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A Comparative Study of Frequency-domain Finite Element Updating Approaches Using Different Optimization Procedures

Xinjun DONG, Yang WANG*

School of Civil and Environmental Engineering, Georgia Institute of Technology
Atlanta, GA 30332, USA *yang.wang@ce.gatech.edu*

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Abstract
In order to achieve a more accurate finite element (FE) model for an as-built structure, experimental data collected from the actual structure can be used to update selected parameters of the FE model. The process is known as FE model updating. This research compares the performance of two frequency-domain model updating approaches. The first approach minimizes the difference between experimental and simulated modal properties, such as natural frequencies and mode shapes. The second approach minimizes modal dynamic residuals from the generalized eigenvalue equation involving stiffness and mass matrices. Both model updating approaches are formulated as an optimization problem with selected updating parameters as optimization variables. This research also compares the performance of different optimization procedures, including a nonlinear least-square, an interior-point and an iterative linearization procedure. The comparison is conducted using a numerical example of a space frame structure. The modal dynamic residual approach shows better performance than the modal property difference approach in updating model parameters of the space frame structure.

1 INTRODUCTION

During the past few decades, significant achievements have been made in developing high-resolution finite element (FE) models of various engineering structures. However, owing to the complexity of civil structures, structural behavior predicted by the FE simulation models (usually built according to design drawings) is generally different from the behavior of the actual constructed structure. For higher simulation accuracy, it is necessary to update the finite element model based on experimental measurements on the actual structure. Numerous FE model updating algorithms have been developed and practically applied in the past few decades [1]. Most algorithms can be categorized into two groups, i.e. frequency-domain approaches and time-domain approaches. Frequency-domain approaches update an FE model using frequency-domain structural characteristics extracted from experimental measurements, such as natural frequencies and vibration mode shapes [2, 3]. On the other hand, time-domain approaches directly utilize measured time histories for model updating, with estimator techniques such as extended Kalman filters [4].

This research compares the performance of two frequency-domain model updating approaches. The first approach being studied attempts to minimize the difference between experimental and simulated modal properties. This approach will be referred as the modal property difference approach. For example, Jaishi & Ren [2] proposed an objective function consisting of difference in simulated and experimentally-measured modal flexibilities for updating the model of a beam structure. Another category of model updating approach, referred as the modal dynamic residual approach, minimizes modal dynamic residuals from the generalized eigenvalue equation involving stiffness and mass matrices. For example, Farhat and Hemez [3] proposed an iterative least-square algorithm for updating element stiffness and mass properties, which effectively minimizes the 2-norm of the modal dynamic residual vector. In essence, with selected updating parameters as optimization variables, both modal property difference and modal dynamic residual approaches are formulated as an optimization problem. Each optimization problem can be solved by various optimization procedures. This research compares the performance of a number of optimization procedures, including a nonlinear least-square approach, an interior-point approach [5], and an iterative linearization approach [6].

The rest of the paper is organized as follows. The formulations of both model updating approaches are presented first. Multiple relevant optimization procedures are then described. Performance of both model updating approaches using different optimization procedures is compared with a numerical example of a space frame structure. Finally, a summary and discussion are provided.

2 MODEL UPDATING APPROACHES AND OPTIMIZATION PROCEDURES

2.1 Model updating approaches

For updating the stiffness parameters of a linear structure, the stiffness matrix can be formulated as a matrix function of the parameter vector $\boldsymbol{\alpha} \in \mathbb{R}^{n_\alpha}$. Notation n_α represents the total number of updating parameters; the j-th ($j = 1,\ldots, n_\alpha$) entry of $\boldsymbol{\alpha}$, α_j, represents a stiffness parameter to be updated (e.g. a Young's modulus value or the stiffness value of a support spring), which is to be treated as an optimization variable in the updating process. In this study, $\boldsymbol{\alpha}$ represents the relative change percentage from initial value of each parameter;

$$\mathbf{K}(\boldsymbol{\alpha}) = \mathbf{K_0} + \sum_{j=1}^{n_\alpha} \alpha_j \mathbf{K}_{0,j} \tag{1}$$

where $\mathbf{K}$ denotes the structural stiffness matrix; $\mathbf{K}_0$ is the initial nominal stiffness matrix prior to model updating (usually generated based on design drawings and nominal material properties); $\mathbf{K}_{0,j}$ is a constant matrix that corresponds to the contribution of the associated updating parameter α_j. In addition, it is assumed that the structural mass matrix is accurate enough and does not require updating.

