localization property of RBF NNs [12], for any bounded trajectory $Z(k)$ within the compact set Ω_Z , $f(Z)$ can be approximated by using a limited number of neurons located in a local region along the trajectory: $f(Z) = W_\zeta^* S_\zeta(Z) + \varepsilon_\zeta$, where ε_ζ is the approximation error, with $\varepsilon_\zeta = O(\varepsilon) = O(\varepsilon^*)$, $S_\zeta(Z) = [s_{j1}(Z), \dots, s_{jN_\zeta}(Z)]^T \in \mathbb{R}^{N_\zeta}$, $W_\zeta^* = [w_{j1}^*, \dots, w_{jN_\zeta}^*]^T \in \mathbb{R}^{N_\zeta}$, $N_\zeta < N_n$, and the integers $j_i = j_1, \dots, j_{N_\zeta}$ are defined by $|s_{j_i}(Z_p)| > \theta$ ($\theta > 0$ is a small positive constant) for some $Z_p \in Z(k)$.

It is shown in [12] that for a localized RBF network $W^T S(Z)$ whose centers are placed on a regular lattice, almost any recurrent trajectory¹ $Z(k)$ can lead to the satisfaction of the PE condition of the regressor subvector $S_\zeta(Z)$. This result is formally summarized in the following lemma.

Lemma 1 ([15, 16]). *Consider any recurrent trajectory $Z(k): \mathbb{Z}_+ \rightarrow \mathbb{R}^q$. $Z(k)$ remains in a bounded compact set $\Omega_Z \subset \mathbb{R}^q$, then for RBF network $W^T S(Z)$ with centers placed on a regular lattice (large enough to cover compact set Ω_Z), the regressor subvector $S_\zeta(Z)$ consisting of RBFs with centers located in a small neighborhood of $Z(k)$ is persistently exciting (PE).*

2.2 Problem Formulation

Consider the following discrete-time nonlinear uncertain system:

$$\begin{cases} x_i(k+1) = x_{i+1}(k), & i = 1, 2, \dots, n-1, \\ x_n(k+1) = f^j(x(k)) + g^j(x(k))u(k), \end{cases} \quad (1)$$

where $x = [x_1, \dots, x_n]^T \in \mathbb{R}^n$, $u \in \mathbb{R}$ are system state and input, respectively, $f^j(x) \in \mathbb{R}$, $g^j(x) \in \mathbb{R}$ are unknown nonlinear functions, the superscript denotes different system dynamics under different control situations with $j \in J = \{1, \dots, N\}$.

Assumption 1. *For $\forall j \in J$, the sign of $g^j(x)$ is known and positive, and there exist constants $g_1^j > g_0^j > 0$ such that $g_1^j > g^j(x) > g_0^j$, $\forall x \in \mathbb{R}^n$.*

Assumption 2. *For $\forall j \in J$, the nominal models of $f^j(x)$, $g^j(x)$ in system (1) are known, denoted by $\bar{f}^j(x) \in \mathbb{R}$, $\bar{g}^j(x) \in \mathbb{R}$, respectively, which satisfy: (i) $\frac{\bar{g}^j(x)}{g^j(x)} > \frac{1}{2}$ and (ii) there exists a known constant $H > 0$ and a certain u such that $|((f^j(x) - \bar{f}^j(x)) + (g^j(x) - \bar{g}^j(x))u)| < H$.*

Remark 1. *In (1), the effects of multiple environments $j \in J$ are reflected by the nonlinear uncertain functions $f^j(x)$ and $g^j(x)$. Different j indicates changes of the system dynamics due to the changing environments, which could be for example, faults in the system, external disturbances, and changes in system parameters. A typical example is the robot manipulator [17], which is*

built to carry up and lay down some specific objects repeatedly. The changing loads and friction would result in changing system dynamics with parameter value jumps and varying structures.

Consider the following reference model:

$$\begin{cases} x_{d_i}(k+1) = x_{d_{i+1}}(k), & i = 1, 2, \dots, n-1, \\ x_{d_n}(k+1) = f_d(x_d(k)), \end{cases} \quad (2)$$

where $x_d = [x_{d_1}, \dots, x_{d_n}]^T \in \mathbb{R}^n$ is the reference state, $f_d(x_d) \in \mathbb{R}$ is a known nonlinear function.

Assumption 3. *All state signals of the reference model (2) are bounded and recurrent.*

In this paper, we assume that the plant (1) initially operates in a known control situation $s \in J$, and is abruptly switched to an unknown control situation l ($l \in J, l \neq s$) at an unknown time instant k_c , as shown in Fig. 1. Our objective is to drive the system state x of (1) to track over the reference signal x_d of (2). For this purpose, as shown in Fig. 1, we seek to: (i) design a set of EBCs, each of which is tailored to an individual control situation for all $s, l \in J$; (ii) develop a robust QSMC to stabilize the overall system during the transition process of control situation switching; (iii) devise a detection mechanism (to detect the change of control situations) and a recognition mechanism (to recognize the active control situation l), so as to determine the activation of the multiple controllers (QSMC or EBCs).

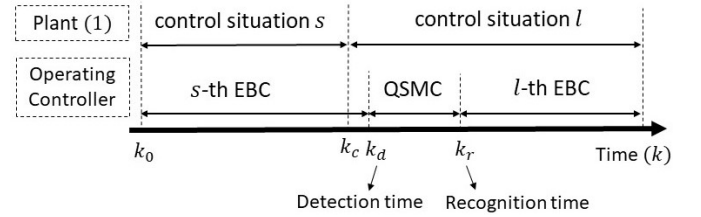


FIGURE 1: Schematic of the real-time control process under multiple environments. k_0 : initial time; k_c : occurrence time of environment changing; k_d : detection time of environment changing from $s \rightarrow l$; k_r : recognition time of the active operating environment l .

