Voltage Stability Based Placement of Distributed Generation Against Extreme Events

Ahmet Öner, Ali Abur
Department of Electrical and Computer Engineering
Northeastern University
Boston, MA
aoner@ece.neu.edu, abur@ece.neu.edu

***Abstract*—This paper is concerned about improving the resilience of power grids against extreme events which may lead to the line and generator outages and subsequent voltage stability problems and blackouts. The reported study investigates ways of eliminating or substantially reducing the chances of having such voltage stability problems during expected extreme events, by strategically placing a few distributed generators in the system. The problem is addressed in two stages, where a reasonably inclusive list of credible contingencies are individually considered first. A minimum number of distributed generators are selected and placed in order to maintain voltage stability under each considered contingency. In the second stage, the number of generators is minimized by the strategic selection of locations to reach a solution that ensures voltage stability under all considered contingencies in the system. Effectiveness and computational performance of the developed strategy are illustrated by simulating several outage scenarios using the IEEE 118-bus system.**

***Index Terms*—Distributed generation (DG) placement, extreme events, resilience, voltage stability**

I. Introduction

As the world's overall energy demand is continually increasing [1], so do the frequency and impact of extreme events on power grids. There are many power systems which are forced to operate closer to their limits due to the changing power flow patterns resulting from increased number of renewable sources at remote locations. Operators of such systems find it very challenging to maintain voltage stability during line or generator outages caused by such extreme events and look for ways of easing this challenge by investing in appropriate tools to eliminate or significantly reduce the probability of having voltage problems during or after such outages. It should also be noted that according to the Department of Energy, the average annual cost of power outages caused by severe weather is estimated to be between \$18 billion and \$33 billion per year [2]. Moreover, the frequency of extreme events is expected to increase in the future [3]. Therefore, there is sufficient evidence that points to the need to urgently invest in tools and strategies to improve the resilience of power grids.

This work considers distributed generators (DG) which are commonly used in both distribution and transmission systems and provide both reactive power support as well as alternative real power dispatch options in order to address both voltage and line flow limit violations, minimize system losses, improve system stability and power quality. There are numerous studies reported in the literature on DG placement strategies [4]–[9]. The authors of [6], [7] solve the DG placement problem by considering the voltage stability of the power grid. If more than one DG is required to be installed, they are selected and installed one DG at a time, thus not guaranteeing the optimality of the result. The authors of [8] allocate PV generation and battery storage to restore the system following an extreme event. Rather than using DGs in restoring the system, DGs can be placed strategically ahead of the event using outage forecasts based on historical outage data. Results of a preliminary study using this approach are reported in [9]. In this paper, this study is further extended by considering a possible set of outage scenarios and determining a minimum number of required DG placements to maintain system feasibility for all considered scenarios.

The developed approach aims to eliminate the potential adverse effects of possible outages by strategic placement of new generators. These new DGs are introduced into the system model by switching the bus types from PQ bus to PV bus in the problem formulation. In the process, their real power outputs are arbitrarily set equal to zero in this study. While this yields a solution where generators essentially act as synchronous condensers, the same approach can be applied equally effectively to solve the problem with any predetermined real power output assigned to the placed generators.

The proposed approach starts out by selecting a set of outage scenarios that are credible and significant based on the historical records of the given power grid. Also, a subset of the load (or PQ) buses is chosen as candidate locations for placing new DGs. In this study, all PQ buses are assumed to belong to this set, but in an actual system it is likely that some of the load buses may be removed from this list due to other considerations or physical limitations. Next, a voltage stability constrained optimal power flow (VSCOPF) problem is repeatedly solved for each outage scenario by placing a

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single DG at a time at the candidate PQ bus locations. If a feasible solution is obtained for a DG location, then it will be assigned as a candidate DG for the corresponding outage. If among the single DGs considered for a given outage, none of them manages to yield a feasible solution then the VSCOPF solution for this specific outage scenario will be attempted to be obtained by using a pair of DGs. This will typically require a large number of solutions each corresponding to one of the possible combinations of a pair of DG locations among the chosen set of candidate PQ buses. At the end of this process, each outage will be assigned a set of single and possibly pairs of candidate DG locations. These candidate DG locations are collectively saved in a binary matrix relating outages to corresponding DG candidate buses. This matrix is then used to formulate an integer programming (IP) problem whose solution will yield the desired numbers and locations of DGs to be placed in order to maintain feasibility under all considered outage scenarios.

