Enhancing Conceptual Knowledge in Early Algebra Through Scaffolding Diagrammatic Self-explanation

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Abstract: Many studies have shown that visual representations can enhance student understanding of STEM concepts. However, prior research suggests that visual representations alone are not necessarily effective across a broad range of students. To address this problem, we created a novel, scaffolded form of diagrammatic self-explanation in which students *explain* their problem-solving steps in the form of diagrams. We used contrasting cases to support students' sense-making between algebraic equations and diagrams in the self-explanation activity. We conducted a classroom experiment with 41 students in grades 5 and 6 to test the effectiveness of this strategy when embedded in an Intelligent Tutoring System for algebra. We found that scaffolded diagrammatic self-explanation enhanced conceptual knowledge for students who did not have prior knowledge of formal equation-solving strategies. The study is the first experimental study showing that visual representations can enhance conceptual knowledge in early algebra.

Introduction

Visual representations for learning

Visual representations, such as diagrams and illustrations, can help students understand complex concepts in many STEM domains (Rau, 2017). Visual representations can depict information that is difficult to express directly or through verbal means and they can make important information salient. Many studies have found that visual representations can help foster student learning and/or problem-solving performance (e.g., Hegarty & Kozhevnikov, 1999).

However, it has also been found that the benefits of visual representations are not universal. Some prior studies have found detrimental or mixed effects of visual representations (e.g., Magner, Schwonke, Aleven, Popescu, & Renkl, 2014) and other studies have found that visual representations are not effective for all subgroups of students (e.g., Cooper, Sidney, & Alibali, 2018; Van Garderen & Montague, 2003). Many factors moderate the effects of learning with visual representations, including prior knowledge, ability, and age. Thus, careful design of visual representations and the accompanying instruction may be essential to meaningfully support student interactions with visual representations, especially for younger, lower-achieving students, and those who have lower prior knowledge (Davenport, Klahr, & Koedinger, 2008). Because these groups of students are the ones who need the most instructional support to achieve their learning goals (Clark & Feldon, 2014), it is important that visual representations, as an instructional support, are designed to help these groups of students.

One domain in which visual representations are frequently used is middle-school algebra. As visual representations can capture relational information and highlight deep structure in algebra problems (e.g., Booth & Koedinger, 2012), visual representations may support students' conceptual understanding in algebra. Conceptual knowledge is important because it underpins students' problem-solving skills (Crooks & Alibali, 2014). Despite its importance, however, there is little evidence regarding the effectiveness of visual representations for supporting conceptual knowledge in algebra.

Although no research has directly addressed whether visual representations can help students *learn* conceptual knowledge in algebra, a few studies have tested their effects on problem-solving *performance* across subgroups of students (e.g., Booth & Koedinger, 2012; Chu, Rittle-Johnson, & Fyfe, 2017). Some studies used tape diagrams, a type of visual representation frequently used in countries where mathematics performance is high, including Japan and Singapore (Figure 1) (Murata, 2008). In the aforementioned studies, tape diagrams were presented alongside story problems and algebraic equations, respectively. Both studies assessed problem-solving accuracy with and without diagrams. Their findings show that, across subgroups of students, there were differences in how students made sense of diagrams and in how much they benefited from diagrams, consistent with other studies mentioned above (e.g., Cooper et al., 2018). Booth and Koedinger (2012) observed performance benefits of diagrams only for higher-grade students (7th and 8th, but not 6th-graders). Chu et al. (2017) found no subgroup differences in the effects of diagrams on equation-solving performance, but found that students in regular classes performed significantly worse than students in advanced classes on a diagram translation assessment. In other words, many of the students in the regular classes were not able to make sense of tape diagrams, even though students received instruction on the diagrams at the beginning of the study.

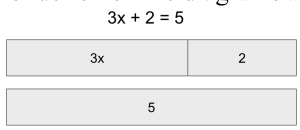


Figure 1. Example tape diagrams.