Remark 2. *For the above control problem, conventional control methods may not be competent. For example, robust control methods (e.g., [18]) can be used to accommodate the system uncertainties due to multiple environments, which however usually lead to conservative control performance. Adaptive switching control methods (e.g., [17]), dealing with the abruptly-changing environments, would necessitate computationally-expensive adaption and switching. In this paper, to overcome these deficiencies, we will propose a novel intelligent control scheme consisting of multiple controllers and a mechanism of detection and recognition of active control situation.*

¹A recurrent trajectory represents a large set of periodic and periodic-like trajectories generated from linear/nonlinear dynamics systems. A detailed characterization of recurrent trajectories can be found in [12].

3 Adaptive Learning Control Under Individual Environment

In this section, we aim at designing multiple EBCs for each individual control situation $j \in J$. An adaptive learning controller based on the deterministic learning theory [12] will be proposed to achieve stable tracking control and accurate learning of associated uncertain system dynamics. Specifically, consider that the plant (1) operating in any fixed individual control situation, say $j \in J$, if the system nonlinear functions $f^j(x)$, $g^j(x)$ are known, one can design an ideal stabilizing control strategy as:

$$u^{j*} = \frac{1}{g^j(x)} (f_d(x_d) - f^j(x) - \lambda_1 z_n - \dots - \lambda_{n-1} z_2), \quad (3)$$

where $f_d(x_d)$ is a known function in (2), $z_i = x_i - x_{d_i}$ ($i = 1, \dots, n$) is state tracking error, $\lambda_1, \dots, \lambda_{n-1}$ are design constants such that $z^{n-1} + \lambda_1 z^{n-2} + \dots + \lambda_{n-1} z$ is a Schur polynomial. It is easy to prove that with the above controller, the filtered tracking error

$$r = z_n + \lambda_1 z_{n-1} + \dots + \lambda_{n-1} z_1, \quad (4)$$

will converge to zero exponentially, and thus the state tracking error z_i will be guaranteed to be also exponentially converging to zero.

However, since the nonlinear functions $f^j(x)$ and $g^j(x)$ are uncertain, making the above ideal controller not implementable, to overcome this issue, we will resort to artificial neural networks. Specifically, according to the NN approximation results as introduced in Section 2.1, for the unknown nonlinear function u^{j*} in (3), there exists an ideal constant NN weight $W^{j*} \in \mathbb{R}^{N_n}$ (with N_n denoting the number of NN nodes) such that

$$u^{j*} = W^{j*T} S(Z) + \varepsilon^j, \quad (5)$$

where $Z = [x^T, x_z^T]^T \in \mathbb{R}^{n+1}$ with $x_z = f_d(x_d) - \lambda_1 z_n - \dots - \lambda_{n-1} z_2$ is the input vector of RBF NNs; $S(Z) \in \mathbb{R}^{N_n}$ is a smooth RBF vector; ε^j is an ideal approximation error satisfying $|\varepsilon^j| < \varepsilon^*$, with ε^* being a positive constant that can be made arbitrarily small by constructing sufficiently large number of neurons. Based on this, the adaptive learning controller is proposed as:

$$\hat{u}^j(k) = \hat{W}^{jT}(k) S(Z(k)), \quad (6a)$$

$$\hat{W}^j(k+1) = \hat{W}^j(k) - \gamma^j r(k+1) S(Z(k)), \quad (6b)$$

with γ^j being a design constant, \hat{W}^j an estimate of W^{j*} in (5).

The rigorous analysis of closed-loop stability, tracking, and learning performance achieved by the controller (6) can be summarized in the following theorem.

Theorem 1. Consider the closed-loop system consisting of the plant (1) with any fixed $j \in J$, the reference model (2), the adaptive learning controller (6). Under Assumptions 1 and 3, given initial conditions $x(0) \in \Omega_0$ (where Ω_0 is a compact set) and $\hat{W}^j(0) = 0$, if the controller coefficient γ^j satisfies $0 < \gamma^j < \frac{1}{1+g_1^j S_M^2}$, with g_1^j and S_M respectively being the upper bounds of $g^j(x)$ and $\|S(Z)\|$, then, it is guaranteed that: (i) all signals in the

closed-loop system remain uniformly ultimately bounded (UUB); (ii) there exists a finite time K_c such that for all $k > K_c$, the state tracking error $x(k) - x_d(k)$ converges to a small neighborhood around the origin, and the network input $Z = [x^T, x_z^T]^T$ converges to a small neighborhood of recurrent orbit $Z_d = [x_d^T, f_d(x_d)]^T$; (iii) a locally accurate approximation for the unknown dynamics u^{j*} in (3) is obtained by $\hat{W}^{jT} S(Z)$ as well as $\bar{W}^{jT} S(Z)$ along the NN input orbit Z , where

$$\bar{W}^j := \frac{1}{K_2 - K_1 + 1} \sum_{k=K_1}^{K_2} \hat{W}^j(k) \quad (7)$$

with $K_2 > K_1 > K_c$ being a time segment after transient process.

Proof of the above theorem follows a similar line of that for [19, Th. 1], readers are referred to this reference for details, which will thus be omitted here.

Remark 3. Under Assumption 3, the system states x in (1) tracking the reference signals x_d in (2) is guaranteed to be recurrent. Thus, the RBF NN input $Z = [x^T, x_z^T]^T$ in (6) will also be guaranteed to be recurrent. From Lemma 1, the satisfaction of PE condition in the design of our approach will be guaranteed, such that accurate parameter convergence can be ensured.

