

## Examining Links Between Arguments and Representations in Pre-Service Teachers' Pedagogical Content Knowledge

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**Abstract:** We examine how the intertwined relationship between arguments and representations can provide opportunities for investigating pedagogical content knowledge (PCK). Our context is pre-service elementary teachers who are engaged in discussions of open-ended hypothetical math and science teaching scenarios that were designed to elicit PCK in particular content areas. Using verbal data from their discussions, we present a mini-theory about this intertwined relationship and then pose new discussion questions pertaining to the relationship between arguments and representations in teacher education.

### Pedagogical content knowledge related to arguments and representations

In this project our goal is to apply existing results from research in the Learning Sciences that focuses on the practices of argumentation and representations to investigate pre-service teachers' (PSTs) pedagogical content knowledge (PCK). We are working to build a theoretical understanding of the intertwined relationship between arguments and representations to highlight new directions for examining pre-service teachers' knowledge and teacher education.

A significant body of work over the past three decades has examined teachers' specialized knowledge for teaching, known as PCK. This specialized knowledge includes not only traditional content knowledge, but also knowledge of specific instructional practices and knowledge of student thinking in particular content areas (e.g. Ball, Thames, & Phelps, 2008). Education researchers have worked to identify, assess, and enhance the development of this knowledge, often through various interventions with in- and pre-service teachers while also working to identify or assess those individuals' PCK. While some tools for assessing PCK emphasize the union of the content and pedagogy, there is a growing focus on disciplinary practices, such as argumentation, across mathematics and science. This focus on PCK pertaining to practices builds on recent policy documents and standards emphasizing these practices in addition to traditional content (e.g. National Governors Association, 2010; Next Generation Science Standards, 2013). This trend has contributed to the uptake of practices in teacher education, but challenges still persist. For instance, research on argumentation has found that despite high quality professional development, science teachers struggle with applying argumentation components to classrooms (McNeill & Knight, 2013). Despite a growing focus in this area, there still persists a limited body of research on PCK for scientific practices compared to the body of research on PCK for disciplinary core ideas (Osborne, 2014).

Here we examine PCK that involves both content and practices by investigating teachers' productive knowledge resources at the nexus of two intertwined practices, arguments and representations, that often occur in tandem in classroom instruction. We build on research that has begun exploring the complex blending of arguments and representations with pre-service teachers (Namder & Shen, 2016), as well as work that highlights the role of learners' productive knowledge resources in future learning (e.g. Smith, diSessa, & Roschelle, 1994). Existing research has documented students' productive knowledge resources about representations (e.g. diSessa, 2004) and disciplinary argumentation (Engle & Conant, 2002), which in turn suggests potential for this line of inquiry. Finally, this connection has power for potentially crossing the traditional math/science divide, which is especially important for elementary teachers who are generalists.

To operationalize this approach, we view representations as visual two- or three-dimensional displays that can involve inscriptions, manipulatives, or other objects. We build on literature that recognizes the generative nature of joint representations in collaboratively building an understanding of the external world (e.g. Nussbaum, 2008). The importance of not only generating but also critiquing, comparing, connecting, and using multiple representations of concepts and relationships is apparent in both mathematics and science learning (e.g., Ainsworth, 1999; Bowen, Roth & McGinn, 1999). Although there are different perspectives on arguments, we view arguments as lines of reasoning that typically involve claims supported by evidence for the purposes of making sense of an idea and that can also involve critique of others' reasoning. Constructing and critiquing arguments is central to supporting mathematics and science learning (Ball & Bass, 2003; Driver, Newton & Osborne, 2000). In studies of elementary mathematics and science instruction, representations are important for students' sensemaking (Lehrer & Schauble, 2004) and for student's explaining their reasoning to peers (e.g., Cobb

et al., 1991). Knowing the importance of both arguments and representations, our goal is to examine the intertwined relationship between arguments and representations in a relatively new setting, pre-service teacher education. We examine how this relationship interacts with aspects of PCK, thereby highlighting new ways for examining PSTs' knowledge.