II. Problem Statement

Given a large power grid and a set of outage scenarios which may involve multiple line and/or generator losses, this study is concerned about placing distributed generators at a selected subset of candidate load (PQ type) buses in order to maintain feasibility under each of the considered outage scenarios. Here, feasibility primarily refers to voltage stability since such outage scenarios are becoming more commonly observed at the sub-transmission or distribution systems which are more often subjected to extreme weather events causing such outages. Hence, the study uses voltage stability constrained optimal power flow as the main tool for restoring feasibility, but it is worth mentioning that this could be replaced by other feasibility restoration tools of choice without changing the steps of the proposed approach.

III. Problem Formulation

The above described problem is formulated and solved in multiple stages involving both nonlinear programming and integer programming (IP) tools. The voltage stability constrained optimal power flow part (nonlinear programming part) is utilized to find the binary feasibility decision matrix and the cost vector considering all the selected outage scenarios. IP solver is used to find an optimal solution for the final set of DG placements. The details of voltage stability constrained optimal power flow problem will be described in section III-A, building the required incidence matrices for the DG placement problem will be described in section III-B and finally the use of IP method to strategically assign DGs in a given power grid will be explained in section III-C.

A. Voltage Stability Constrained Optimal Power Flow (VSCOPF)

The core of the proposed DG placement approach relies heavily on efficiently and reliably determining whether or not a given outage scenario will lead to voltage instability in the power grid. If the identified infeasible cases can be made feasible by introducing a DG into the system, then this will also have to be verified. Both of these tasks require a reliable optimization tool which can solve the well studied nonlinear optimal power flow problem while ensuring voltage stability under the operating conditions corresponding to the obtained solution. In this work, such an optimization problem is formulated and solved repeatedly for all considered outage scenarios and assumed DG locations. The details of the problem formulation including the objective function and the constraint equations are provided below.

1) Objective Function: The objective function to be minimized is chosen as the total production cost of generators:

$$\min\ C_1 P_G^2 + C_2 P_G \tag{1}$$

where P_G is the vector of generator active power outputs, C_1 and C_2 are the vector cost coefficients.

2) Constraints: There are 5 sets of constraints associated with this problem. These constraints not only account for network operation limits but also ensure voltage stability at the resulting solution. These sets of constraints involve the following equations:

a) Power Balance Equations: The first set reflects the power balance equations at each bus expressed for both active and reactive power flows. For a given bus i, these equations will be given by:

$$P_G(i) - P_D(i) = \sum_{j=1}^{n_B} \Big(G_{ij} |\ V_i \ || \ V_j \ | cos(\theta_{ij}) + B_{ij} |\ V_i \ || \ V_j \ | sin(\theta_{ij}) \Big) \tag{2}$$

$$Q_G(i) - Q_D(i) = \sum_{j=1}^{n_B} \Big(G_{ij} |\ V_i \ || \ V_j \ | sin(\theta_{ij}) - B_{ij} |\ V_i \ || \ V_j \ | cos(\theta_{ij}) \Big) \tag{3}$$

where:
i and j are bus numbers,
n_B represents the total number of buses considered for DG placement. While it designates all the system buses in this formulation, in practice buses with existing generation as well as some other buses which do not lend themselves to DG connection will be excluded from this list, making this number much less than the total number of buses in the system,
G_{ij} and B_{ij} represent i,jth elements of the real and imaginary parts of the bus admittance matrix, respectively,
$|\ V_i\ |$ and $|\ V_j\ |$ represent the voltage magnitudes at bus-i and bus-j respectively,
θ_{ij} is the phase angle difference between bus-i and bus-j,
$P_G(i)$ $P_D(i)$, $Q_G(i)$ and $Q_D(i)$ are the active/reactive power generation/demand, at bus-i, respectively.

