
PC-Fairness: A Unified Framework for Measuring Causality-based Fairness

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Abstract

Fair machine learning studies how to construct predictive machine learning models such that decisions made with their assistance fairly treat all groups of people. A recent trend in this field is to define fairness as causality-based notions which concern the causal connection between protected attributes and decisions. However, one common challenge of all causality-based fairness notions is identifiability, i.e., whether they can be uniquely measured from observational data, which is a critical barrier to applying these notions to real-world situations. In this paper, we develop a framework for measuring different causality-based fairness. We propose a unified definition that covers most of previous causality-based fairness notions, namely the path-specific counterfactual fairness (PC fairness). Based on that, we propose a general method in the form of a constrained optimization problem for bounding PC fairness under all unidentifiable situations. Experiments on synthetic and real-world datasets show the correctness and effectiveness of our method.

1 Introduction

Fair machine learning is now an important research field which studies how to develop predictive machine learning models such that decisions made with their assistance fairly treat all groups of people irrespective of their protected attributes such as gender, race, etc. A recent trend in this field is to define fairness as causality-based notions which concern the causal connection between protected attributes and decisions. Based on Pearl’s structural causal models [8], a number of causality-based fairness notions have been proposed for capturing fairness in different situations, including total effect [19, 16, 20], direct/indirect discrimination [19, 16, 7, 20], and counterfactual fairness [5, 14, 15, 9].

One common challenge of all causality-based fairness notions is identifiability, i.e., whether they can be uniquely measured from observational data. As causality-based fairness notions are defined based on different types of causal effects, such as total effect on interventions, direct/indirect discrimination on path-specific effects, and counterfactual fairness on counterfactual effects, their identifiability depends on the identifiability of these causal effects. Unfortunately, in many situations these causal effects are in general unidentifiable, referred to as unidentifiable situations [12]. Identifiability is a critical barrier for the causality-based fairness to be applied to real applications. In previous works, simplifying assumptions are proposed to evade this problem [5, 19, 4]. However, these simplifications may severely damage the performance of predictive models. In [20] the authors propose a method to bound indirect discrimination as the path-specific effect in unidentifiable situations, and in [14] a method is proposed to bound counterfactual fairness. Nevertheless, the tightness of these methods is

not analyzed. In addition, it is not clear whether these methods can be applied to other unidentifiable situations, and more importantly, a combination of multiple unidentifiable situations.

In this paper, we propose a framework for handling different causality-based fairness notions. We first propose a general representation of all types of causal effects, i.e., the path-specific counterfactual effect, based on which we define a unified fairness notion that covers most previous causality-based fairness notions, namely the path-specific counterfactual fairness (PC fairness). We summarize all unidentifiable situations that are discovered in the causal inference literature. Then, we develop a constrained optimization problem for bounding the PC fairness, which is motivated by the method proposed in [2] for bounding confounded causal effects. The key idea is to parameterize the causal model using so-called response-function variables, whose distribution captures all randomness encoded in the causal model, so that we can explicitly traverse all possible causal models to find the tightest possible bounds. In the experiments, we evaluate the proposed method and compare it with previous bounding methods using both synthetic and real-world datasets. The results show that our method is capable of bounding causal effects under any unidentifiable situation or combinations. When only path-specific effect or counterfactual effect is considered, our method provides tighter bounds than methods in [20] or [14].

The proposed framework settles a general theoretical foundation for causality-based fairness. We make no assumption about the hidden confounders so that hidden confounders are allowed to exist in the causal model. We also make no assumption about the data generating process and whether the observation data is generated by linear or non-linear functions would not introduce bias into our results. We only assume that the causal graph is given, which is a common assumption in structural causal models. One limitation of the framework is that currently it can only deal with discrete variables with finite domains. We will extend the framework to continuous variables with infinite domains in the future work.

2 Preliminaries

In our notations, an uppercase denotes a variable, e.g., X ; a bold uppercase denotes a set of variables, e.g., $\mathbf{X}$; and a lowercase denotes a value or a set of values of the variables, e.g., x and $\mathbf{x}$.

2.1 Causal Model and Causal Graph

Definition 1 (Structural Causal Model [8]). *A structural causal model $\mathcal{M}$ is represented by a quadruple $\langle \mathbf{U}, \mathbf{V}, \mathbf{F}, P(\mathbf{U}) \rangle$ where*

1. $\mathbf{U}$ is a set of exogenous variables that are determined by factors outside the model.
2. $P(\mathbf{U})$ is a joint probability distribution defined over $\mathbf{U}$.
3. $\mathbf{V}$ is a set of endogenous variables that are determined by variables in $\mathbf{U} \cup \mathbf{V}$.
4. $\mathbf{F}$ is a set of structural equations from $\mathbf{U} \cup \mathbf{V}$ to $\mathbf{V}$. Specifically, for each $V \in \mathbf{V}$, there is a function $f_V \in \mathbf{F}$ mapping from $\mathbf{U} \cup (\mathbf{V} \setminus V)$ to V , i.e., $v = f_V(\mathbf{pa}_V, u_V)$, where $\mathbf{pa}_V$ is a realization of a set of endogenous variables $\mathbf{PA}_V \in \mathbf{V} \setminus V$ that directly determines V , and u_V is a realization of a set of exogenous variables that directly determines V .