2.1.1 Modal property difference approach

The modal property difference approach is usually formulated as an optimization problem that attempts to minimize the difference between experimental and simulated eigenvalues and eigenvectors of the structural system. In comparison with experimental modal properties ($\boldsymbol{\lambda}$ and $\boldsymbol{\psi}$) obtained from dynamic testing on the actual structure in the field, the simulated modal properties ($\boldsymbol{\lambda}^{\mathrm{FE}}$ and $\boldsymbol{\psi}^{\mathrm{FE}}$) are generated by the FE model. In practice, not all degrees of freedom (DOFs) can be instrumented and measured. To reflect this in the formulation, $\boldsymbol{\psi}^{\mathrm{m}}$ represents entries in $\boldsymbol{\psi}$ that can be measured, and $\boldsymbol{\psi}^{\mathrm{u}}$ represents these not measured. Eq. 2 shows the optimization problem for the modal property difference approach. The optimization variables include the vector $\boldsymbol{\alpha}$ containing stiffness parameters to be updated, and the simulated modal properties ($\boldsymbol{\lambda}^{\mathrm{FE}}$, $\boldsymbol{\psi}^{\mathrm{FE,m}}$ and $\boldsymbol{\psi}^{\mathrm{FE,u}}$).

$$\text{minimize} \sum_{i=1}^{n_{\text{modes}}} \left\{ \left(\frac{\lambda_i^{\text{FE}} - \lambda_i}{\lambda_i} \right)^2 + \left(\frac{1 - \sqrt{\text{MAC}_i}}{\text{MAC}_i} \right)^2 \right\} \tag{2a}$$

$$\text{subject to } \left[\mathbf{K}(\boldsymbol{\alpha}) - \lambda_i^{\text{FE}} \mathbf{M} \right] \begin{Bmatrix} \boldsymbol{\psi}_i^{\text{FE,m}} \\ \boldsymbol{\psi}_i^{\text{FE,u}} \end{Bmatrix} = \{\mathbf{0}\} \tag{2b}$$

$$\text{MAC}_i = \frac{\left\{ (\boldsymbol{\psi}_i^{\text{m}})^{\text{T}} \boldsymbol{\psi}_i^{\text{FE,m}} \right\}^2}{\|\boldsymbol{\psi}_i^{\text{m}}\|_2^2 \left\|\boldsymbol{\psi}_i^{\text{FE,m}}\right\|_2^2}, \quad i = 1, \cdots, n_{\text{modes}} \tag{2c}$$

$$\text{L}_\alpha \leq \boldsymbol{\alpha} \leq \text{U}_\alpha \tag{2d}$$

$$L_{\boldsymbol{\psi}_i^{\text{FE,m}}} \leq \boldsymbol{\psi}_i^{\text{FE,m}} \leq U_{\boldsymbol{\psi}_i^{\text{FE,m}}}, \quad i = 1, \cdots, n_{\text{modes}} \tag{2e}$$

$$L_{\boldsymbol{\psi}_i^{\text{FE,u}}} \leq \boldsymbol{\psi}_i^{\text{FE,u}} \leq U_{\boldsymbol{\psi}_i^{\text{FE,u}}}, \quad i = 1, \cdots, n_{\text{modes}} \tag{2f}$$

where n_{modes} denotes the number of available experimentally-measured modes being used for model updating; λ_i^{FE} and λ_i represent the i-th simulated (from FE model) and experimental eigenvalues, respectively; $\|\cdot\|_2$ denotes the 2-norm of a vector. MAC_i represents the modal assurance criterion evaluating the difference between the i-th simulated and experimental eigenvectors at the measured DOFs, i.e. between $\boldsymbol{\psi}_i^{\text{FE,m}}$ and $\boldsymbol{\psi}_i^{\text{m}}$; L_α and U_α denote the lower and upper bounds for the optimization variable vector $\boldsymbol{\alpha}$ Similarly, $L_{\boldsymbol{\psi}_i^{\text{FE,m}}}$, $L_{\boldsymbol{\psi}_i^{\text{FE,u}}}$ and $U_{\boldsymbol{\psi}_i^{\text{FE,m}}}$, $U_{\boldsymbol{\psi}_i^{\text{FE,u}}}$ represents the lower and upper bounds for the $\boldsymbol{\psi}_i^{\text{FE,m}}$ and $\boldsymbol{\psi}_i^{\text{FE,u}}$. Note that the sign "≤" is overloaded to represent element-wise inequality; $\mathbf{M}$ denotes the structural mass matrix.