Self-explanation as sense-making support

The studies discussed above raise the question of how visual representations might support learning of concepts for a wide range of students. We take the perspective that visual representations may need to be designed so students can engage in sense making, such as making connections between the visual representations and the target content or concepts (Booth & Koedinger, 2010; Davenport et al., 2008; Rau, 2017). One approach to integrating sense-making support into the use of visual representations is through self-explanation. Self-explanation is an established learning strategy in which learners attempt to make sense of what they learn by generating explanations to themselves, which helps integrate new information with their own prior knowledge (e.g., Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Rittle-Johnson, Loehr, & Durkin, 2017). Self-explanation has been found to enhance conceptual understanding across many areas in mathematics (Rittle-Johnson et al., 2017). Moreover, prior studies have demonstrated the effectiveness of self-explanation prompts when learning with visual representations (e.g., Aleven & Koedinger, 2002; Rau, Aleven, & Rummel, 2015). These studies used verbal or text-based self-explanation prompts (i.e., explaining visual representations verbally, by typing in a text box or by selecting written explanations from multiple options).

An alternative form of self-explanation, namely, *explaining student's own problem solving in the form of visual representations (diagrammatic self-explanation)*, has not been extensively studied (but see Butcher & Aleven, 2013). Given that visual representations can contain information that is hard to represent in text and can help humans process information efficiently by grouping important and related information together (Larkin & Simon, 1987), it is worth investigating whether explaining through visual representations can foster learning of conceptual knowledge effectively and efficiently. Attempts have been made to integrate visual representations into self-explanation in the form of drawing or sketching, but the evidence as to whether explaining through drawing can enhance learning is mixed (e.g., Scheiter, Schleinschok, & Ainsworth, 2017; Zhang & Fiorella, 2019). Studies have also found that the quality of student-generated drawings is correlated with students' spatial ability, suggesting that generating drawings may not be beneficial for all students. (Zhang & Fiorella, 2019). Because drawing or sketching may increase cognitive load due to the unstructured nature of the prompts (Wu & Rau, 2018), a more scaffolded form of diagrammatic self-explanation may better support students, particularly those who need help the most. Scaffolded, structured verbal self-explanation prompts (e.g., selecting an appropriate explanation from several options) in Intelligent Tutoring Systems (ITSs) have been shown to be effective in enhancing conceptual knowledge in mathematics (Aleven & Koedinger, 2002; Rau et al., 2015; Rittle-Johnson et al., 2017). Therefore, we hypothesize that scaffolded diagrammatic self-explanation in an ITS environment could potentially promote students' conceptual knowledge in algebra.

In the present study, we investigate the effectiveness of scaffolding self-explanation in the form of visual representations for enhancing conceptual knowledge in the domain of algebra. We ask the following research questions: 1) Does scaffolded diagrammatic self-explanation help students gain conceptual knowledge in algebra? 2) Is scaffolded diagrammatic self-explanation effective for students at different grade levels? We hypothesize that scaffolded diagrammatic self-explanation will foster conceptual knowledge and that it will be beneficial for students across the middle grades, due to its scaffolding support.

Method

To investigate the effectiveness of scaffolded diagrammatic self-explanation on enhancing conceptual knowledge in algebra, we conducted a 2 × 2 (Diagram/No-Diagram: diagrammatic self-explanation or no diagrammatic self-explanation, Grade: 5th or 6th grade) between-subjects classroom experiment.

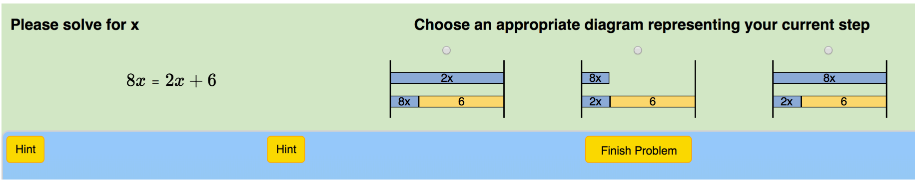
Participants

A total of 45 students participated in the study (19 fifth and 26 sixth graders). Four students in the 6th grade were in Pre-algebra and two were in Algebra I. All others were in their grade-level math class.