In general, $f_V(\cdot)$ can be an equation of any type. In some cases, people may assume that $f_V(\cdot)$ is of a specific type, e.g., the nonlinear additive function if $v = f_V(\mathbf{pa}_V) + u_V$. On the other hand, if all exogenous variables in $\mathbf{U}$ are assumed to be mutually independent, then the causal model is called a *Markovian model*; otherwise, it is called a *semi-Markovian model*. In this paper, we don't make assumptions about the type of equations and independence relationships among exogenous variables.

The causal model $\mathcal{M}$ is associated with a causal graph $\mathcal{G} = \langle \mathcal{V}, \mathcal{E} \rangle$ where $\mathcal{V}$ is a set of nodes and $\mathcal{E}$ is a set of edges. Each node of $\mathcal{V}$ corresponds to a variable of $\mathbf{V}$ in $\mathcal{M}$. Each edge in $\mathcal{E}$, denoted by a directed arrow $\rightarrow$, points from a node $X \in \mathbf{U} \cup \mathbf{V}$ to a different node $Y \in \mathbf{V}$ if f_Y uses values of X as input. A *causal path* from X to Y is a directed path which traces arrows directed from X to Y . The causal graph is usually simplified by removing all exogenous variables from the graph. In a Markovian model, exogenous variables can be directly removed without loss of information. In a semi-Markovian model, after removing exogenous variables we also need to add dashed bi-directed

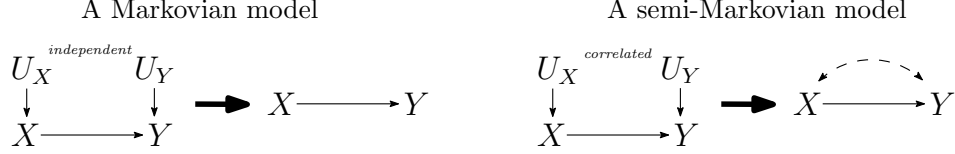


Figure 1: Causal graphs of a Markovian model and a semi-Markovian models

edges between the children of correlated exogenous variables to indicate the existence of unobserved common cause factors, i.e., hidden confounders. Examples are demonstrated in Figure 1.

2.2 Causal Effects

Quantitatively measuring causal effects in the causal model is facilitated with the *do*-operator [8] which forces some variable X to take certain value x , formally denoted by $do(X = x)$ or $do(x)$. In a causal model $\mathcal{M}$, the intervention $do(x)$ is defined as the substitution of structural equation $X = f_X(\text{PA}_X, U_X)$ with $X = x$. For an observed variable Y ($Y \neq X$) which is affected by the intervention, its interventional variant is denoted by Y_x . The distribution of Y_x , also referred to as the post-intervention distribution of Y under $do(x)$, is denoted by $P(Y_x = y)$ or simply $P(y_x)$.

By using the *do*-operator, the total causal effect is defined as follows.

Definition 2 (Total Causal Effect [8]). *The total causal effect of the value change of X from x_0 to x_1 on $Y = y$ is given by*

$$\text{TCE}(x_1, x_0) = P(y_{x_1}) - P(y_{x_0}).$$

The total causal effect is defined as the effect of X on Y where the intervention is transferred along all causal paths from X to Y . If we force the intervention to be transferred only along a subset of all causal paths from X to Y , the causal effect is then called the path-specific effect, defined as follows.

Definition 3 (Path-specific Effect [1]). *Given a causal path set π , the π -specific effect of the value change of X from x_0 to x_1 on $Y = y$ through π (with reference x_0) is given by*

$$\text{PE}_\pi(x_1, x_0) = P(y_{x_1|\pi, x_0|\bar{\pi}}) - P(y_{x_0}),$$

where $P(y_{x_1|\pi, x_0|\bar{\pi}})$ represents the post-intervention distribution of Y where the effect of intervention $do(x_1)$ is transmitted only along π while the effect of reference intervention $do(x_0)$ is transmitted along the other paths.

Definition 2 and 3 consider the average causal effect over the entire population without any prior observations. If we have certain observations about a subset of attributes $\mathbf{O} = \mathbf{o}$ and use them as conditions when inferring the causal effect, then the causal inference problem becomes a *counterfactual inference* problem meaning that the causal inference is performed on the sub-population specified by $\mathbf{O} = \mathbf{o}$ only. Symbolically, the distribution of Y_x conditioning on factual observation $\mathbf{O} = \mathbf{o}$ is denoted by $P(y_x|\mathbf{o})$. The counterfactual effect is defined as follows.

Definition 4 (Counterfactual Effect [12]). *Given a factual condition $\mathbf{O} = \mathbf{o}$, the counterfactual effect of the value change of X from x_0 to x_1 on $Y = y$ is given by*

$$\text{CE}(x_1, x_0|\mathbf{o}) = P(y_{x_1}|\mathbf{o}) - P(y_{x_0}|\mathbf{o}).$$

3 Path-specific Counterfactual Fairness

In this section, we define a unified fairness notion for representing different causality-based fairness notions. The key component of our notion is a general representation of causal effects. Consider an intervention on X which is transmitted along a subset of causal paths π to Y , conditioning on observation $\mathbf{O} = \mathbf{o}$. Based on that, we define path-specific counterfactual effect as follows.