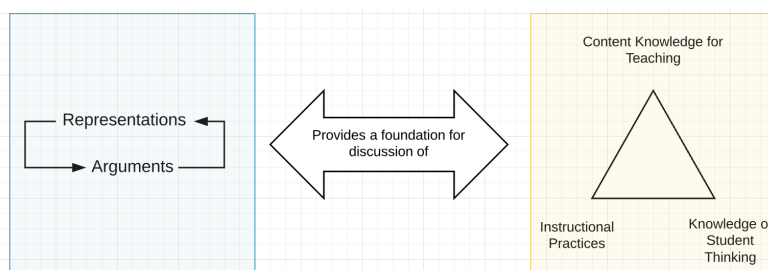
## Research setting

The setting for this work is an ongoing design-based research project (e.g. Design-Based Research Collective, 2003) that has been aiming to build coherence between mathematics and science content and methods classes for PSTs. The pillars for that coherence building have been arguments and representations as they cut across courses. We started with initial ideas and questions about how these pillars might drive course modifications and, in turn, how PSTs might conceptualize the relationship between arguments and representations. Building on that initial understanding, we are taking a progressive refinement of hypothesis approach to develop increasingly specific hypotheses about these relationships based on our data (Engle, Conant, & Greeno, 2007).

Data informing this study comes from PSTs in our newly modified elementary math content courses who are engaged in small group discussions of hypothetical teaching scenarios. These hypothetical teaching scenarios were designed to elicit pre-service teachers' PCK in the context of real-life problems of practice in mathematics or science that also involve arguments and representations. These scenarios depict elementary teachers and their students engaged in mathematics or science lessons in which the students struggle with aspects of content related to both arguments and representations. The scenarios loosely exist within a long data collection tradition in teacher education research of using what has gone by a variety of names, including case-based scenarios, vignettes, and contrasting activities (e.g. Derry, Wilsman, & Hackbarth, 2007). We iteratively developed a series of these scenarios that pertain to topics in elementary math and science education (e.g. subtraction, fractions, ecosystems, weight and volume).

We collected audio data from 44 PSTs discussing the scenarios in 15 small groups that were comprised of 2-5 individuals in math content courses over two semesters. Thus, our unit of analysis is the small group conversation. After data collection we engaged in iterative qualitative analysis that involved developing codes related to PCK, arguments, and representations. In developing these codes, we considered nuanced differences across the scenarios and various ways in which argument and representations could be used in the PSTs' discussions; for example, we recognized differences between generating and critiquing a representation. We identified excerpts in the discussions where the PSTs were focused on only arguments or representations and instances where both practices were utilized. For excerpts that involved both, we examined the potential relationship between how the arguments and representations were used, which resulted in refinement of our initial ideas. Our mini-theory (Figure 1) focuses on the intertwined relationship between arguments and representations and how this can provide opportunities for examining PCK in this setting. Importantly, this mini-theory is a localized theory of this relationship. Although theory sometimes get limited attention in educational research, theory is important in dealing with foundational problems (Lester, 2005), and small sets of interconnected ideas that aim to be coherent can also become tractable in ways that will increase the likelihood of future use. An eventual outcome of the mini-theory may be a more directed means for developing instruction aiming to bolster pre- and in-service teachers' PCK through an emphasis on arguments and representations.

## Mini-theory about the relationship between arguments, representations, and PCK



**Figure 1.** The schematized relationship for how arguments and representations can provide a foundation for discussion of pre-service teachers' math or science content knowledge for teaching, instructional practices, and knowledge of student thinking.

We view there to be an intertwined relationship between arguments and representations, which in turn can provide a foundation for discussion of PCK, including instructional practices, student thinking, and relevant mathematical

or scientific content knowledge (Figure 1). In this relationship, one might generate a new representation in response to an argument, or one can generate an argument based on an existing representation. Either way, this relationship can provide a foundation for discussion and reflection on instructional practices, student thinking, or the mathematical or scientific content. Through that discussion and reflection, we can potentially gain insights into one's PCK. Furthermore, the other direction is possible. Discussion about instructional practices, student thinking, or relevant content can provide a foundation for generating a new representation or argument based on an existing one.

To illustrate this developing mini-theory we present an example transcript excerpt in which three PSTs discuss one of the hypothetical teaching scenarios. In this scenario, a hypothetical class of 3rd graders (8-9 yrs. old) is doing an investigation that involves measuring the weight and volume of a series of hollow and solid spheres and cones (*representations*). The hypothetical students are asked to predict what will happen when a hollow sphere is placed in a tub of water. One student (Clayton) thinks it will float, while another student (Aiden) argues (incorrectly) that the object will sink ("Teacher: What do you think will happen to Object B [the large hollow plastic sphere] when it is placed in the water? Clayton: I think it will float cause it's clear. Aiden: No, it's gonna sink cause it's big and the small ball is gonna float.") Then, the PSTs were told: "Imagine that you are the teacher. Given the two student responses, how would you continue the conversation given the previously stated goals?" The stated goals focused on having the class produce data that can be evidence in a scientific argument and listening and responding to student arguments. Thus, in designing the scenario, the intention was that the PSTs had to simultaneously draw on their existing knowledge resources of arguments, representations, and PCK.