b) Line Flow Limits: Line flow equations for both active and reactive power flows can be written as:

$$P_{ij} = g_{ij}(\mid V_i \mid^2 - \mid V_i \mid\mid V_j \mid cos(\theta_{ij})) - b_{ij}\mid V_i \mid\mid V_j \mid sin(\theta_{ij}) \quad (4)$$

$$Q_{ij} = -b_{ij}(\mid V_i \mid^2 - \mid V_i \mid\mid V_j \mid cos(\theta_{ij})) - g_{ij}\mid V_i \mid\mid V_j \mid sin(\theta_{ij}) \quad (5)$$

where:
P_{ij}, Q_{ij}: active/reactive power flow on branch (i, j),
g_{ij}, b_{ij}: conductance/susceptance of branch (i, j).

The apparent power flow for branch (i, j), $\mid S_{ij} \mid$ which is given by:

$$\mid S_{ij} \mid = \sqrt{P_{ij}^2 + Q_{ij}^2} \quad (6)$$

will be limited by the power transfer capacity, hence:

$$\mid S_{ij} \mid \leq \overline{S_{ij}} \quad (7)$$

where, $\overline{S_{ij}}$ represents the upper limit for power transfer on branch (i, j).

c) Generator Limits: Each generator is assigned upper and lower power output limits. The upper limit is related to the unit's power generation capacity. The lower limit is used to enforce mandatory power generation while operating a boiler (if exists). The associated constraints for power generation limits are given below:

$$\underline{P} \leq P_G \leq \overline{P} \quad (8)$$

$$\underline{Q} \leq Q_G \leq \overline{Q} \quad (9)$$

where, $\underline{P}$, $\overline{P}$, $\underline{Q}$ and $\overline{Q}$ are the vectors that represent lower and upper limits for the active and reactive power generation.

d) Voltage Limits: Upper and lower voltage limits are introduced for all buses in order to avoid damage to voltage sensitive loads and equipment.

$$\underline{V} \leq \mid V \mid \leq \overline{V} \quad (10)$$

where, $\underline{V}$ and $\overline{V}$ are the vectors that represent lower and upper limits for voltage magnitudes.

e) Voltage Stability Limit: Depending on the considered scenario, line and generator outages may cause voltage stability problems which may then trigger cascading failures and partial/complete blackouts in the system. The commonly encountered reason for voltage instability is lack of reactive power support. While line outages may limit transfer of reactive power, generator outages may lead to reactive power shortage due to the missing reactive power source. The problem of voltage stability is quite challenging and has been the topic of numerous studies in the past. Several of these studies focus on the development of relevant indices which can then be used as proxies in formulating optimization problems requiring voltage stability constraints. In this work, a similar approach will be taken and a voltage stability index will be used for this purpose. While this index is well documented in the literature, in order to make the paper self contained, its derivation will be briefly reviewed below.

The problem of detecting and quantifying the proximity to voltage instability for all load buses in a given system has been investigated, and an effective and practical method was presented earlier in [10]. The method is based on the calculation of a stability index which can then be used to check voltage stability of a given operating point. The index is computed in three steps using only the bus admittance matrix and the bus voltages. System buses are initially separated into two sets representing the generator and load buses. The load bus voltages and generator bus currents are then expressed in terms of load bus currents and generator bus voltages. Derivation of these equations require partial inversion of the bus admittance matrix and results in the following partitioned matrix equation:

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = \begin{bmatrix} H \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} = \begin{bmatrix} Z_{LL} & F_{LG} \\ K_{GL} & Y_{GG} \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} \quad (11)$$

where, Z_{LL}, F_{LG}, K_{GL} and Y_{GG} are the submatrices of H matrix, V_L, I_L, V_G and I_G are load bus voltages, load bus currents, generator bus voltages and generator bus currents, respectively.