Through the learning process, the knowledge of the associated uncertain dynamics u^{j*} in (3) can be obtained and stored in the constant RBF NN model $\bar{W}^{jT} S(Z)$ (with constant weight \bar{W}^j given in (7)). According to [19, 20], such an approximation can be achieved in a local region Ω_c^j along the trajectory Z_d , which can be described as:

$$\Omega_c^j = \{Z \mid \text{dist}(Z, Z_d) < d_c^* \Rightarrow |\bar{W}^{jT} S(Z) - u^{j*}| < \varepsilon_c^*\}. \quad (8)$$

where the constant d_c^* characterizes the size of NN approximation region, and ε_c^* is the approximation error within Ω_c^j ($\forall j \in J$), which can be pre-specified and obtained by constructing a sufficiently large number of neurons according to [19].

Consequently, with the models $\bar{W}^{jT} S(Z)$, a set of multiple EBCs can be designed as follows:

$$u^j(k) = \bar{W}^{jT} S(Z(k)), \quad j = 1, \dots, N \quad (9)$$

where the constant weight \bar{W}^j is given in (7), and j denotes the associated control situation. The closed-loop stability and tracking performance of such controllers are summarized as follows.

Theorem 2. Consider the closed-loop system consisting of the plant (1) with a fixed $j \in J$, the reference model (2), and the associated j -th EBC in the form of (9). Under Assumptions 1 and 3, given the same recurrent reference orbits $Z_d = [x_d^T, f_d(x_d)]^T$, and with initial condition $Z(0)$ in a close vicinity of Z_d , we have that: (i) all signals in the closed-loop system remain UUB; (ii) the state tracking error $x - x_d$ converges to a small neighborhood around the origin; (iii) the filtered tracking error in (4) after transient process satisfies:

$$|r| < g_1^j \epsilon_c^*, \quad (10)$$

where g_1^j is the upper bound of $g^j(x)$, ϵ_c^* is the NN approximation error within Ω_c^j .

Proof. For the plant (1) under control situation j , once the system trajectory Z enters a vicinity of the experienced tracking orbit Z_d , i.e., Ω_c^j in (8), the j -th EBC in (9) will quickly recall the stored knowledge to provide accurate approximation for the dynamics u^{j*} in (3), as verified in (8), so as to achieve stable tracking control without online adaptation. The detailed analysis can be conducted by following the same process for [19, Th. 2]. Moreover, from (1)–(4), (8) and (9), after the transient control process, the filtered tracking error r is guaranteed to satisfy

$$|r(k+1)| = |g^j(x(k))(\bar{W}^{jT} S(Z(k)) - u^{j*}(k))| < g_1^j \epsilon_c^*, \quad (11)$$

which ends the proof.

4 Quasi Sliding Mode Control During Transition of Multiple Environments

In this section, a robust quasi sliding mode controller (QSMC) will be proposed to guarantee the system stability in the transition process of EBCs-switching, i.e., the time duration between k_d and k_r as shown in Fig. 1. Specifically, consider the plant (1) and reference model (2), a sliding surface is first designed as:

$$s(k) = s(k-2) + \alpha_0 z_n(k) + \cdots + \alpha_{n-1} z_1(k), \quad (12)$$

where $z_i = x_i - x_{d_i}$ ($i = 1, \dots, n$) are tracking errors, the parameters $\alpha_0, \dots, \alpha_{n-1}$ are selected to make $\alpha_0 z_n + \cdots + \alpha_{n-1} z_1$ a Schur polynomial. We then design the robust QSMC as:

$$u^0(k) = -\frac{1}{\alpha_0 \bar{g}(x(k))} (\alpha_0 (\bar{f}(x(k)) - f_d(x_d(k))) + \alpha_1 z_n(k) + \cdots + \alpha_{n-1} z_2(k) + s(k-1) + \rho \cdot \text{sgn}(s(k))), \quad (13)$$

where $\bar{f}(x)$, $\bar{g}(x)$ are known functions as specified in Assumption 2, $\rho > \alpha_0 H$ is a design constant, and H is a known constant defined in Assumption 2. With the controller (13), from (1)–(2), the function $s(k)$ exhibits the zigzag behavior:

$$s(k+1) = \alpha_0 (e^j(k+1)) - \rho \cdot \text{sgn}(s(k)) \quad (14)$$

where $e^j(k+1) := f^j(x(k)) + g^j(x(k))u(k) - (\bar{f}(x(k)) + \bar{g}(x(k))u(k))$ is bounded by $|e^j(k+1)| < H$, as given in Assumption 2. The boundedness can thus be obtained as: $|s(k)| \leq \alpha_0 H + \rho$, according to [18].

Following the analysis of [18], it can be demonstrated that the QSMC in (13) will ensure that the system state x of (1) will stay within a close vicinity of the reference orbit x_d of (2), guaranteeing overall stability of the system during the transition process of control situation changing.

5 Control Situation Detection And Recognition

In this section, according to Fig. 1, a rapid pattern detection and recognition mechanism will be further developed to determine controller activation among multiple controllers.

5.1 Detection of Environment Changing

We first design a detection mechanism for detecting the occurrence of changes of the operating control situations. Recall Fig. 1, suppose that the operating control situation is abruptly changed from $s \in J$ to $l \in J$ ($l \neq s$) at time k_c , i.e., the controlled plant's dynamics obeys

$$\begin{cases} x_i(k+1) = x_{i+1}(k), & i = 1, 2, \dots, n-1 \\ x_n(k+1) = \begin{cases} f^s(x(k)) + g^s(x(k))u^s(k), & k_0 \leq k < k_c \\ f^l(x(k)) + g^l(x(k))u^s(k), & k_c \leq k \end{cases} \end{cases} \quad (15)$$

where k_0 is initial time, u^s is the s -th EBC of (9).