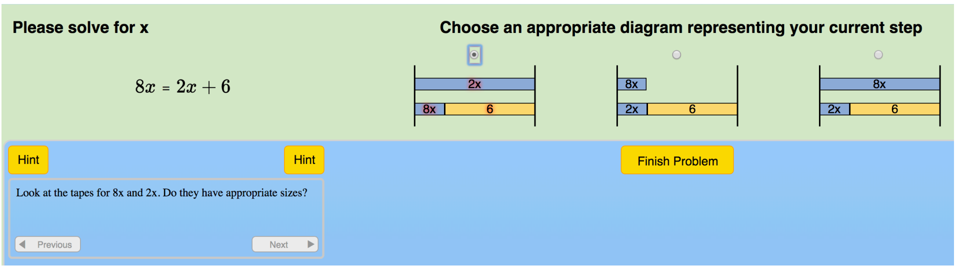
Materials

<u>Equation-solving tutor with scaffolded diagrammatic self-explanation</u>

Figures 2-5 show how we implemented diagrammatic self-explanation in an ITS for equation solving called Lynnette. As is typical of ITSs, Lynnette supports students' problem solving by providing immediate and targeted feedback and adaptive hints (Figure 3). It can recognize a wide range of correct solution steps that students could take. For our study, we created a version of Lynnette in which diagrammatic self-explanation is interleaved with problem solving: students translate their algebraic solution steps into visual representations, namely, tape diagrams, which were designed and tested with students and teachers (Nagashima et al., 2020). To scaffold this strategy, we use contrasting cases (Schwartz, Chase, Oppezo, & Chin, 2011), an instructional strategy in which multiple examples that differ in an important aspect are shown next to each other to help learners notice important features. First, students select an appropriate diagrammatic representation for the given equation from among three contrasting cases presented by the system (Figure 2). They then solve the equation on the left-hand side of the screen (Figures 4, 5). For each of the equation transformations they perform (e.g., subtract 2 from both sides of $3x + 2 = 8$), they are then asked to *explain* the transformation by selecting the appropriate diagrammatic representation from among three options given (Figure 5). Diagram choices are generated automatically based on the equation transformation input that the student provides. These diagram options were designed so that each two diagrams differ in a single conceptual aspect (Schwartz et al., 2011). For example, diagrams in Figure 5 show the action of dividing the equation by 2. The diagrams on the left and in the middle differ in terms of the lengths of tapes on each side in the resulting diagrams (i.e., whether the tape on the top and bottom have the same length) whereas the diagrams on the left and those on the right have the same length but differ in the action performed: division (left) or subtraction (right). Incorrect diagram options model common conceptual mistakes students make, which we found as we worked with middle school mathematics teachers prior to the present study (Nagashima et al., 2020). The features of the diagrams (e.g., color-coding variables and constants differently) were designed based on the pedagogical affordances and constraints of tape diagrams that we identified in prior work (Nagashima et al., 2020). The tutor included 20 equation problems that varied in their complexity (Table 1).



<u>Figure 2</u>. Student selects a diagrammatic representation that corresponds to the given equation.



<u>Figure 3</u>. Feedback message is given when an incorrect attempt is made. The message here says, "Look at the tapes for $8x$ and $2x$. Do they have appropriate sizes?"

<u>Figure 4</u>. Student solves a first step of the given equation.

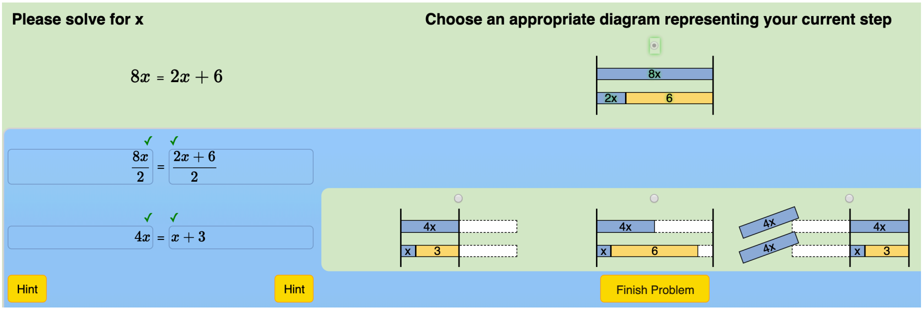


Figure 5. Student *explains* the problem-solving step by selecting an appropriate diagrammatic representation.