Using the top part of the above equation, an equivalent voltage V_{0j} for each load bus j is calculated using the elements of the submatrix F_{LG} and the generator bus voltages V_G [10]:

$$V_{0j} = -\sum_{i \in G} F_{ji} V_i \quad (12)$$

Finally, a voltage instability proximity index L_j for all load buses is calculated using V_{0j} and load bus voltages V_j:

$$L_j = \left| 1 + \frac{V_{0j}}{V_j} \right| \quad (13)$$

where:
G and L refer to generator and load buses,
i and j are generator and load bus indices,
V_i and V_j are complex voltage phasors.

Large number of simulated cases confirmed the reliability of the index in predicting the voltage stability of a given system and associated operating point. It is proven in [10] that the voltage stability will be guaranteed if $L_j < 1$. It turns out that this condition is sufficient, but not necessary, i.e. systems may still remain stable even when the calculated index violates above inequality. Hence, it is a conservative index but a reliable one if the main concern is avoiding voltage instability. It is therefore adopted in this work as a proxy for a constraint that will maintain voltage stability when considering an outage scenario. The constraint can then be simply included as:

$$L_j \leq L_{objective} \quad (14)$$

Choosing $L_{objective}$ as equal to 1.0 will yield voltage stable solutions as proven in [10]. On the other hand, choice of values less than 1.0 for $L_{objective}$ will yield more conservative solutions with respect to proximity to voltage instability.

Given the above description of various constraints needed to be incorporated into the optimization problem, the VSCOPF problem can now be written in compact form as follows:

$$\min_{P_G, Q_G, |V|, \theta} \; C_G^T P_G \tag{15}$$

subject to:
1. Power Balance Equations: (2), (3)
2. Line Flow Limits: (7)
3. Generator Limits: (8), (9)
4. Voltage Limits: (10)
5. Voltage Stability Limit: (14)

Active power generation (P_G), reactive power generation (Q_G), bus voltage magnitudes ($\mid V \mid$) and phase angles (θ) at each bus will be the control variables in the above problem.

B. Forming Matrices for DG Placement Method

The impact of each considered outage scenario on the power grid will be different and may require DGs to be placed at different buses. In order to determine all possible options of placing a single DG for a given outage scenario, the VSCOPF problem will be solved as many times as the number of candidate locations for DGs (typically only PQ buses or a subset of them) in the system. These solutions will then be repeated in a similar fashion for each outage scenario. At the end of this process, one can build an assignment matrix where for each outage scenario a set of candidate locations for placing a single DG will be marked. A similar matrix L can be created where for each of these DG placements, the maximum element of the corresponding stability index vector L_j for bus j will be recorded. The elements of L matrix will be used to gauge the effectiveness of a given DG placement at bus j for the outage scenario i, i.e. the smaller $L(i, j)$, the more effective the placement.

The DG assignment table is stored as a binary incidence matrix A where the rows and columns correspond to outage scenarios and placed DGs respectively. Note that this is very similar to the incidence matrix formed for optimal meter placement in [11]. If the VSCOPF problem cannot be solved, i.e. it cannot find a feasible solution by placing a single DG at any of the possible locations, then the corresponding row of A will be null indicating that for that outage scenario voltage instability cannot be avoided by placing a single DG anywhere in the system. In this work, such cases are marked and subsequently processed by considering placement of pairs of DGs instead of a single DG.

Consider a power system having n_B buses and M outage scenarios, where each outage case may involve multiple line and/or generator outages. A flowchart of the proposed approach is given in Fig. 1. Note that the index i refers to the candidate buses to place DGs and the index k refers to the considered outage scenarios.

For certain considered outage scenarios, VSCOPF may fail to find a feasible solution no matter where DG is placed. Such cases will be indicated by a null row in the binary assignment matrix A. Such cases will then be studied by considering placement of pairs of DGs instead of a single DG. Thus, all possible combinations of DG pairs that can be placed for a given set of n_B buses will have to be calculated as:

$$\binom{n_B}{2} = \frac{n_B!}{(n_B-2)!2!} \tag{16}$$

and placement of these possible pairs will have to be tested one pair at a time for feasibility. Note that, if none of the pair combinations yields a feasible solution, then simultaneous placement of larger numbers of DGs will have to be investigated. In this work, only single DG and pairs of DGs are considered. Fig. 2 shows the way candidate DG pairs are selected and placed when certain rows of initial A matrix are null indicating the infeasibility of solving the optimization problem by placing only single DGs. Details of extending the rows of A matrix to account for simultaneous placement of DG pairs will be described in section III-C below.