2.1.2 Modal dynamic residual approach

In comparison with the modal property difference approach, the modal dynamic residual approach attempts to minimize the residuals of the generalized eigenvalue equations. Matrices given by the FE model are used in combination with experimentally measured modal properties for calculating the modal dynamic residuals during evaluation of the objective function.

$$\text{minimize} \sum_{i=1}^{n_{\text{modes}}} \left\| \left[\mathbf{K}(\boldsymbol{\alpha}) - \lambda_i \mathbf{M} \right] \begin{Bmatrix} \boldsymbol{\psi}_i^{\text{m}} \\ \boldsymbol{\psi}_i^{\text{FE,u}} \end{Bmatrix} \right\|^2 \tag{3a}$$

$$\text{subject to } \text{L}_\alpha \leq \boldsymbol{\alpha} \leq \text{U}_\alpha \tag{3b}$$

$$L_{\boldsymbol{\psi}_i^{\text{FE,u}}} \leq \boldsymbol{\psi}_i^{\text{FE,u}} \leq U_{\boldsymbol{\psi}_i^{\text{FE,u}}}, \quad i = 1, \cdots, n_{\text{modes}} \tag{3c}$$

where $\|\cdot\|$ denotes any vector norm; λ_i and $\boldsymbol{\psi}_i^{\text{m}}$ denote the i-th experimental eigenvalue and eigenvector entries corresponding to measured DOFs; $\boldsymbol{\psi}_i^{\text{FE,u}}$ corresponds to the unmeasured DOFs of the i-th eigenvector. In addition to stiffness parameters ($\boldsymbol{\alpha}$), unmeasured entries in mode shape vectors ($\boldsymbol{\psi}^{\text{FE,u}}$), are also treated as optimization variables in the modal dynamic residual approach.

2.2 Optimization procedures

Given that both model updating approaches are formulated as an optimization problem, a number of optimization procedures can be used for solving the problem. For example, MATLAB optimization solvers are commonly adopted for solving optimization problems. This research mainly focuses on two MATLAB optimization solvers, i.e. `lsqnonlin` and `MultiStart`. Furthermore, for the modal dynamic residual approach, an iterative linearization procedure can also be used and is added into the comparison [6].

2.2.1 MATLAB lsqnonlin

In this research, a nonlinear least-square optimization solver, '`lsqnonlin`' in MATLAB optimization toolbox [5], is first adopted to numerically solve the optimization problems. In Eq. 2, for the modal property difference approach, the optimization variable contains the stiffness parameters, $\boldsymbol{\alpha}$, and the simulated modal properties, $\boldsymbol{\lambda}^{\mathrm{FE}}$, $\boldsymbol{\psi}^{\mathrm{FE,m}}$ and $\boldsymbol{\psi}^{\mathrm{FE,u}}$. In Eq. 3, for the modal dynamic residual approach, the optimization variables include both the updating parameter, $\boldsymbol{\alpha}$, and the mode shape entries that correspond to the unmeasured DOFs, $\boldsymbol{\psi}^{\mathrm{FE,u}}$.

The optimization solver seeks a minimum of the objective function through Levenberg-Marquardt algorithm, which is a combination of the steepest descent and the Gauss-Newton algorithm [7]. At every iteration, the algorithm linearizes the objective function (Eq. 2 or Eq. 3) with respective to the corresponding optimization parameters to determine the searching direction. When determining the step size at every iteration, the Levenberg-Marquardt algorithm includes a damping term to balance the contribution from the steepest descent and Gauss-Newton algorithm. When the current solution is far from a local optimum, the damping term value is set to be small, so that the algorithm is close to the steepest descent algorithm. On the other hand, when the current solution is close to a local optimum, the damping term value will be increased, and it becomes closer to the Gauss-Newton algorithm. The drawback of Levenberg-Marquardt algorithm implemented in MATLAB is that it does not allow to set the upper and lower bounds for the optimization variables.