Definition 5 (Path-specific Counterfactual Effect ¹). *Given a factual condition $\mathbf{O} = \mathbf{o}$ and a causal path set π , the path-specific counterfactual effect of the value change of X from x_0 to x_1 on $Y = y$ through π (with reference x_0) is given by*

$$\text{PCE}_\pi(x_1, x_0|\mathbf{o}) = P(y_{x_1|\pi, x_0|\bar{\pi}}|\mathbf{o}) - P(y_{x_0}|\mathbf{o}).$$

¹In [6], the conditional path-specific effect is different from our notion in that, for the former the condition is on the post-intervention distribution, and for the latter, the condition is on the pre-intervention distribution.

Description	References	Relating to PC fairness
Total effect	[19, 16]	$\mathbf{O} = \emptyset$ and $\pi = \Pi$
(System) Direct discrimination	[19, 7, 16, 3]	$\mathbf{O} = \emptyset$ and $\pi = \pi_d = \{S \rightarrow \hat{Y}\}$
(System) Indirect discrimination	[19, 7, 16, 3]	$\mathbf{O} = \emptyset$ and $\pi = \pi_i \subset \Pi$
Individual direct discrimination	[17]	$\mathbf{O} = \{S, \mathbf{X}\}$ and $\pi = \pi_d = \{S \rightarrow \hat{Y}\}$
Group direct discrimination	[18]	$\mathbf{O} = \mathbf{Q} = \text{PA}_Y \setminus \{S\}$ and $\pi = \pi_d = \{S \rightarrow \hat{Y}\}$
Counterfactual fairness	[5, 9, 14]	$\mathbf{O} = \{S, \mathbf{X}\}$ and $\pi = \Pi$
Counterfactual error rate	[15]	$\mathbf{O} = \{S, Y\}$ and $\pi = \pi_d$ or π_i

Table 1: Connection between previous fairness notions and PC fairness

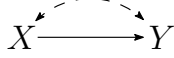


Figure 2: The “bow graph”.

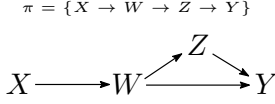


Figure 3: The “kite graph”.

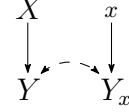


Figure 4: The “w graph”.

In the context of fair machine learning, we use $S \in \{s^+, s^-\}$ to denote the protected attribute, $Y \in \{y^+, y^-\}$ to denote the decision, and $\mathbf{X}$ to denote a set of non-protected attributes. The underlying mechanism of the population over the space $S \times \mathbf{X} \times Y$ is represented by a causal model $\mathcal{M}$, which is associated with a causal graph $\mathcal{G}$. A historical dataset $\mathcal{D}$ is drawn from the population, which is used to construct a predictor $h : \mathbf{X}, S \rightarrow \hat{Y}$. The causal model for the population over space $S \times \mathbf{X} \times \hat{Y}$ can be considered the same as $\mathcal{M}$ except that function f_Y is replaced with a predictor h . We use Π to denote all causal paths from S to $\hat{Y}$ in the causal graph.

Then, we define the path-specific counterfactual fairness based on Definition 5.

Definition 6 (Path-specific Counterfactual Fairness² (PC Fairness)). *Given a factual condition $\mathbf{O} = \mathbf{o}$ where $\mathbf{O} \subseteq \{S, \mathbf{X}, Y\}$ and a causal path set π , predictor $\hat{Y}$ achieves the PC fairness if $\text{PCE}_\pi(s_1, s_0 | \mathbf{o}) = 0$ where $s_1, s_0 \in \{s^+, s^-\}$. We also say that $\hat{Y}$ achieves the τ -PC fairness if $|\text{PCE}_\pi(s_1, s_0 | \mathbf{o})| \leq \tau$.*

We show that previous causality-based fairness notions can be expressed as special cases of the PC fairness. Their connections are summarised in Table 1, where π_d contains the direct edge from S to $\hat{Y}$, and π_i is a path set that contains all causal paths passing through any redlining attributes (i.e., a set of attributes in $\mathbf{X}$ that cannot be legally justified if used in decision-making). Based on whether $\mathbf{O}$ equals $\emptyset$ or not, the previous notions can be categorized into the ones that deal with the system level ($\mathbf{O} = \emptyset$) and the ones that have certain conditions ($\mathbf{O} \neq \emptyset$). Based on whether π equals Π or not, the previous notions can be categorized into the ones that deal with the total causal effect ($\pi = \Pi$), the ones that consider the direct discrimination ($\pi = \pi_d$), and the ones that consider the indirect discrimination ($\pi = \pi_i$).

In addition to unifying the existing notions, the notion of PC fairness also resolves new types of fairness that the previous notions cannot do. One example is individual indirect discrimination, which means discrimination along the indirect paths for a particular individual. Individual indirect discrimination has not been studied yet in the literature, probably due to the difficulty in definition and identification. However, it can be directly defined and analyzed using PC fairness by letting $\mathbf{O} = \{S, \mathbf{X}\}$ and $\pi = \pi_i$.

4 Measuring Path-specific Counterfactual Fairness

In this section, we develop a general method for bounding the path-specific counterfactual effect in any unidentifiable situation. In the causal inference field, researchers have studied the reasons for unidentifiability under different cases. When $\mathbf{O} = \emptyset$ and $\pi \subset \Pi$, the reason for unidentifiability can be the existence of the “kite graph” (see Figure 3) in the causal graph [1]. When $\mathbf{O} \neq \emptyset$ and

²In [3] the authors use the same term path-specific counterfactual fairness but with different meanings. Their fairness notion is defined only based on the path-specific effect and is a special case covered by our definition.