Table 1: Types of equations the tutor contained and the number of problems in the tutor

	Equation type	Example	Number of problems in the tutor
Level 1	$x + \mathrm{a} = \mathrm{b}$	$x + 3 = 5$	6
Level 2	$\mathrm{a}x + \mathrm{b} = \mathrm{c}$, $\mathrm{a}x = \mathrm{b}$	$2x + 3 = 7$, $6x = 12$	6
Level 3	$\mathrm{a}x + \mathrm{b} = \mathrm{c}x$	$5x - 2 = 3x$	4
Level 4	$\mathrm{a}x + \mathrm{b} = \mathrm{c}x + \mathrm{d}$	$5x - 2 = 3x + 8$	4

Test instruments

We developed pretest and posttest assessments to assess students' conceptual and procedural knowledge of basic algebra. The test contained several items drawn from prior studies (e.g., Rittle-Johnson, Matthews, Taylor, & McEldoon, 2011) as well as some new items. The conceptual knowledge items consisted of seven multiple-choice questions and four open-ended questions, which assessed a wide range of conceptual knowledge constructs, including equality, like terms, inverse operations, isolating variables, and the concept of keeping both sides of an equation equal. We also included three procedural knowledge items, including two problem-solving items (e.g., "solve for x: $3x + 2 = 8$") and one multiple-choice item. In addition, to assess students' knowledge about tape diagrams, we included two tape diagram items (one multiple-choice, one open-ended). We developed two isomorphic versions of the test that varied only with respect to the specific numbers used in the items; participants received one form as pretest and the other as posttest (with versions counterbalanced across subjects).

Procedure

The study took place during regular mathematics class in the students' classrooms. Students within each grade were randomly assigned to either the Diagram condition or the No-Diagram condition. The advanced students (four students in Pre-algebra and two students in Algebra I among 6^{th} graders) were separately randomly assigned so that each condition would have two Pre-algebra students and one Algebra I student. In the Diagram condition, students practiced with the Lynnette version with scaffolded diagrammatic self-explanation. In the No-Diagram condition, students used a version of the tutor showing the equation-solving part only. Thus, the only difference between the Diagram and No-Diagram conditions was whether students *self-explained* their solution steps in the form of diagrams or not. Per teacher report, the students had never used tape diagrams in class.

All students first worked on the paper-based pretest for 20 minutes. Then a teacher in each of the classrooms passed out two instructional handouts, one about how to use the tutor and the other describing tape diagrams, and read them out loud. Next, students practiced equation solving using their randomly-assigned tutor version using school-provided computers. After 40 minutes of working with the tutor, students took the paper-based posttest for 20 minutes. Students were given access to both tutor versions after the study.

Results

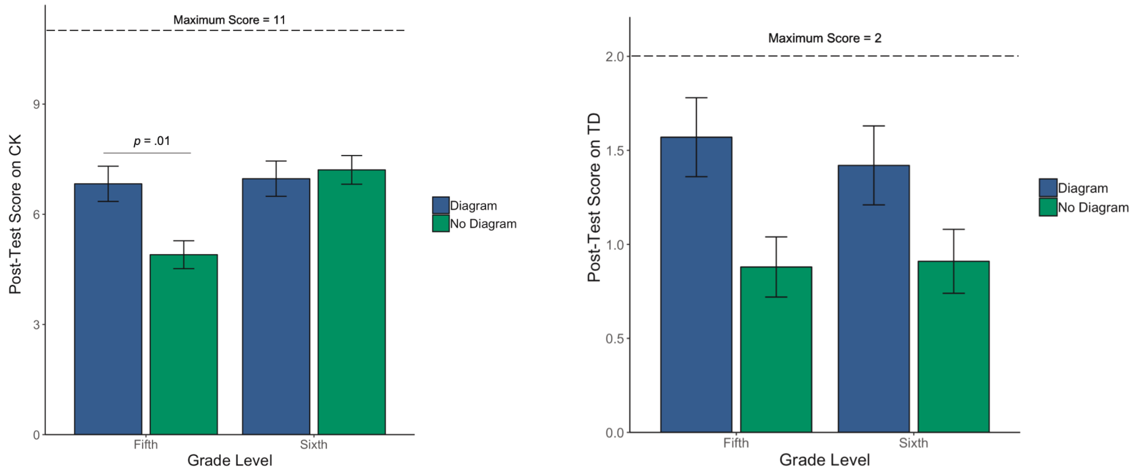
Open-ended items were coded for whether student answers were correct or incorrect by two researchers (*Cohen's kappa* = .83). Data from three 5^{th}-graders and one 6^{th}-grader were excluded because they did not complete the study; therefore, we analyzed data from 41 students, namely, 16 5^{th}-graders (8 Diagram, 8 No-Diagram) and 25 6^{th}-graders (13 Diagram, 12 No-Diagram). Table 2 presents raw pretest and posttest performance on conceptual knowledge (CK), procedural knowledge (PK), and understanding of tape diagrams (TD) items. The maximum scores for CK, PK, TD were 11, 3, and 2, respectively.

