

## WHERE TO START? THIRD GRADERS' MEASUREMENT CRITIQUES

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When measuring with nonstandard, discrete units, some early elementary students will leave gaps between the units (Clements & Sarama, 2009). When measuring with rulers, some students report the number at the end of the object as its length regardless of the starting position of the object (Bragg & Outhred, 2004; Nunes, Light, & Mason, 1993), interpret the numbers on the ruler as points as opposed to accumulating units, and count from one instead of zero (Bragg & Outhred, 2004; Clements & Sarama, 2009). One method that can illuminate students' conceptions about mathematics problems and potentially help them confront their confusions is analyzing incorrect worked examples (e.g., Booth, Lange, Koedinger, & Newton, 2013; Durkin & Rittle-Johnson, 2012). Therefore, in this study, we investigated the following research questions: When analyzing worked examples of incorrect measurements, what did third graders identify as incorrect? How did this relate to their ability to identify the correct measurements?

Participants included 37 third graders who saw pictures of items incorrectly measured with broken rulers (or cubes) and were asked to identify and correct the mistakes: (#1): egg spanning 4 to 7 on the ruler, incorrect answer: 7 units; (#2): screwdriver spanning 2 to 4, incorrect answer: 4 units; (#3): screwdriver spanning 1 to 9, incorrect answer: 9 units; (#4): screwdriver spanning the edge of ruler to 4.5, incorrect answer: 4.5 units; (#5): screwdriver measured with four blocks with gaps, incorrect answer: four blocks long. The starting points of the objects on the rulers were varied to probe students' conceptions of where to start measuring. We recorded the mistakes they identified and their reasoning for what the measurements should be.

Students' performance in identifying the length of the objects improved as the starting point of the object being measured was positioned closer to zero on the ruler. Students who thought measurements should start at one on the ruler thought the answer for (#2) would be three units, because if the screwdriver had started at one on the ruler, it would end at three. However, students who originally said the object should start at *one* or *the edge* (which never aligned with zero) when measuring, changed their answer when they saw a screwdriver positioned at one; in these cases, they said it should be positioned at zero. Surprisingly, 54% of students, many of whom said the screwdriver positioned at one should start at zero in the previous problem (#3), thought the screwdriver in (#4) correctly showed a measurement of 4.5 units. With the blocks, only 54% of the students were able to correctly determine that the screwdriver would be longer than four blocks once the gaps between the four blocks were removed; six students said that after the gap or alignment problem was fixed, the screwdriver would still be four blocks long, and surprisingly, four students said it would be less than four blocks long! Overall, students' conceptions of zero seem tied to the ruler's edge even when it was not marked at the edge.

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