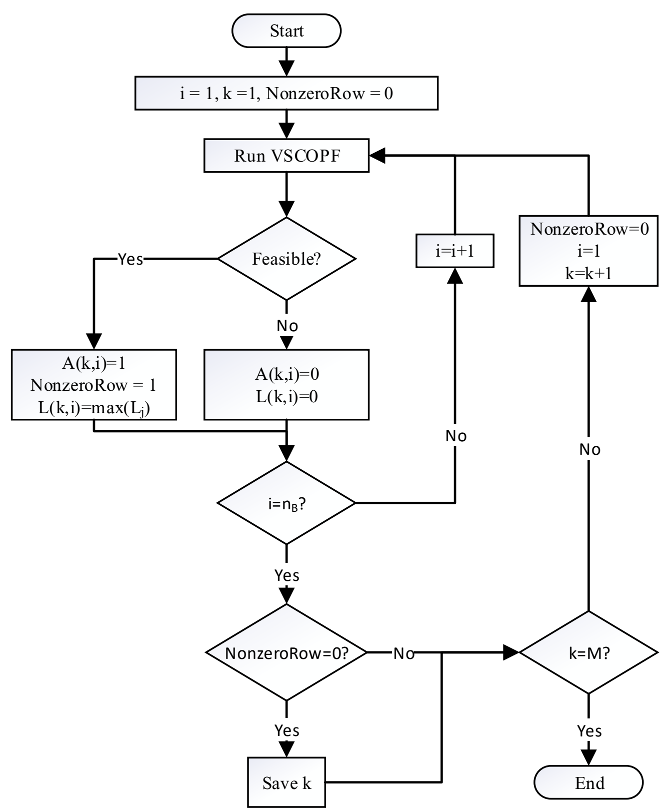

Fig. 1. Formation of A *and* L matrices for only single DG

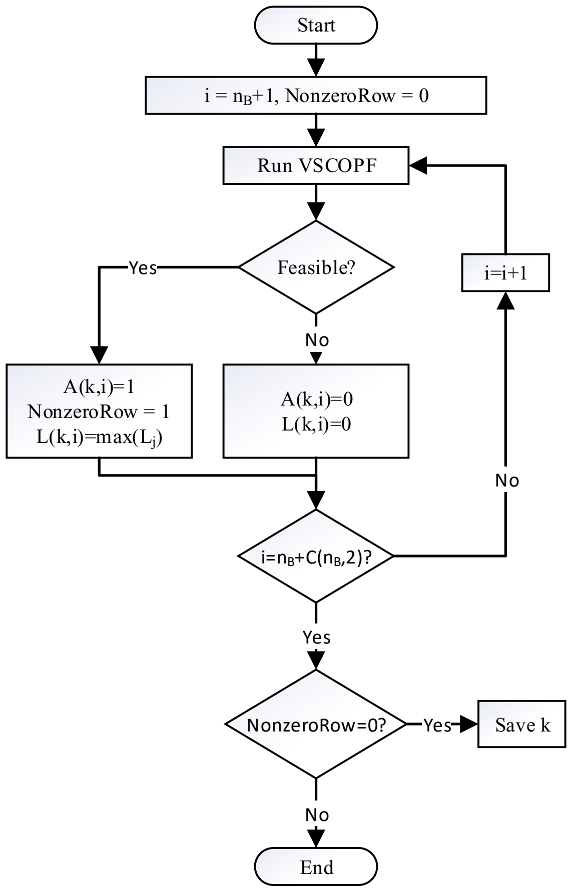

Fig. 2. Formation of A *and* L matrices for pairs of DGs

C. DG Placement Method

Let us assume that the repeated VSCOPF solutions are carried out and assignment matrix A and stability index matrix L are formed for a considered set of outage scenarios and candidate load buses for placing DGs as described above. Then, the following integer programming (IP) problem can be formulated [11] to determine the minimum set of DGs that are required to be placed in order to ensure feasible operating conditions under all considered outage scenarios:

$$\begin{aligned} \min \; & c^T x \\ s.t. \; & Ax \geq b \end{aligned} \tag{17}$$

where:
c is the cost vector which is formed based on the entries of the matrix L considering all M outage scenarios as follows:

$$c_i = \|(L_i)\| = \sqrt{\sum_{k=1}^{M} \{L(k,i)\}^2} \tag{18}$$

x is a binary vector whose dimension depends on matrix A and defined as:

$$x_i = \begin{cases} 1 & \text{if a DG or DG pair is assigned} \\ & \text{to bus } i \text{ or bus pair } i \\ 0 & \text{otherwise} \end{cases}$$

Note that in the case of DG pair placements, paired buses will be associated with a separate fictitious bus whose number will be larger than n_B and the corresponding entry in x will be assigned a 1.
b is a binary vector with the first n_B elements equal to 1, and the remaining ones are 0.
A is a matrix which contains the candidacy information for all single and paired DGs. Details of forming this coefficient matrix will be explained next.

Forming the [A] Matrix:
If VSCOPF finds at least one feasible solution for each outage scenario by placing a single DG, then the algorithm in Fig. 1 will be sufficient for building A matrix. Else, some rows of A may remain null requiring simultaneous placement of more than one DG to maintain voltage stability. In this work it was sufficient to consider only up to two (or a pair of) DGs to find feasible solutions. The flowchart in Fig. 2 shows how to form A using pairs of DGs for those infeasible outage scenarios. Moreover, the constraints will be augmented by additional equations and variables increasing the row and column sizes of A as illustrated below using a tutorial example. Augmented matrix A can be written in partitioned form as:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{19}$$

Note that A_{11} refers to the initial A matrix formed considering single DG placements only. The augmented rows and columns will be provided by the sub-matrices A_{12}, A_{21} and A_{22} whose entries will be defined below:
i) A_{11}: Apply the algorithm in Fig. 1 to form the initial A matrix, let it be A_{11} as shown below.

$$A_{11} = \begin{array}{cl} \begin{array}{ccccccc} \text{Bus-1} & \text{Bus-2} & \text{Bus-3} & \dots & \text{Bus-i} & \dots & \text{Bus-}n_B \end{array} & \\ \left[\begin{array}{ccccccc} 0 & 1 & 0 & & 1 & & 0 \\ 1 & 0 & 0 & & 0 & & 0 \\ 0 & 0 & 0 & & 0 & & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & A(k,i) & & \vdots \\ \vdots & \vdots & \vdots & & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & & 0 & & 1 \end{array}\right] & \begin{array}{l} \text{Outage-1} \\ \text{Outage-2} \\ \text{Outage-3} \\ \vdots \\ \text{Outage-k} \\ \vdots \\ \text{Outage-M} \end{array} \end{array}$$

Note that the third (3^{rd}) row is null, indicating that no feasible solution can be found for outage scenario-3 considering single DG placements. Therefore, for this scenario, in addition to single DGs, DG pair placements should also be investigated in order to maintain voltage stability. This requires finding all possible DG pair combinations, sequentially assigning them fictitious bus numbers starting from $n_B + 1$ and solving the

VSCOPF problem by placing these pairs one at a time. This will extend the columns of A matrix by the number of newly assigned fictitious bus numbers and creates a submatrix A_{12} representing the newly added columns.

ii) A_{12}: Assume that applying the flowchart in Fig. 2 yields:

$$A_{12} = \begin{array}{c} \begin{array}{ccccccc} n_B+1 & n_B+2 & n_B+3 & \dots & n_B+i & \dots & n_B+\binom{n_B}{2} \end{array} \\ \left[\begin{array}{ccccccc} 0 & 0 & 0 & & 0 & & 0 \\ 0 & 0 & 0 & & 0 & & 0 \\ 0 & 1 & 0 & & 1 & & 0 \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & 0 & & 0 & & 0 \end{array}\right] \end{array} \begin{array}{l} \\ \text{Outage-1} \\ \text{Outage-2} \\ \text{Outage-3} \\ \\ \text{Outage-M} \end{array}$$