2.2.2 MATLAB MultiStart

Because both optimization problems, Eq. 2 and Eq. 3, are non-convex, the nonlinear least-square solver can be easily trapped into a local optimum near the starting point (i.e. $\boldsymbol{\alpha} = \mathbf{0}$). To increase the chance of finding a more optimal solution, the search can be started at other values of $\boldsymbol{\alpha}$ within the bounds of $\boldsymbol{\alpha}_L$ and $\boldsymbol{\alpha}_U$ (Eq. 2 and Eq. 3). Designed towards this purpose, the '`MultiStart`' optimization solver in MATLAB randomly generates a number of starting points within the assigned bounds based on the uniform distribution. A local solver, i.e. `lsqnonlin` and `fmincon`, is then adopted to find a local minimum from each starting point. The `fmincon` optimization solver seeks a minimum of the objective function through the interior-point algorithm, which performs a direct-step or a conjugate-gradient search at each iteration [5]. Finally, the smallest value among all the local minima is determined as the best solution. The more starting points that the `MultiStart` solver generates and searches from, the higher possibility that the final solution is closer to the global optimum. The associated downside, as expected, is the increased computational effort. Therefore, the selection of number of starting points is usually based on experience.

2.2.3 Iterative linearization procedure

As previously described, Eq. 3 leads to a non-convex optimization problem that is generally difficult to solve. However, if mode shape entries for unmeasured DOFs, $\boldsymbol{\psi}^{\mathrm{FE,u}}$ were held constant and not treated as an optimization variable, Eq. 3 degenerates to a convex optimization problem, for which global optimality can be guaranteed by convex optimization algorithms [8-10]. The only optimization variable is stiffness parameter vector, $\boldsymbol{\alpha}$, and the problem can be efficiently solved. Likewise, if the system parameter vector, $\boldsymbol{\alpha}$, were held constant, Eq. 3 also degenerates to a convex optimization problem with optimization variable $\boldsymbol{\psi}^{\mathrm{FE,u}}$ only. Therefore, an iterative linearization procedure can be adopted for finding a solution of the optimization problem in Eq. 3. The pseudo code and more detailed description of the procedure can be found in a previous study [6].

3 NUMERICAL EXAMPLE

In this section, the performance of two model updating approaches through different optimization procedures is evaluated using a space frame structure example (Figure 1). The space frame model contains 46 nodes, each node with 6 DOFs. Although mainly a frame structure, the segmental cross bracings in top plane and two side planes are truss members. Transverse and vertical springs (k_y and k_z) are allocated at both ends of the bridge to simulate non-ideal boundary conditions. Detailed description of the structural stiffness parameters can be found in a previous study [6]. It is assumed that 14 tri-axial accelerometers are instrumented for model updating. The 14 accelerometers are uniformly spaced on the structure, as shown in Figure 1. Section 3.1 describes the structural model updating using the presented model updating approaches and optimization procedures. Section 3.2 describes a sensitivity analysis of the updating parameters for the two model updating approaches.

3.1 Model updating

The main model updating variable, $\boldsymbol{\alpha}$, includes all the stiffness parameters to be updated. Shown in Zhu, et al [6], these parameters are divided into three categories. The first category includes six parameters, which are elastic moduli of the frame members along the entire length of the frame structure (E_1~ E_5) and the diagonal bracing truss members in top plane (E_6). The second category contains ten parameters, which are the elastic moduli of diagonal bracing truss members in two side planes for different segments (E_{S2}~ E_{S11}). The third category contains stiffness parameters of the four types of support springs (k_{y1}, k_{z1}, k_{y2}, and k_{z2}). In total, twenty stiffness parameters will be updated, i.e. n_α = 20. Table 1 lists the actual correct value of stiffness parameter, $\boldsymbol{\alpha}$. These values are the ideal correct solutions of the model updating processes.

Modal properties of the structure with actual values of $\boldsymbol{\alpha}$ are used as the "experimental" properties, i.e. λ_i and $\boldsymbol{\psi}_i^{\mathrm{m}}$ in Eq. 2 and Eq. 3. Each model updating starts with nominal stiffness parameter values, i.e. 0% for all entries in $\boldsymbol{\alpha}$. Using different updating approaches and optimization procedures, model updating results from five cases are studied (Table 2). MATLAB `lsqnonlin` and `MultiStart` are applied to both modal property difference and