$\pi = \Pi$, the reason for unidentifiability can be the existence of the “w graph” (see Figure 4) [11]. In any situation, as long as there exists a “hedge graph” (where the simplest case is the “bow graph” as shown in Figure 2), then the causal effect is unidentifiable [12]. Obviously, all above unidentifiable situations can exist in the path-specific counterfactual effect.

Our method is motivated by [2] which formulates the bounding problem as a constrained optimization problem. The general idea is to parameterize the causal model and use the observational distribution $P(\mathbf{V})$ to impose constraints on the parameters. Then, the path-specific counterfactual effect of interest is formulated as an objective function of maximization or minimization for estimating its upper or lower bound. The bounds are guaranteed to be tight as we traverse all possible causal models when solving the optimization problem. Thus, a byproduct of the method is a unique estimation of the path-specific counterfactual effect in the identifiable situation.

For presenting our method, we first introduce a key concept called the response-function variable.

4.1 Response-function Variable

Response-function variables are proposed in [2] for parameterizing the causal model. Consider an arbitrary endogenous variable denoted by $V \in \mathbf{V}$, its endogenous parents denoted by PA_V , its exogenous parents denoted by U_V , and its associated structural function in the causal model denoted by $v = f_V(\text{pa}_V, u_V)$. In general, U_V can be a variable of any type with any domain size, and f_V can be any function, making the causal model very difficult to be handled. However, we can note that, for each particular value u_V of U_V , the functional mapping from PA_V to V is a particular deterministic response function. Thus, we can map each value of U_V to a deterministic response function. Although the domain size of U_V is unknown which might be very large or even infinite, the number of different deterministic response functions is known and limited, given the domain sizes of PA_V and V . This means that the domain of U_V can be divided into several equivalent regions, each corresponding to the same response function. As a result, we can transform the original non-parameterized structural function to a limited number of parameterized functions.

Formally, we represent equivalent regions of each endogenous variable V by the *response-function variable* $R_V = \{0, \dots, N_V - 1\}$ where $N_V = |V|^{\text{PA}_V}$ is the total number of different deterministic response functions mapping from PA_V to V ($N_V = |V|$ if V has no parent). Each value r_V represents a pre-defined response function. We also denote the mapping from U_V to R_V as $r_V = \ell_V(u_V)$. Then, for any $f_V(\text{pa}_V, u_V)$, it can be re-formulated as

$$f_V(\text{pa}_V, u_V) = f_V(\text{pa}_V, \ell_V^{-1}(r_V)) = f_V \circ \ell_V^{-1}(\text{pa}_V, r_V) = g_V(\text{pa}_V, r_V),$$

where g_V is the composition of f_V and ℓ_V^{-1} , and denotes the response functions represented by r_V . We denote the set of all response-function variables by $\mathbf{R} = \{R_V : V \in \mathbf{V}\}$.

Next, we show how joint distribution $P(\mathbf{v})$ can be expressed as a linear function of $P(\mathbf{r})$. According to [13], $P(\mathbf{v})$ can be expressed as the summation over the probabilities of certain values $\mathbf{u}$ of $\mathbf{U}$ that satisfy following corresponding requirements: for each $V \in \mathbf{V}$, we must have $f_V(\text{pa}_V, u_V) = v$ where v, pa_V are specified by $\mathbf{v}$ and u_V is specified by $\mathbf{u}$. In other words, denoting by $V(\mathbf{u})$ the value that V would obtain if $\mathbf{U} = \mathbf{u}$, we have $P(\mathbf{v}) = \sum_{\mathbf{u}: V(\mathbf{u})=\mathbf{v}} P(\mathbf{u})$. Then, by mapping from $\mathbf{U}$ to $\mathbf{R}$, we accordingly obtain $P(\mathbf{v}) = \sum_{\mathbf{r}: V(\mathbf{r})=\mathbf{v}} P(\mathbf{r})$, where for each $V \in \mathbf{V}$, $V(\mathbf{r}) = v$ means that $g_V(\text{pa}_V, r_V) = v$. As a result, by defining an indicator function

$$\mathbb{I}(v; \text{pa}_V, r_V) = \begin{cases} 1 & \text{if } g_V(\text{pa}_V, r_V) = v, \\ 0 & \text{otherwise,} \end{cases}$$

we obtain

$$P(\mathbf{v}) = \sum_{\mathbf{r}} P(\mathbf{r}) \prod_{V \in \mathbf{V}} \mathbb{I}(v; \text{pa}_V, r_V), \quad (1)$$

which is a linear expression of $P(\mathbf{r})$.

Example 1. Consider the causal graph shown in Figure 1 with two endogenous variables X and Y , and two exogenous variables U_X and U_Y with unknown domains. Assume that both X and Y are binary, i.e., $X \in \{x_0, x_1\}$ and $Y \in \{y_0, y_1\}$, and denote their response variables as R_X and R_Y . For Y , since there are a total number of $2^2 = 4$ response functions, response-function variable R_Y and response function g_Y can be defined as follows:

$$r_Y = \ell_Y(u_Y) = \begin{cases} 0 & \text{if } f_Y(x_0, u_Y) = y_0, f_Y(x_1, u_Y) = y_0; \\ 1 & \text{if } f_Y(x_0, u_Y) = y_0, f_Y(x_1, u_Y) = y_1; \\ 2 & \text{if } f_Y(x_0, u_Y) = y_1, f_Y(x_1, u_Y) = y_0; \\ 3 & \text{if } f_Y(x_0, u_Y) = y_1, f_Y(x_1, u_Y) = y_1. \end{cases} \quad g_Y(x, r_Y) = \begin{cases} y_0 & \text{if } r_Y = 0; \\ y_0 & \text{if } x = x_0, r_Y = 1; \\ y_1 & \text{if } x = x_1, r_Y = 1; \\ y_1 & \text{if } x = x_0, r_Y = 2; \\ y_0 & \text{if } x = x_1, r_Y = 2; \\ y_1 & \text{if } r_Y = 3. \end{cases}$$