One group of PSTs, when discussing this scenario, generated a new representation in response to one of the hypothetical student's arguments to support that student's ongoing understanding:

- PST 1: (*Quoting Clayton*) I think it will float cause it's clear. (*Slightly raised voice, as though directly responding to Clayton*) Clayton, why do you think it's gonna float...why do you think it's.. be.. uh, do you think it'd float it wasn't clear? (*Laughs*)
- PST 2: (*Acting as Clayton*) No. (*Acting as themselves now*) Then we could find another object that's clear too. That's heavier. [PST 1: Yeah] (PST 1 laughing a little) = and put it in the water, and like, oh just kidding. (*soft laughing from PST 1*) (.7) No.
- PST 1: The kicker is that that they're filled with air, so like, the ones that I think that would be.. hollow, I think they would float. Just cause they're hollow. Like a boat. [PST 2 & PST 3: Yeah. Yeah]
- PST 2: [Yeah, I think (.05) what he could be, (.3) like, um, starting to get to, is like clear equals hollow. [PST 1: Oh, yeah.] And he might not be able to articulate that, but, but because the ball is clear, he knows nothing's inside of it. [PST 2: Right.] =It's hollow. So that [So that--
- PST 1: [So we could say, what if it was hollow, but it wasn't clear? Like, would it sink? [PST 3: Yeah] Would it still float?
- PST 3: Yeah, yeah, that's a great follow-up question. Yeah.

At first the PSTs generate a new representation about a heavy clear object that sinks. This suggestion aims to directly counter the student's argument that clear objects float, and is likely based in their deduction that he might be thinking that clear equals hollow and floating. The PSTs suggest asking a follow-up question (instructional strategy) about the representation that directly addresses the student's current thinking. Then another PST connects that idea with the student's possible understanding of this new object being "like a boat." The PSTs recognize that the student might not be able to articulate his or her idea well, but likely has prior understandings about hollow objects. At this point the three PSTs have recognized the value in this hypothetical student's thinking, and then one of them suggests a follow-up question that directly targets this underlying issue of assuming that clear objects float. They suggest asking about a hollow object that isn't clear, likely one that would float and thus would challenge the student's thinking. While the impact of this instructional approach is unknown, the important point is that the PSTs are building from students' thinking in their instruction. To summarize, the three PSTs generated a representation to use in response to a student's argument that in turn aims to support the development of a new argument that can shift the student's understanding towards the normative physics.

From the perspective of our developing mini-theory, the scenario included a visual representation (of spheres and cones), and the hypothetical student used that representation to generate an initial (incorrect) argument. The PSTs used that argument as a foundation, along with their own existing knowledge of this science

content and their knowledge of student thinking to value that hypothetical student's thinking (the incorrect argument) and generate a new representation and new questions (an instructional strategy) that they thought would have the direct effect of nudging the student's thinking in the correct direction. Here we see an interplay between the representation and the argument as the PSTs generated a new representation in response to the student's argument. There is also an interplay between those two practices and the PSTs' specific PCK in this instance that pertained to their knowledge of student thinking, content knowledge in physics, and knowledge of relevant instruction. All these dimensions worked as their reasoning process played out in this short discussion.

## Discussion

The development of this mini-theory and our use of empirical data to illustrate its potential allows us to now ask further questions about these relationships. Our empirical data showed a case that started from a visual representation that contributed to an argument, both of which were used to launch a discussion that involved PCK. However, other relationships are possible; for instance, an interaction might begin with a stated argument and then incorporate a representation to launch further discussions. Additionally, there might be cases in which the PSTs have difficulties accessing relevant knowledge about arguments, representations, or student thinking. Although we illustrated this mini-theory with data from elementary PSTs, relationships might play out differently with in-service teachers or secondary teachers who have different knowledge bases. Finally, this work opens the door to more broadly investigate other potential relationships between arguments and representations and teachers' (and students') understanding of those relationships, especially across various learning environments, as representations and arguments can serve a variety of purposes within instruction.

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