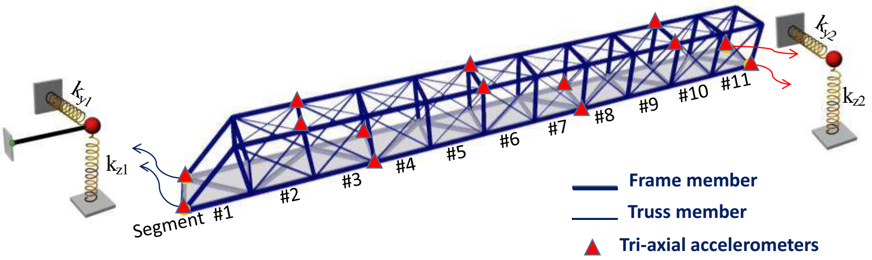

Figure 1. Illustration of space frame structure and sensor instrumentation

Table 1 Actual values of updating stiffness parameters $\boldsymbol{\alpha}$ (relative change percentage from initial parameter values: %)

Quantities throughout the entire bridge						Spring			
E_1	E_2	E_3	E_4	E_5	E_6	k_{y1}	k_{z1}	k_{y1}	k_{z2}
5.00	5.00	-5.00	-10.00	5.00	-5.00	-30.00	60.00	-30.00	60.00
Segmental quantities									
E_{S2}	E_{S3}	E_{S4}	E_{S5}	E_{S6}	E_{S7}	E_{S8}	E_{S9}	E_{S10}	E_{S11}
-10.00	-10.00	-10.00	-5.00	-5.00	-5.00	-15.00	-10.00	-5.00	-5.00

Table 2 Model updating cases for comparison

Case #	Objective function	Optimization procedure
1(a)	Modal property difference (Eq. 2)	`lsqnonlin`
1(b)		`MultiStart` with `fmincon`
2(a)	Modal dynamic residual (Eq. 3)	`lsqnonlin`
2(b)		`MultiStart` with `lsqnonlin`
2(c)		Iterative linearization

modal dynamic residual approaches, while the iterative linearization procedure is only applied to the modal dynamic residual approach. The updated results of five cases will be compared with accurate values in Table 1 to evaluate the performance of the model updating approaches and the optimization procedures. For MATLAB `lsqnonlin`, the initial values of updating parameters are set to be zero. The upper bound and lower bounds of all parameters are set to be 1 and -1, respectively, which means that error of nominal stiffness parameters is assumed to be no larger than ±100%. When utilizing MATLAB `MultiStart`, the number of starting points is selected to be 30. For each approach, the updating is performed assuming different numbers of measured modes are available (i.e. modes corresponding to the lowest 3 to 6 natural frequencies).

In Case 1(a), MATLAB `lsqnonlin` solver is applied on the modal property difference approach with all initial values of updating parameters, $\boldsymbol{\alpha}$, set to zero. The updating results show that the updated optimal values of parameters all stay very close to zero, with a standard deviation of 0.0002%. This means the optimization solver stopped at a local minimum near the starting point. As a result, errors of the updated parameters are large and the updating process provides little benefit.

In Case 1(b), the MATLAB `MultiStart` solver is applied on the modal property difference approach, and MATLAB `fmincon` is selected as the local solver for better performance. The `fmincon` solver is selected over `lsqnonlin` because the Levenberg-Marquardt algorithm implemented in MATLAB `lsqnonlin` does not allow to set the lower and upper bounds for the optimization variables. As a result, the updating results of some starting points turns out to be beyond the assigned bounds. Table 3 summarizes the model updating results. The `MultiStart` solver generates updated values of $\boldsymbol{\alpha}$ away from zero, i.e. the optimal solution has gone farther away from initial starting point, compared with Case 1(a). However, all the updated parameter changes are apparently different from the corresponding correct values shown in Table 1. For clear demonstration of updating accuracy, Figure 2 plots the relative errors of the updating results. The figure shows that the updating results have large errors, particularly for support spring stiffness (k_{y1}, k_{z1}, k_{y2}, and k_{z2}). In addition, by observing the 30 sets of optimal results from the 30 starting points generated by `MultiStart`, most of the optimal solutions do not converge to the same set of value $\boldsymbol{\alpha}$.