Similarly, response-function variable R_X and response function g_X can be defined as

$$r_X = \ell_X(u_X) = \begin{cases} 0 & \text{if } f_X(u_X) = x_0; \\ 1 & \text{if } f_X(u_X) = x_1. \end{cases} \quad g_X(r_X) = \begin{cases} x_0 & \text{if } r_X = 0; \\ x_1 & \text{if } r_X = 1. \end{cases}$$

As a result, the joint distribution over X, Y is given by

$$P(x, y) = \sum_{r_X, r_Y} P(r_X, r_Y) \mathbb{I}(x; r_X) \mathbb{I}(y; r_Y).$$

4.2 Expressing Path-specific Counterfactual Fairness

For bounding the path-specific counterfactual effect, i.e., $\text{PCE}_\pi(s_1, s_0 | \mathbf{o}) = P(\hat{y}_{s_1 | \pi, s_0 | \bar{\pi}} | \mathbf{o}) - P(\hat{y}_{s_0 | \pi, s_0 | \bar{\pi}} | \mathbf{o})$, we also apply response-function variables to express it. We focus on the expression of $P(\hat{y}_{s_1 | \pi, s_0 | \bar{\pi}} | \mathbf{o})$, and the expression of $P(\hat{y}_{s_0 | \pi, s_0 | \bar{\pi}} | \mathbf{o})$ can be similarly obtained as a simpler case. Similar to the previous section, we first express $P(\hat{y}_{s_1 | \pi, s_0 | \bar{\pi}} | \mathbf{o})$ as the summation over the probabilities of certain values of $\mathbf{U}$ that satisfy corresponding requirements. However, as described below, the requirements are much more complicated than previous ones due to the integration of intervention, path-specific effect, and counterfactual.

Firstly, since the path-specific counterfactual effect is under a factual condition $\mathbf{O} = \mathbf{o}$, values $\mathbf{u}$ must satisfy that $\mathbf{O}(\mathbf{u}) = \mathbf{o}$, i.e., for each $O \in \mathbf{O}$, we must have $f_O(\text{pa}_O, u_O) = o$. Secondly, the path-specific counterfactual effect is transmitted only along some path set π . According to [20], for the variables of $\mathbf{X}$ that lie on both π and $\bar{\pi}$, referred to as *witness variables/nodes* [1], we need to consider two sets of values, one obtained by treating them on π and the other obtained by treating them on $\bar{\pi}$. Formally, non-protected attributes $\mathbf{X}$ are divided into three disjoint sets. We denote by $\mathbf{W}$ the set of witness variables, denote by $\mathbf{A}$ the set of non-witness variables on π , and denote by $\mathbf{B}$ the set of non-witness variables on $\bar{\pi}$. A simple example is given in Figure 5. We denote the interventional variant of $\mathbf{A}$ by $\mathbf{A}_{s_1 | \pi}$, the interventional variant of $\mathbf{B}$ by $\mathbf{B}_{s_0 | \bar{\pi}}$, the interventional variant of $\mathbf{W}$ treated on π by $\mathbf{W}_{s_1 | \pi}$, and the interventional variant of $\mathbf{W}$ treated on $\bar{\pi}$ by $\mathbf{W}_{s_0 | \bar{\pi}}$. Then, $P(\hat{y}_{s_1 | \pi, s_0 | \bar{\pi}} | \mathbf{o})$ can be written as

$$P(\hat{y}_{s_1 | \pi, s_0 | \bar{\pi}} | \mathbf{o}) = \sum_{\mathbf{a}, \mathbf{b}, \mathbf{w}_1, \mathbf{w}_0} P(\hat{Y}_{s_1 | \pi, s_0 | \bar{\pi}} = y, \mathbf{A}_{s_1 | \pi} = \mathbf{a}, \mathbf{B}_{s_0 | \bar{\pi}} = \mathbf{b}, \mathbf{W}_{s_1 | \pi} = \mathbf{w}_1, \mathbf{W}_{s_0 | \bar{\pi}} = \mathbf{w}_0 | \mathbf{o}).$$

To obtain the above joint distribution, in addition to $\mathbf{O}(\mathbf{u}) = \mathbf{o}$, values $\mathbf{u}$ must also satisfy that:

1. $\mathbf{A}_{s_1 | \pi}(\mathbf{u}) = \mathbf{a}$, which means for each $A \in \mathbf{A}$, we must have $f_A(\text{pa}_A^1, u_A) = a$, where pa_A^1 means that if PA_A contains S or any witness node W , its value is specified by s_1 or w_1 if edge $S/W \rightarrow Y$ belongs to a path in π , and specified by s_0 or w_0 otherwise;
2. $\mathbf{B}_{s_0 | \bar{\pi}}(\mathbf{u}) = \mathbf{b}$, which means for each $B \in \mathbf{B}$, we must have $f_B(\text{pa}_B^0, u_B) = b$, where pa_B^0 means that if PA_B contains S or any witness node W , its value is specified by s_0 or w_0 ;
3. $\mathbf{W}_{s_1 | \pi}(\mathbf{u}) = \mathbf{w}_1$, which means for each $W \in \mathbf{W}$, we must have $f_W(\text{pa}_W^1, u_W) = w_1$;
4. $\mathbf{W}_{s_0 | \bar{\pi}}(\mathbf{u}) = \mathbf{w}_0$, which means for each $W \in \mathbf{W}$, we must have $f_W(\text{pa}_W^0, u_W) = w_0$.