In (15), for $k_0 \leq k < k_c$, the operating controller u^s matches the current control situation s . The filtered tracking error r of (4) is guaranteed smaller than $g_1^s \epsilon_c^*$, as verified in Theorem 2. Once the control situation is switched to l ($l \neq s$) at time k_c , the filtered tracking error r could increase and become possibly larger than $g_1^s \epsilon_c^*$. Based on this, a threshold (denoted by \bar{e}_c) can be designed:

$$\bar{e}_c = q_c \epsilon_{cm}^*, \quad (16)$$

where $\epsilon_{cm}^* := \max_{j \in J} \{g_1^j \epsilon_c^*\}$, in which the value $g_1^j \epsilon_c^*$ is given in (10) and can be obtained in the training phase of designing EBCs; $q_c \geq 1$ is an auxiliary parameter designed to prevent the misjudgment of detection.

Detection decision making scheme: Consider the system (15) and the reference model (2). Compare the filtered tracking error r of (4) with the threshold \bar{e}_c of (16). If there exists a finite time $k_d > k_c$ such that $|r(k_d)| > \bar{e}_c$, occurrence of the change of active control situations can be detected at time k_d .

Remark 4. With the above detection mechanism, stability of the system (15) can be guaranteed. It is noted that the threshold \bar{e}_c in (16) can be considered as the maximum tolerable tracking error level under the proposed EBCs of (9). As long as the filtered tracking error $|r|$ of (4) remains smaller than the designed threshold \bar{e}_c , it can be deduced that the system (15) operates in a stable fashion and the active EBC matches the current control situation.

5.2 Recognition of Active Operating Environment

As shown in Fig. 1, the recognition mechanism will be activated once occurrence of control situation changes is detected at time k_d . Note that after time k_d , the operating controller is switched to QSMC u^0 of (13), in this case the monitored system is governed by the following dynamics:

$$\begin{cases} x_i(k+1) = x_{i+1}(k), & i = 1, 2, \dots, n-1 \\ x_n(k+1) = f^l(x(k)) + g^l(x(k))u^0(k), & k_r > k > k_d \end{cases} \quad (17)$$

Development of the recognition mechanism will first involve a training phase. Specifically, we consider that the monitored plant operates in any fixed individual control situation, say $j \in J$, and is controlled by the robust QSMC u^0 of (13), i.e.,

$$\begin{cases} x_i(k+1) = x_{i+1}(k), & i = 1, 2, \dots, n-1 \\ x_n(k+1) = f^j(x(k)) + g^j(x(k))u^0(k). \end{cases} \quad (18)$$

For the unknown nonlinear function $f^j(x) + g^j(x)u^0$ in (18), according to the RBF NN approximation methods introduced in Section 2.1, there exists an ideal constant NN weight $W_r^{j*} \in \mathbb{R}^{N_n}$ (with N_n denoting the number of NN nodes) such that

$$f^j(x) + g^j(x)u^0 = W_r^{j*T} S_r(x, u^0) + \varepsilon_{r,0}^j \quad (19)$$

where (x, u^0) is the input of RBF NNs generated from system (18), $S_r(x, u^0)$ is a smooth RBF NN vector, $\varepsilon_{r,0}^j$ is an ideal approximation error satisfying $|\varepsilon_{r,0}^j| < \varepsilon_r^*$, and ε_r^* is a positive constant that can be made arbitrarily small by constructing large number of neurons. Based on this, we first propose to design an identifier as follows to learn the above unknown function:

$$\hat{x}_r^j(k+1) = a^j(\hat{x}_r^j(k) - x_n(k)) + \hat{W}_r^{jT}(k) S_r(x(k), u^0(k)), \quad (20a)$$

$$\hat{W}_r^j(k+1) = \hat{W}_r^j(k) - c^j \tilde{x}_n(k+1) S_r(x(k), u^0(k)), \quad (20b)$$

where a^j, c^j are design parameters, $\tilde{x}_r^j = \hat{x}_r^j - x_n$, and \hat{W}_r^j is an estimate of the ideal constant weight W_r^{j*} in (19).

Learning performance of the identifier (20) is summarized in the following theorem.

Theorem 3. Consider the adaptive learning system consisting of the plant (18), the identifier (20). Under Assumption 3, given the recurrent orbit (x, u^0) , with initial conditions $(x(0), u^0(0)) \in \Omega_{r,0}$ (where $\Omega_{r,0}$ is a compact set) and $\hat{W}_r^j(0) = 0$, if the associated coefficients in (20) satisfy $0 < a^j < \frac{\sqrt{5}-1}{2}$ and $0 < c^j < \frac{1}{S_{r,M}^2(2+a^j)}$ with $S_{r,M}$ being upper bound of $\|S_r(x, u^0)\|$, then, we have: (i) all signals in the system remain UUB; (ii) there exists a finite time K_r such that for all $k > K_r$, the estimation error $\hat{x}_r^j(k) - x_n(k)$ converges to a small neighborhood around the origin; (iii) a locally accurate approximation for the system dynamics $f^j(x) + g^j(x)u^0$ can be obtained by $\hat{W}_r^{jT} S_r(x, u^0)$ as well as $\bar{W}_r^{jT} S_r(x, u^0)$ along the NN input orbit (x, u^0) , with $\bar{W}_r^j = \frac{1}{K_2} \sum_{k=K_1}^{K_1+K_2-1} \hat{W}_r^j(k)$, and $K_2 > K_1 > K_r$ representing a time segment after the transient process.

Proof of the above theorem follows a similar line of that in [21, Th. 1], readers are referred to this reference for details, which are thus omitted here.