Overall pretest performance did not differ either between the diagram conditions, $F(1, 37) = .27$, $p = .60$, or between grades, $F(1, 37) = .94$, $p = .33$; scores on each of the item subsets (CK, PK, TD) also did not differ between conditions or grades. The average total number of tutor problems attempted also did not differ either between diagram conditions (Diagram: $M = 16.14$, $SD = 3.35$, No-Diagram: $M = 17.75$, $SD = 3.61$), $F(1, 37) = .51$, $p = .48$, or grades (5th: $M = 16.13$, $SD = 3.77$, 6th: $M = 17.44$, $SD = 3.34$), $F(1, 37) = .44$, $p = .51$.

To analyze how the conditions may have affected students' posttest performance, we conducted three ANCOVAs, each with the same independent variables (diagram condition and grade) and covariate (pretest, to control for prior knowledge), but with a different dependent variable (CK, PK, or TD). First, we investigated the effect of diagrammatic self-explanation on posttest performance on the conceptual knowledge items (CK). For scores on CK items, we found a main effect of diagrammatic self-explanation, $F(1, 36) = 8.01$, $p < .01$, *partial* $\eta^2 = .06$, and a significant interaction between diagram condition and grade, $F(1, 36) = 6.18$, $p = .02$, *partial* $\eta^2 = .15$. A post-hoc analysis revealed that 5th-graders benefited from diagrammatic self-explanation ($F[1, 13] = 8.31$, $p = .01$, *partial* $\eta^2 = .39$), whereas no difference between the Diagram and No-Diagram conditions was found for 6th-graders (Figure 6, panel a). No significant main effects nor interactions were found for students' performance on PK items. For TD items, there was a main effect in favor of diagrammatic self-explanation, $F(1, 36) = 5.58$, $p = .02$, *partial* $\eta^2 = .22$, but no main effect of grade nor any interaction (Figure 6, panel b).

Table 2: Means and standard deviations (in parenthesis) for CK, PK, and TD on the pretest and posttest

	CK (maximum score: 11)		PK (maximum score: 3)		TD (maximum score: 2)	
	pretest	posttest	pretest	posttest	pretest	posttest
5th Diagram	4.75 (1.28)	6.38 (1.85)	2.13 (.99)	2.00 (.93)	0.88 (.35)	1.50 (.54)
5th No-Diagram	5.38 (2.07)	4.88 (1.89)	2.38 (.74)	2.50 (1.07)	0.75 (.46)	0.88 (.64)
6th Diagram	6.46 (2.15)	7.23 (2.09)	1.62 (.96)	2.08 (.96)	0.92 (.28)	1.46 (.52)
6th No-Diagram	6.50 (2.71)	7.25 (2.42)	1.33 (.98)	1.67 (1.23)	0.83 (.39)	0.92 (.80)



(a) (b)

Figure 6. Adjusted means for CK (a) and TD (b) on the posttest (error bars show standard errors).