Then, by mapping from $\mathbf{U}$ to $\mathbf{R}$, we can obtain the requirements for $\mathbf{R}$ accordingly. Finally, denoting the values of $\mathbf{R}$ that satisfy $\mathbf{O}(\mathbf{r}) = \mathbf{o}$ by $\mathbf{r}_\mathbf{o}$, we obtain

$$P(\hat{y}_{s_1 | \pi, s_0 | \bar{\pi}} | \mathbf{o}) = \sum_{\substack{\mathbf{a}, \mathbf{b}, \mathbf{w}_1 \\ \mathbf{w}_0, \mathbf{r} \in \mathbf{r}_\mathbf{o}}} \frac{P(\mathbf{r})}{P(\mathbf{o})} \mathbb{I}(\hat{y}; \text{pa}_Y^1, r_Y) \prod_{A \in \mathbf{A}} \mathbb{I}(a; \text{pa}_A^1, r_A) \prod_{B \in \mathbf{B}} \mathbb{I}(b; \text{pa}_B^0, r_B) \prod_{W \in \mathbf{W}} \mathbb{I}(w_1; \text{pa}_W^1, r_W) \mathbb{I}(w_0; \text{pa}_W^0, r_W), \quad (2)$$

which is still a linear expression of $P(\mathbf{r})$.

Similarly, we can obtain

$$P(\hat{y}_{s_0}|\mathbf{o}) = \sum_{\mathbf{v}', \mathbf{r} \in \mathbf{r}_{\mathbf{o}}} \frac{P(\mathbf{r})}{P(\mathbf{o})} \mathbb{I}(\hat{y}; \text{pa}_{\hat{Y}}, r_{\hat{Y}}) \prod_{V \in \mathbf{V}'} \mathbb{I}(v; \text{pa}_V, r_V), \quad (3)$$

where $\mathbf{V}' = \mathbf{V} \setminus \{S, Y\}$.

Example 2. Consider causal graphs shown in Figures 2, 3, 4 and following unidentifiable causal effects: total causal effect $\text{TCE}(x_1, x_0)$ in Figure 2, path-specific effect $\text{PE}_\pi(x_1, x_0)$ in Figure 3, and counterfactual effect $\text{CE}(x_1, x_0|x_0, y_0)$ in Figure 4. By similarly defining response functions as in Example 1, for Figure 2 with $\mathbf{R} = \{R_X, R_Y\}$, we have

$$\text{TCE}(x_1, x_0) = \sum_{r_X, r_Y} P(r_X, r_Y) \mathbb{I}(y; x_1, r_Y) - \sum_{r_X, r_Y} P(r_X, r_Y) \mathbb{I}(y; x_0, r_Y),$$

for Figure 3 with $\mathbf{R} = \{R_X, R_W, R_Z, R_Y\}$, we have

$$\begin{aligned} \text{PE}_\pi(x_1, x_0) &= \sum_{z, w_1, w_0, \mathbf{r}} P(\mathbf{r}) \mathbb{I}(y; z, w_0, r_Y) \mathbb{I}(z; w_1, r_Z) \mathbb{I}(w_1; x_1, r_W) \mathbb{I}(w_0; x_0, r_W) \\ &\quad - \sum_{z, w, \mathbf{r}} P(\mathbf{r}) \mathbb{I}(y; z, w, r_Y) \mathbb{I}(z; w, r_Z) \mathbb{I}(w; x_0, r_W), \end{aligned}$$

for Figure 4 with $\mathbf{R} = \{R_X, R_Y\}$, we have

$$\text{CE}(x_1, x_0) = \sum_{r_X, r_Y \in \mathbf{r}_{\mathbf{o}}} \frac{P(r_X, r_Y)}{P(x_0, y_0)} \mathbb{I}(y; x_1, r_Y) - \sum_{r_X, r_Y \in \mathbf{r}_{\mathbf{o}}} \frac{P(r_X, r_Y)}{P(x_0, y_0)} \mathbb{I}(y; x_0, r_Y).$$

Note that in Figures 2, the total causal effect is identifiable if U_X and U_Y are independent. This is reflected in our formulation such that when R_X and R_Y are independent, we have $P(y_{x_1}) = \sum_{r_X, r_Y} P(r_X)P(r_Y) \mathbb{I}(y; x_1, r_Y) = P(y|x_1)$, which can be directly measured from observational data. Similar phenomenons can be observed in other identifiable situations.

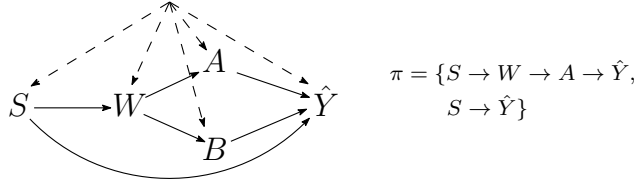


Figure 5: A causal graph with unidentifiable path-specific counterfactual fairness.