Remark 5. Under Assumption 3, the system state x and reference signal x_d are guaranteed to be recurrent. Note that the control signal u^0 of (13) is designed as a function of x and x_d , which

thus is also recurrent. From Lemma 1, given the recurrent orbit (x, u^0) , the associated PE condition in the adaptive learning control system will be satisfied, ensuring convergence of associated adaptation parameters to their true values.

Through the above learning/training phase, the uncertain dynamics $f^j(x) + g^j(x)u^0$ in (18) can be identified and represented by constant RBF NN models $\bar{W}_r^{jT} S_r(x, u^0)$. Moreover, the approximation is achieved in a local region Ω_r^j along the experienced recurrent trajectory (denoted by ϕ_r^j) of (18) with

$$\Omega_r^j := \{(x, u) \mid \text{dist}((x, u), \phi_r^j) < d_r^* \Rightarrow |f^j(x) + g^j(x)u^0 - \bar{W}_r^{jT} S_r(x, u)| < \varepsilon_r^*\}, \quad (21)$$

where d_r^* characterizes the size of NN approximation region, ε_r^* is the NN approximation error within the region Ω_r^j .

Based on the obtained models $\bar{W}_r^{jT} S_r(x, u^0)$, for the monitored system (17), we are ready to design the following system for real-time recognition of active operating environment:

$$e_r^j(k) = b_r e_r^j(k-1) + |\bar{W}_r^{jT} S(x(k-1), u^0(k-1)) - x_n(k)|, \quad (22)$$

where $j = 1, \dots, N$, $0 < b_r < 1$ is a design parameter, (x, u^0) is the real-time trajectory of (17), x_n is the system state in (17), and e_r^j is a residual signal with $e_r^j(k_d) = 0$, which can be taken as the similarity measure between the operating control situation l of (17) and each trained control situation j in (18), as argued in [22, 23].

The recognition decision making is based on the smallest residual principle, that is, the active operating control situation l of (17) is recognized when the corresponding residual e_r^l becomes the smallest one. Following the methodologies from [22, 23], to achieve rapid recognition, the following assumption is made.

Assumption 4. For different control situations $l, \bar{l} \in J$ ($\bar{l} \neq l$), the system dynamics in the form of (18) satisfies

$$|(f^{\bar{l}}(x) + g^{\bar{l}}(x)u^0) - (f^l(x) + g^l(x)u^0)| > \varepsilon_r^*, \quad (23)$$

where ε_r^* is the ideal approximation error given in (21).

The following theorem summarizes the recognition results.

Theorem 4. Consider the monitored system (17) and the recognition systems (22). Under Assumption 4, for $l \in J, \forall \bar{l} \in J \setminus \{l\}$, there must exist a finite time $k_r > k_d$ such that $e_r^l(k) < e_r^{\bar{l}}(k)$ holds for $k \geq k_r$, and the control situation l of (17) can be recognized at time k_r .

Proof of the above theorem follows similar line of that of [22, Th. 1].

6 Summary of the Overall Controller Structure

In this section, we summarize the overall intelligent controller structure that consists of all the control and recognition components proposed from the previous sections, and present the real-time control process for the plant (1) under multiple environments. Specifically, as shown in Fig. 1, when the operating control situation is abruptly changed from s to l ($l \neq s$) at time k_c , it will be rapidly detected at time k_d . Then, the operating controller is switched to QSMC, and the recognition process is activated. The rapid recognition for the control situation l is achieved at time k_r . Subsequently, the operating controller is switched from QSMC to the l -th EBC that matches the active control situation. Consequently, the overall control system can be summarized as:

$$\begin{cases} x_i(k+1) = x_{i+1}(k), & i = 1, 2, \dots, n-1 \\ x_n(k+1) = \begin{cases} f^s(x(k)) + g^s(x(k))u^s(k), & k_0 \leq k < k_c \\ f^l(x(k)) + g^l(x(k))u^s(k), & k_c \leq k \leq k_d \\ f^l(x(k)) + g^l(x(k))u^0(k), & k_d < k \leq k_r \\ f^l(x(k)) + g^l(x(k))u^l(k), & k_r < k, \end{cases} \end{cases} \quad (24)$$

with $s, l \in J$ ($l \neq s$) representing different control situations, u^s, u^l being the EBCs of (9), u^0 the QSMC of (13), k_0 the initial time, k_c the occurrence time of control situation l , k_d the detection time, and k_r the recognition time.

We have the following theorem to summarize the stability property of the overall system (24).

Theorem 5. *Consider the closed-loop system consisting of the plant (24), the reference model (2), the EBCs (9), the QSMC (13), and the recognition systems (22). Under Assumptions 1–4, given the recurrent reference orbit $Z_d = [x_d^T, f_d(x_d)]^T$, and with initial condition $Z(k_0) = [x(k_0)^T, x_z(k_0)^T]^T$ in a close vicinity of Z_d , we have: (i) all signals in the closed-loop system remain UUB; (ii) the state tracking error $x - x_d$ converges to a small neighborhood around zero.*

Proof. When $k_0 \leq k < k_c$, the plant (24) operates in control situation s and is controlled by the s -th EBC u^s . According to Theorem 2, with the given initial conditions, we have that all signals in the closed-loop system remains UUB, and the state tracking error $x - x_d$ converges to a small neighborhood around the origin.

At time k_c , in (24), the operating control situation is switched from s to l ($l \neq s$). Such a change is detected at time k_d , and then, the QSMC u^0 of (13) is selected in operation for time $k_d < k \leq k_r$. As analyzed in Section 4, with the controller u^0 , the system stability can be guaranteed and the system trajectory x of (24) stays close to the reference orbit x_d of (2). Moreover, for time $k_d < k \leq k_r$, the signal $Z = [x^T, x_z]^T$ is guaranteed to stay in a close vicinity of $Z_d = [x_d^T, f_d(x_d)]^T$, which enables the switching process of EBCs ($u^s \rightarrow u^l$) to be completed in a stable fashion.