To further investigate differences between 5th- and 6th-graders, we examined the strategies that students used to solve the procedural items on the pretest and posttest. We expected that students in Grade 5 and 6 would have had different levels of exposure to equation solving prior to the study. We coded the strategies by adapting the coding scheme used by Chu et al. (2017) and Koedinger, Alibali, and Nathan (2008), which includes both formal (algebraic) and informal (non-algebraic) ways of solving equations (Table 3; *Cohen's kappa* = .82). We performed the strategy coding independent of the correctness coding. We were primarily interested in the Algebra strategy because the goal of our tutor (and of the equation-solving instruction in the school where the study took place) was to help students learn the formal algebraic strategy. For the two problem-solving items on the pretest, there was a significant difference between the two grades, but not between the diagram conditions, in use of the Algebra strategy. Whereas 80% of 6th-graders used the Algebra strategy at least once, only 6% of 5th-graders used it ($\chi^2[1, n = 41] = 21.24$, $p < .01$) (Figure 7). The difference shrank dramatically at posttest, but remained significant: 88% of 6th-graders used Algebra strategy, whereas 56% of 5th-graders used it ($\chi^2[1, n = 41] = 5.33$, $p = .02$). We used McNemar's test to compare the frequency of Algebra strategy use at pretest and posttest. Fifth-grade students increased their use of the Algebra strategy significantly from pretest to posttest ($p = .01$), but 6th-grade students did not ($p = .72$) (Figure 7). These findings suggest that equation-solving practice with the ITS helped the 5th-graders transition from informal to formal strategy use. This shift is masked in the raw pretest to posttest accuracy data on equation-solving items, but it is visible in the strategy choices that students made.

Table 3: Strategies used to solve equations, adapted from Chu et al. (2017) and Koedinger et al. (2008)

Strategy name	Description	Example answer for $3x + 2 = 8$
Algebra	Student uses algebraic manipulations to find an answer	$3x = 6$ $x = 6/3 = 2$
Unwind	Student works backward using inverse operations to find an answer	$8 – 2 = 6$ $6/3 = 2$
Guess and Check	Student tests potential solutions by substituting different values	$3*2 + 2 = 8$ $6 + 2 = 8$
Other	Student uses other non-algebraic strategies	$3 + 2 = 5$ $8/5 = 1.6$
Answer Only	Student provides an answer without showing any written work	$x = 2$
No Attempt	Student leaves problem blank or explicitly indicates that she/he does not know how to solve the problem	"I don't know"

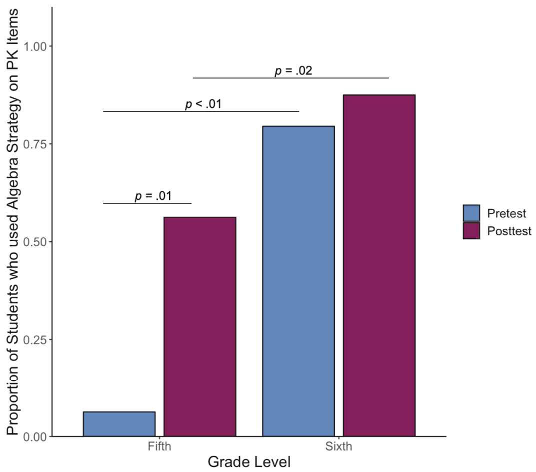


Figure 7. Proportions of students who used the Algebra strategy at least once on the problem-solving items.

Discussion and conclusion

We investigated whether scaffolded diagrammatic self-explanation, embedded in an intelligent tutor, can enhance the learning of conceptual knowledge in early algebra. We combined prompts for diagrammatic self-explanation with hints and feedback. The prompts showed contrasting cases designed to support connection-making between diagrams and equations. The results of our classroom experiment partially supported our hypotheses: Scaffolded diagrammatic self-explanation supported student learning of conceptual knowledge, but only among lower-grade students. The effect of diagrammatic self-explanation on learning of conceptual knowledge was observed independent of students' prior knowledge, as measured by pretest scores. Also, the post-test performance on TD items showed that diagrammatic self-explanation helped students, regardless of their grade or prior knowledge, to correctly understand the tape diagram representation. These findings differ from those reported in prior literature on using diagrams in algebra, which showed increased performance with diagrams and correct understanding of diagrammatic representations only among higher-grade or higher-ability students (e.g., Booth & Koedinger, 2012; Chu et al., 2017). Regarding students' procedural knowledge, however, we did not find that diagrammatic self-explanation helped students solve problems more accurately. To the best of our knowledge, the present study is the first experimental study showing that diagrams can be used to enhance learning of conceptual knowledge in early algebra.