Example 3. Consider a causal graph shown in Figure 5, and the path-specific counterfactual effect $\text{PCE}_\pi(s_1, s_0|\mathbf{o})$ where $\pi = \{S \rightarrow \hat{Y}, S \rightarrow W \rightarrow A \rightarrow \hat{Y}\}$ and $\mathbf{o} = \{s_0, w', a', b'\}$. Any pair of exogenous variables can be correlated. Response-function variables are given by $\mathbf{R} = \{R_S, R_W, R_A, R_B, R_{\hat{Y}}\}$. By similarly defining response functions as in Example 1, we can obtain

$$P(\hat{y}_{s_1}|\pi, s_0|\bar{\pi}|\mathbf{o}) = \sum_{\substack{a, b, w_1, w_0 \\ \mathbf{r} \in \mathbf{r}_{\mathbf{o}}}} \frac{P(\mathbf{r})}{P(\mathbf{o})} \mathbb{I}(\hat{y}; a, b, s_1, r_{\hat{Y}}) \mathbb{I}(a; w_1, r_A) \mathbb{I}(b; w_0, r_B) \mathbb{I}(w_1; s_1, r_W) \mathbb{I}(w_0; s_0, r_W),$$

and

$$P(\hat{y}_{s_0}|\mathbf{o}) = \sum_{a, b, w, \mathbf{r} \in \mathbf{r}_{\mathbf{o}}} \frac{P(\mathbf{r})}{P(\mathbf{o})} \mathbb{I}(\hat{y}; a, b, s_0, r_{\hat{Y}}) \mathbb{I}(a; w, r_A) \mathbb{I}(b; w, r_A) \mathbb{I}(w; s_0, r_W).$$

4.3 Bounding Path-specific Counterfactual Fairness

In above two sections we express both joint distribution $P(\mathbf{v})$ and the path-specific counterfactual effect as linear functions of $P(\mathbf{r})$. All causal models (represented by different $P(\mathbf{r})$) that agree with

the distribution of observational data $\mathcal{D}$ cannot be distinguished and should be considered in bounding PC fairness. Therefore, finding the lower or upper bound of the path-specific counterfactual effect is equivalent to finding the $P(\mathbf{r})$ that minimizes or maximizes the path-specific counterfactual effect, subject to that the derived joint distribution $P(\mathbf{v})$ agrees with the observational distribution $P(\mathcal{D})$. This fact results in the following linear programming problem for deriving the lower/upper bound of path-specific counterfactual effect.

$$\begin{aligned} \min/\max \quad & P(\hat{y}_{s_1}|\pi, s_0|\bar{\pi}|\mathbf{o}) - P(\hat{y}_{s_0}|\mathbf{o}), \\ \text{s.t.} \quad & P(\mathbf{V}) = P(\mathcal{D}), \quad \sum_{\mathbf{r}} P(\mathbf{r}) = 1, \quad P(\mathbf{r}) \geq 0, \end{aligned} \quad (4)$$

where $P(\hat{y}_{s_1}|\pi, s_0|\bar{\pi}|\mathbf{o})$ is given by Eq. (2), $P(\hat{y}_{s_0}|\mathbf{o})$ is given by Eq. (3), and $P(\mathbf{v})$ is given by Equation (1).

The lower and upper bounds derived by solving the above optimization problem is guaranteed to be the tightest, since the response function is an equivalent mapping that covers all possible causal models thus we can explicitly traverse all possible causal models.

We use the derived bounds for examining τ -PC fairness: if the upper bound is less than τ and the lower bound is greater than $-\tau$, then τ -PC fairness must be satisfied; if the upper bound is less than $-\tau$ or the lower bound is greater than τ , τ -PC fairness must not be satisfied; otherwise, it is uncertain and cannot be determined from data.

5 Experiments

Datasets. For synthetic datasets, we manually build a causal model with complete knowledge of exogenous variables and equations using Tetrad [10] according to the causal graphs. The causal model consists of 4 endogenous variables, $S, W, A, \hat{Y}$, all of which have two domain values. Then, we consider two versions of the causal model: (1) we assume a shared exogenous variables, i.e., a hidden confounder, with 100 domain values (the causal graph is shown in Figure 6); (2) we assume all exogenous variables are mutually independent (the causal graph is omitted due to the space limit). The distribution of exogenous variables and structural equations of endogenous variables are randomly assigned. Finally, we generate two datasets from each version of the causal model, denoted by $\mathcal{D}_1$ and $\mathcal{D}_2$ respectively.

For the real-world dataset, we adopt the Adult dataset, which consists of 65,123 records with 11 attributes including *edu, sex, income* etc. Similar to [14], we select 7 attributes, binarize their values, and build the causal graph. Fairness threshold τ is set to 0.1. The datasets and implementation are available at <http://tiny.cc/pc-fairness-code>.

Bounding Path-specific Counterfactual Fairness. We use $\mathcal{D}_1$ to validate our method in Eq. (4) for bounding $\text{PCE}_{\pi}(s^+, s^-|\mathbf{o})$ where $\mathbf{O} = \{S, W, A\}$ and $\pi = \{S \rightarrow W \rightarrow A \rightarrow \hat{Y}, S \rightarrow \hat{Y}\}$. The ground truth can be computed by exactly executing the intervention under given conditions using the complete causal model. The results are shown in Table 2, where the first column indicates the indices of $\mathbf{o}$'s value combinations. As can be seen, the true values of $\text{PCE}_{\pi}(s^+, s^-|\mathbf{o})$ fall into the range of our bounds for all value combinations of $\mathbf{O}$, which validates our method.