At time k_r , the control situation l is recognized, and then the l -th EBC u^l of (9) is activated for $k > k_r$. According to the Theorem 2, once the EBC u^l is selected (at time k_r), it will quickly recall the stored knowledge to achieve desired control performance. All signals in the closed-loop system (24) remain UUB, and the state tracking error converges to a small neighborhood around zero. This ends the proof.

7 Simulation Studies

In this section, we will use a numerical example to demonstrate effectiveness of the proposed approach. To this end, we consider the following second-order nonlinear systems:

$$\begin{cases} x_1(k+1) = x_2(k), \\ x_2(k+1) = f^j(x(k)) + g^j(x(k))u(k), \end{cases} \quad (25)$$

where $j \in J = \{1, 2, 3\}$ represents different control situations with $f^1(x) = \frac{x_1}{1+2x_2^2}$, $g^1(x) = 1 + 0.3(\sin x_2)^2$, $f^2(x) = \frac{\sin x_1}{0.8+x_2^2}$, $g^2(x) = 0.8 + 0.3\cos(x_1x_2)$ and $f^3(x) = \frac{x_1+0.8\sin x_1}{1+x_2^2+\cos x_1}$, $g^3(x) = 1.1 + 0.2\sin(x_1x_2)$ being unknown functions. Their nominal models are known and given as: $\bar{f}^j(x) = \frac{x_1}{1+x_2^2}$, $\bar{g}^j(x) = 1$. Consider the reference model:

$$\begin{cases} x_{d1}(k+1) = x_{d2}(k), \\ x_{d2}(k+1) = f_d(x_d(k)), \end{cases} \quad (26)$$

with $f_d(x_d(k)) = \sin(0.5(k+1))$. Given (25) and (26), it is seen that Assumptions 1–4 are all satisfied.

We first design multiple EBCs for each individual control situation $j = 1, 2, 3$. Specifically, for the adaptive learning control in (6), we construct the RBF network $S(Z)$ in a regular lattice, with nodes $N_n = 2197$, the centers evenly spaced on $[-1.2, 1.2] \times [-1.2, 1.2] \times [-1.2, 1.2]$ and the width $\eta_c = 0.2$. The initial conditions are set as $x(0) = [1, 1]^T$, $x_d(0) = [0, 0]^T$, $\hat{W}^j(0) = 0$ and the parameters are set as $\lambda_1 = 0.2$, $\gamma^j = 0.1$. The EBCs in (9) are developed with $\bar{W}^j = \frac{1}{100} \sum_{k=1101}^{1200} \hat{W}^j(k)$ ($j = 1, 2, 3$). Consider the control situation 1, the learning performance achieved by adaptive learning control are plotted in Figs. 2a and 2b, and the tracking performance of EBC is in Fig. 2c. The simulation results of control situations $j = 2, 3$ are similar thus omitted here.

The QSMC in (13) is designed by setting $\alpha_0 = 1$, $\alpha_1 = 0.24$, and $\rho = 5$. Given the initial conditions $x(0) = [1, 1]^T$, $x_d(0) = [0, 0]^T$, the tracking performance for the control situations $j = 1, 2, 3$ are plotted in Fig. 3. It is seen that no matter which control situation occurs, QSMC always guarantees that the system trajectory x of (25) remains within a close vicinity of the reference orbit x_d of (26).

Finally, the detection and recognition mechanisms can be constructed as follows. For detection purpose, the detection

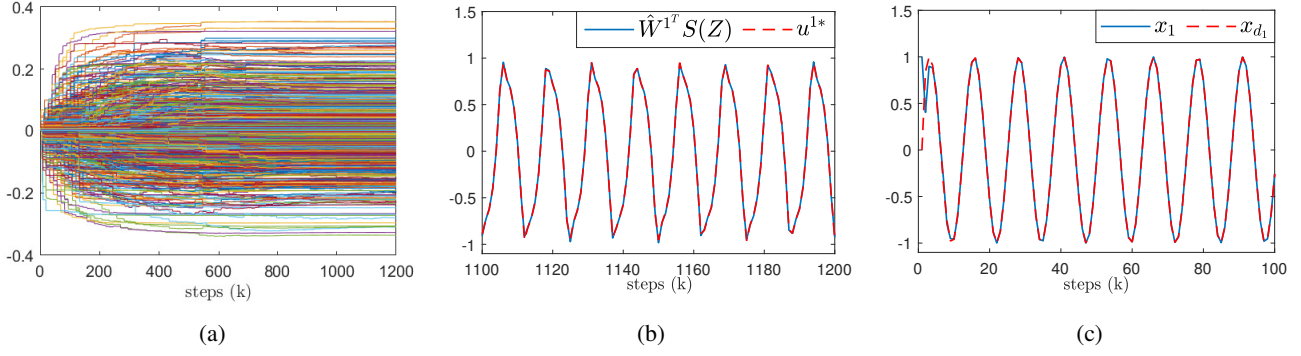


FIGURE 2: Learning and control performance under control situation 1. (a) NN weight convergence of controller (6): \hat{W}^1 . (b) Function approximation of controller (6): $\hat{W}^{1T} S(Z)$ and u^{1*} . (c) Tracking performance of the 1-st EBC of (9): system state x_1 and x_{d1} .