Furthermore, scaffolded diagrammatic self-explanation efficiently supported learning. Despite the additional steps and cognitive processing required for translating between two representations (i.e., algebraic equations and diagrams), the average number of tutor problems attempted by students in the Diagram condition did not differ reliably from the number attempted by students in the No-Diagram condition (with practice time kept constant between conditions). We attribute the effectiveness and efficiency of diagrammatic self-explanation for lower-grade students to the scaffolding support that we designed. Specifically, scaffolding students' connection-making between algebraic equations and diagrammatic representations through contrasting cases, together with targeted hints and feedback available in the intelligent tutor, may have reduced cognitive load in translating between those two types of representations. Our results show that this type of scaffolding support helped lower-grade students successfully understand tape diagram representations.

Further investigation suggested that scaffolded diagrammatic self-explanation is useful when students are transitioning from informal to formal algebraic strategies in equation solving. Overall pretest performance did not differ between 5^{th}- and 6^{th}-graders, but a detailed coding of students' problem-solving strategies revealed that 5^{th}-graders initially relied on informal strategies to solve equations, whereas 6^{th}-graders relied mostly on algebraic strategies. A follow-up interview with a teacher from the school also confirmed that, prior to the study, the 5^{th}-graders had only been exposed to *arithmetic* equation solving whereas 6^{th}-graders had consistently been required to use the algebraic strategy to solve equations. In 5^{th} grade, students' knowledge and understanding of the algebraic notation and the algebraic operations is brittle; the diagrams (which may be more visually intuitive than the algebraic notation) may aid sense-making and for that reason, students may attend to them more so than when their understanding of the algebraic notation and operations is further developed. Also, the diagrammatic self-explanation prompts highlight important conceptual aspects by means of correct and incorrect diagrams that modeled algebraic strategies (e.g., incorrect diagrams: subtracting 2 only from the left-hand side of $3x + 2 = 8$). This information may help students come to understand the meaning of algebraic manipulations, as we saw for the 5^{th}-graders in the Diagram condition. It may be that contrasting cases are particularly helpful in this regard. Further, as Sidney, Hattikudur, and Alibali (2015) argue, contrasting cases may be particularly effective in illustrating differences in novel knowledge, but may not be effective in supporting the revision of knowledge with which students are already familiar. This reasoning might also explain why diagrammatic self-explanation did not help 6^{th}-grade students acquire stronger conceptual knowledge.

Our study has several limitations. First, the study was conducted with one specific type of diagrams, tape diagrams, and on one specific topic, equation solving in algebra, and the sample size was small. To understand how the results could generalize, more research is needed to examine the effects of scaffolded diagrammatic self-explanation with various types of visual representations across topics and domains, and with more students. Also, the fact that the diagrams did not help 6^{th}-graders, together with prior studies on diagram use in algebra, suggests that it may be challenging to find a way to use diagrams that is helpful for the full range of algebra learners (Koedinger & Aleven, 2007; Zhang & Fiorella, 2019). Future studies should explore various forms of scaffolds and should consider how different scaffolds might be used to best support learning with visual representations for students with different characteristics.

In summary, our results suggest that scaffolded diagrammatic self-explanation helps students acquire conceptual knowledge of algebra in the early stages of their learning, as the students are transitioning from informal (non-algebraic) to formal (algebraic) ways of solving. Our findings also contribute to theoretical understanding of how students use visual representations. Specifically, our findings highlight that appropriate connection-making scaffolds that are not cognitively demanding can help students who have little knowledge in the domain. Practically, such scaffolding support, embedded in a learning technology, could effectively help students understand algebra conceptually before they are expected to solve many equations. Moreover, the fact that the diagrams helped 5^{th}-graders indicates that we succeeded in finding a way to use diagrams to help learners with low prior knowledge.

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