In this example, several feasible solutions are assumed to be found by placing DG pairs as indicated by 1's in row 3.

iii) A_{21} *and* A_{22}: In assigning DG pairs, in order to establish consistency between the created fictitious buses (e.g. representing the pair of buses (n_B+2) and (n_B+i)) and actual buses, additional equations are required to indicate which bus pair belongs to which actual buses. Thus, a truth table can be formed as shown in Table I for the example where the fictitious buses (n_B+2) and (n_B+i) refer to bus pairs 2-3 and 2-i respectively.

TABLE I
TRUTH TABLE

Solution 1

Input	Outputs	
n_B+2	Bus-2	Bus-3
1	1	1
0	unknown	unknown

Solution 2

Input	Outputs	
n_B+i	Bus-2	Bus-i
1	1	1
0	unknown	unknown

These tables can now be converted into the following equations:

$$x_2 + x_3 \geq 2x_{n_B+2}$$

$$x_2 + x_i \geq 2x_{n_B+i}$$

and accordingly A_{21} and A_{22} can be formed as follows:

$$A_{21} = \begin{array}{c} \begin{array}{ccccccc} \text{Bus-1} & \text{Bus-2} & \text{Bus-3} & \dots & \text{Bus-i} & \dots & \text{Bus-}n_B \end{array} \\ \left[\begin{array}{ccccccc} 0 & 1 & 1 & & 0 & & 0 \\ 0 & 1 & 0 & & 1 & & 0 \end{array}\right] \end{array}$$

$$A_{22} = \begin{array}{c} \begin{array}{ccccccc} n_B+1 & n_B+2 & n_B+3 & \dots & n_B+i & \dots & n_B+\binom{n_B}{2} \end{array} \\ \left[\begin{array}{ccccccc} 0 & -2 & 0 & & 0 & & 0 \\ 0 & 0 & 0 & & -2 & & 0 \end{array}\right] \end{array}$$

IV. SIMULATION RESULTS

The proposed methodology is simulated in Matlab [12] using the IEEE 118-bus system. The computations are carried out using the Northeastern University Discovery cluster which is housed at the Massachusetts Green High Performance Computing Center (MGHPCC), which provides access to over 20,000 CPU cores and over 200 GPUs.

In this study, 40 extreme event outage scenarios are created. Each scenario involves outages of three random lines. Loads (both active and reactive) are increased by 100% to create highly stressed operating conditions and pushing the system limits. Moreover, $L_{objective}$ is intentionally reduced in order to create cases requiring paired DG placements for some of the simulated scenarios.

Simulation results are shown in Table II. Out of the simulated 40 outage scenarios, 32 of them required only a single DG placement at bus 44 in order to maintain voltage stability. The remaining 8 outage scenarios required three more DGs to be placed in order to maintain a voltage stable system. To obtain those results, initially, the candidate locations are found by using the single DG placement algorithm in Fig. 1. However, voltage instability cannot be avoided for 8 out of 40 scenarios where corresponding rows of A are found to be null. Then, candidate locations are determined to place paired DGs for those remaining 8 scenarios. Finally, DGs are placed at buses 21, 44, 52, and 95 by using the integer programming (IP) problem formulation and solution.

In order to illustrate the overall procedure, one of the outage scenarios is chosen as an example. In this scenario three of the transmission lines are taken out of service, which makes the system voltage unstable. Applying the DG placement procedure, it can be shown that by placing a DG pair at either one of the twelve possible combinations formed among the sets of buses $\{43, 44, 45\}$, and $\{51, 52, 53, 58\}$, the system can be made voltage stable. Among these, since the pair 44-52 also helps other considered outage scenarios, it is chosen as the solution. Fig. 3 shows the voltage magnitude profile of the system buses before and after the placement of DGs at buses 44 and 52. Note that voltage magnitude limits used for this study are between 0.9 and 1.06 per unit in all simulations.