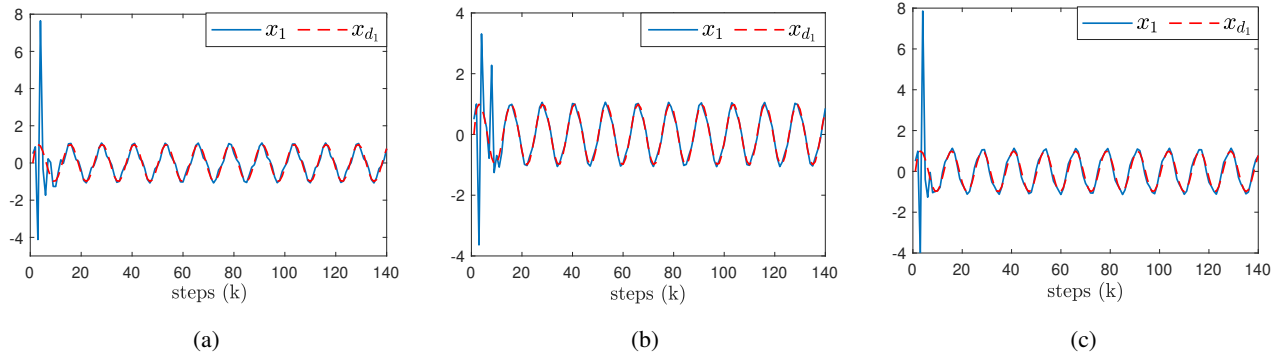


FIGURE 3: Tracking control performance of the QSMC (13) (a) under control situation 1, (b) under control situation 2, (c) under control situation 3.

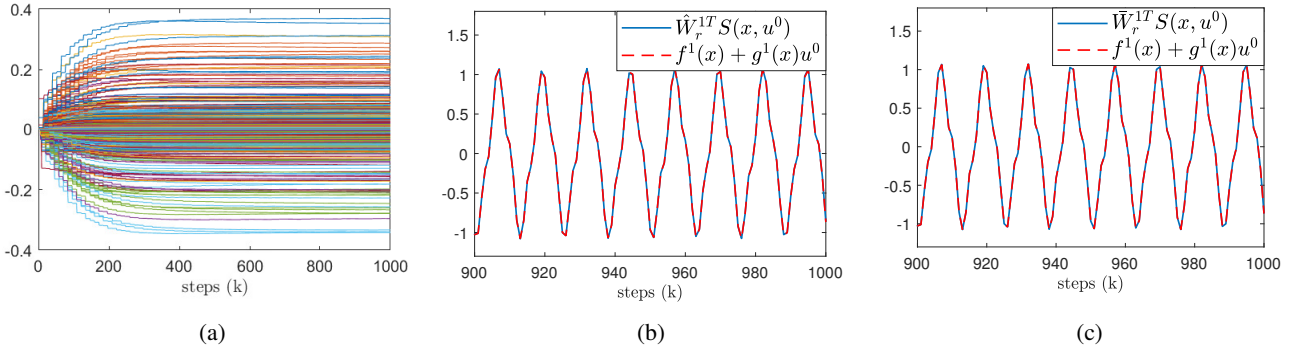


FIGURE 4: Identification performance of identifier (20) under control situation 1. (a) NN weight convergence: \hat{W}_r . (b) Function approximation: $\hat{W}_r^{1T} S_r(x, u)$ and $f^1(x) + g^1(x)u^0$. (c) Function approximation: $\bar{W}_r^{1T} S_r(x, u)$ and $f^1(x) + g^1(x)u^0$.

threshold in (16) is designed by setting $q_c = 10$. The accuracy level of filtered tracking error $|r|$ of (10) can be obtained as $\varepsilon_{cm}^* = 0.0024$ through the training phase of implementing the EBCs. For recognition of using the identifier (20), we construct the RBF network $S_r(x, u^0)$ in a regular lattice with nodes $N_n = 2197$, the centers evenly spaced on $[-1.2, 1.2] \times [-1.2, 1.2] \times [-1.2, 1.2]$ and the width $\eta_d = 0.2$. The parameters are set as $a^j = 0.1$, $c^j = 0.1$, and the initial conditions are $\hat{W}_r^j(0) = 0$, $\hat{x}_r^j(0) = 0$ ($j = 1, 2, 3$). The learned knowledge is stored as

$\bar{W}_r^j = \frac{1}{100} \sum_{k=901}^{1000} \hat{W}_r^j(k)$ ($j = 1, 2, 3$). The recognition systems in (22) are developed by setting $b_r = 0.9$. Considering the control situation 1, the identification performance of the identifier is shown in Fig. 4. The performance of both detection and recognition mechanisms are given in Fig. 5. Note that in the simulation process, the recognition result is observed with a delay of 50 time steps, so as to prevent potential misjudgment of recognition. The simulation results of control situations $j = 2, 3$ are similar thus omitted here.