Comparing with previous bounding methods. We use $\mathcal{D}_2$ to compare with the previous methods [20, 14] which are derived under the Markovian assumption. We compare with [20] for bounding $\text{PE}_{\pi}(s^+, s^-)$ with $\pi = \{S \rightarrow W \rightarrow A \rightarrow \hat{Y}, S \rightarrow \hat{Y}\}$. We also compare with [14] for bounding $\text{CE}(s^+, s^-|\mathbf{o})$ with $\mathbf{O} = \{S, W, A\}$. The results are shown in Table 3 where the bold indicates that our method makes different judgments on discrimination detection due to the tighter bounds. As can be seen, our method achieves much tighter bounds than previous methods, which can be used to examine fairness more accurately. For example, when measuring indirect discrimination using $\text{PE}_{\pi}(s^+, s^-)$ (Row 1 in Table 3), it is uncertain for [20] since the lower and upper bounds are -0.2605 and 0.2656 , but our method can guarantee that the decision is discriminatory as the lower bound 0.1772 is larger than $\tau = 0.1$. As another example, when measuring counterfactual fairness of the 2nd groups of $\mathbf{o}$ using $\text{CE}(s^+, s^-|\mathbf{o})$ (Row 3 in Table 3), the method in [14] is uncertain since the lower and upper bounds are -0.4383 , -0.0212 but our method can guarantee that the decision is fair due to the range of $[-0.0783, -0.0212]$.

We also use the Adult dataset to compare with the method in [14] for bounding $CE(s^+, s^- | \mathbf{o})$ with $\mathbf{O} = \{age, edu, marital-status\}$ and obtain similar results, which are shown in Table 4.

# of $\mathbf{o}$	$PCE_{\pi}(s^+, s^- \mathbf{o})$		
	lb	ub	$Truth$
1	-0.4548	0.5452	0.1507
2	-0.5565	0.4435	-0.0928
3	-0.5065	0.4935	0.0561
4	-0.4598	0.5402	0.0548

Table 2: Bounds and ground truth of PC fairness on $\mathcal{D}_1$.

	# of $\mathbf{o}$	$Truth$	Previous methods		Our method	
			lb	ub	lb	ub
PE	N/A	0.1793	-0.2605	0.2656	0.1772	0.1836
CE	1	0.3438	0.0878	0.5049	0.0878	0.5049
	2	-0.0557	-0.4383	-0.0212	-0.0783	-0.0212
	3	0.2318	-0.1192	0.2979	0.1282	0.2847
	4	0.0800	-0.2101	0.2070	0.0110	0.1499

Table 3: Compare with existing methods in [20, 14] on $\mathcal{D}_2$.

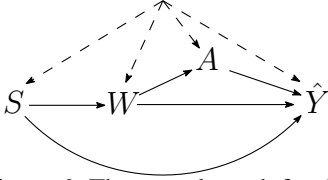


Figure 6: The causal graph for the synthetic dataset $\mathcal{D}_1$.

# of $\mathbf{o}$	Method in [14]		Our Method	
	lb	ub	lb	ub
0	0.0541	0.2946	0.1498	0.1944
1	-0.1314	0.1091	-0.1314	0.1091
2	0.1878	0.3210	0.2507	0.2890
3	-0.0356	0.0976	-0.0356	0.0976
4	0.1676	0.5289	0.4419	0.5289
5	-0.1634	0.1979	-0.0731	0.1979
6	0.1290	0.4689	0.3942	0.4689
7	-0.1808	0.1591	0.0014	0.1591

Table 4: Compare with the existing method in [14] on the Adult dataset.

6 Conclusion

In this paper, we develop a general framework for measuring causality-based fairness. We propose a unified definition that covers most of previous causality-based fairness notions, namely the path-specific counterfactual fairness (PC fairness). Then, we formulate a linear programming problem to bound PC fairness which can produce the tightest possible bounds. Experiments using synthetic and real-world datasets show that, our method can bound causal effects under any unidentifiable situation or combinations, and achieves tighter bounds than previous methods.

As the concern of scalability, the domain size of each response variable is exponential to the number of parents, meaning that the joint domain size of all response variables are exponential to the total in-degree of the causal graph. However, we notice that not all response variables are needed in the formulation, and only those that directly lead to unidentification are needed. For example, when a hidden confounder causes unidentification, only the children of the hidden confounder need to have response variables in the formulation; and when a “kite graph” causes unidentification, only the witness variable need to have a response variable in the formulation. As a result, the total complexity of the problem formulation could be significantly decreased. We will study the decomposition and reduction of response variables in the future work.

How to construct fair predictive models based on the derived bounds is another future research direction. One possible method would be to incorporate the bounding formulation into a post-processing method. The new formulation will be a min-max optimization problem, where the optimization variables will include response variables $P(\mathbf{r})$ as well as a post-processing mapping $P(\hat{y}|\hat{y}, \text{pa}_Y)$. The inner optimization is to maximize the path-specific counterfactual effect to find the upper bound, and the outer optimization is to minimize both the loss function and the upper bound. We will to explore this method in the future work.

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