VSCOPF solution is repeated tens of thousands of times for the IEEE 118-bus system. While it is manageable for such a test system, the computational burden can rapidly increase for much larger size power grids. In anticipation of applying this method to very large scale problems, utilization of the high-performance computing facility of the institution is considered and actually implemented for the 118-bus system. Note the total number of required repeat solutions for the IEEE 118-bus system given in Table III. Implementing the outage scenarios in the Discovery Cluster by accessing 50 of its cpu cores, yields each scenario solution on average in about 5 seconds. It should also be noted that it is possible to implement and execute the proposed algorithm also for very large scale power

TABLE II
DG PLACEMENT RESULTS

Algorithm	No. of Stabilized Scenarios	DG(s) Placed at Bus
Single DG Placement Algorithm	32	44
Multi DG Placement Algorithm	40	21, 44, 52, 95

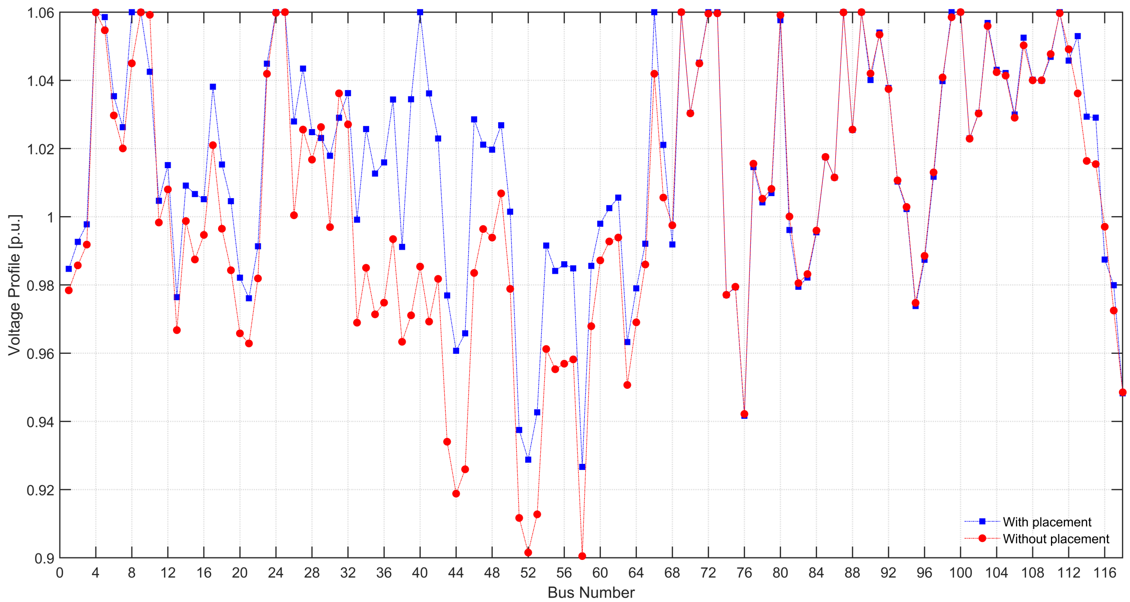

Fig. 3. Voltage Profiles for the Outage Scenario

TABLE III
TOTAL NUMBER OF REPEATED SOLUTIONS

Number of Scenarios	Total Number of Candidate Combinations	Number of Solutions
40	64	2560
8	2016	16128
		18688

grids efficiently by using a lot more of the 20,000 cpu cores of the cluster.

V. CONCLUSION

This paper is concerned about resiliency of power grids during extreme events which may cause multiple outages in the system. The primary focus of the study is on maintaining voltage stability during such events and accomplishing this by installing a small number of DGs in the system. This is proposed to be done in two stages, where in the first stage possible candidate locations for each considered outage scenario are identified via repeated optimization solutions and this candidate list is pruned via an integer programming formulation to reach a final set of recommended DG locations. While being an off-line procedure, its heavy cpu requirements are shown to be readily addressed by using a multi-core high performance computer cluster and one illustrative implementation of it is presented using the 118-bus system as an example.

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