To examine the effectiveness of the overall control scheme,

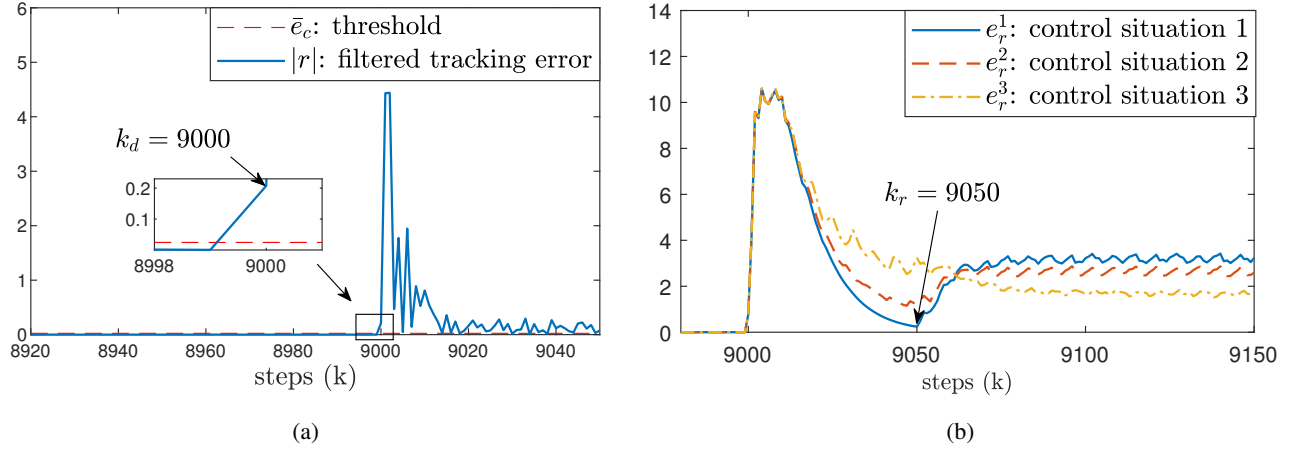


FIGURE 5: Detection and recognition performance under control situation 1 that recurs at time $k_c = 8999$. (a) Detection of the occurrence of control situation 1 at time $k_d = 9000$. (b) Recognition of control situation 1 at time $k_r = 9050$.

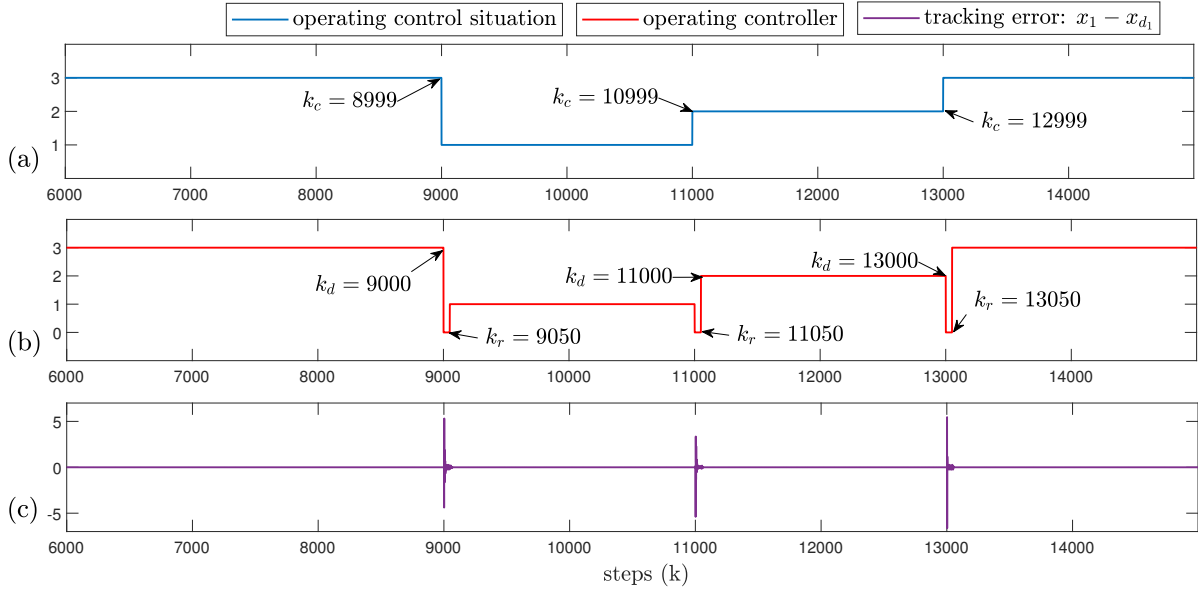


FIGURE 6: Real-time control performance of the overall controller under multiple environments. (a) Operating control situations $j = 1, 2, 3$. (b) Operating controllers with index 0: QSMC in (13); index 1, 2, 3 of EBCs in (9). (c) State tracking error: $x_1 - x_{d1}$.

we assume the operating control situation of system (25) follows a switching sequence: $3 (0 \leq k < 8999) \rightarrow 1 (8999 \leq k < 10999) \rightarrow 2 (10999 \leq k < 12999) \rightarrow 3 (12999 \leq k)$, as shown in Fig. 6a. We first consider the case that the operating control situation is switched from $j = 3$ to $j = 1$ at time $k_c = 8999$. The detection of such a change is achieved at time $k_d = 9000$ as shown in Fig. 5a, and the recognition of control situation $j = 1$ is achieved at time $k_r = 9050$ as in Fig. 5b. Based on the detection and recognition results, the active operating controller at time $k_d + 1 = 9001$ is the QSMC, and switched to the 1-st EBC at time $k_r + 1 = 9051$, see Fig. 6b. Stabilizing tracking control

performance is achieved in Fig. 6c. For the cases that control situation $j = 2$ recurs at time $k_c = 10999$, and $j = 3$ at $k_c = 12999$, the simulation results are similar and also depicted in Fig. 6.

8 Conclusions

In this paper, we have proposed a novel intelligent control scheme for a class of discrete-time nonlinear uncertain systems operating under multiple environments. First, for each individual environment, with a novel adaptive learning control approach, a set of multiple EBCs were designed to achieve stabilizing tracking control performance. In the transition process of multiple environments, a robust QSMC was designed to stabilize the over-

all system. To determine activation among multiple controllers (QSMC or EBCs), a novel detection mechanism (to detect the occurrence of environment changing) and a recognition mechanism (to rapidly recognize the active operating environment) were developed. Extensive simulation studies have been conducted to demonstrate that our approach is able to successfully operate in varying multiple environments with desired tracking control performance guaranteed.

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