In Case 2(a), the optimization problem in the modal dynamic residual approach is solved by MATLAB `lsqnonlin` solver. In Case 2(b), the problem is solved by the MATLAB `MultiStart` solver with `lsqnonlin` as local solver. The results for the two cases are essentially identical, and therefore, both are presented by Table 4. In addition, the results also exactly matches between selecting `lsqnonlin` and `fmincon` as the local solver. For every available number of modes, all the updated parameter changes are close to the ideal correct values shown in Table 1. Figure 3 plots the relative errors of the updating parameters, with different numbers of available modes. It shows that the largest updating error in all four scenarios is less than 0.006%. Furthermore, when using `MultiStart`, searches from all the

Table 3 Updated stiffness parameter changes (%) by minimizing modal property difference (Case 1(b) MATLAB `MultiStart`)

Available modes	Quantities throughout the entire bridge						Spring			
	E_1	E_2	E_3	E_4	E_5	E_6	k_{y1}	k_{z1}	k_{y1}	k_{z2}
3 modes	9.36	0.49	-1.93	-8.84	-1.36	-11.05	-32.79	82.00	-4.63	-13.74
4 modes	-0.13	2.89	0.24	-2.29	-0.41	-0.12	-18.10	23.78	-28.66	19.44
5 modes	9.23	-0.49	4.06	-5.18	-5.90	-5.14	-17.26	-6.85	-4.51	25.08
6 modes	3.72	1.78	-0.91	1.50	-6.68	1.24	-21.57	26.27	6.63	26.27
Available modes	Segmental quantities									
	E_{S2}	E_{S3}	E_{S4}	E_{S5}	E_{S6}	E_{S7}	E_{S8}	E_{S9}	E_{S10}	E_{S11}
3 modes	-13.97	9.4	-3.46	5.88	-6.44	2.93	-3.21	6.06	-0.92	4.03
4 modes	-0.06	-1.77	0.04	0.89	-0.89	1.31	-0.15	-0.67	-4.94	-6.75
5 modes	3.02	6.24	-1.19	-4.04	-1.41	-3.47	-2.36	4.83	6.92	-1.47
6 modes	-0.15	-0.9	-1.54	5.14	3.61	-2.32	-10.39	-2.53	-3.68	2.71

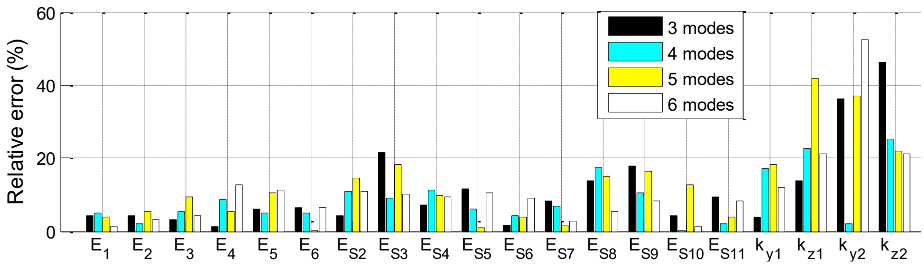

Figure 2. Relative errors of the updated parameters by minimizing modal property difference (Case 1(b) MATLAB `MultiStart`)

Table 4 Updated stiffness parameter changes (%) by minimizing modal dynamic residuals (Case 2(a) MATLAB `lsqnonlin` and Case 2(b) MATLAB `MultiStart`)

Available modes	Quantities throughout the entire bridge						Spring			
	E_1	E_2	E_3	E_4	E_5	E_6	k_{y1}	k_{z1}	k_{y1}	k_{z2}
3 modes	5.00	5.00	-5.00	-10.00	5.00	-5.00	-30.00	60.00	-30.00	60.00
4 modes	5.00	5.00	-5.00	-10.00	5.00	-5.00	-30.00	60.00	-30.00	60.00
5 modes	5.00	5.00	-5.00	-10.00	5.00	-5.00	-30.00	60.00	-30.00	60.00
6 modes	5.00	5.00	-5.00	-10.00	5.00	-5.00	-30.00	60.00	-30.00	60.00
Available modes	Segmental quantities									
	E_{S2}	E_{S3}	E_{S4}	E_{S5}	E_{S6}	E_{S7}	E_{S8}	E_{S9}	E_{S10}	E_{S11}
3 modes	-10.00	-10.00	-10.00	-5.00	-5.00	-5.00	-15.00	-10.00	-5.00	-5.00
4 modes	-10.00	-10.00	-10.00	-5.00	-5.00	-5.00	-15.00	-10.00	-5.00	-5.00
5 modes	-10.00	-10.00	-10.00	-5.00	-5.00	-5.00	-15.00	-10.00	-5.00	-5.00
6 modes	-10.00	-10.00	-10.00	-5.00	-5.00	-5.00	-15.00	-10.00	-5.00	-5.00

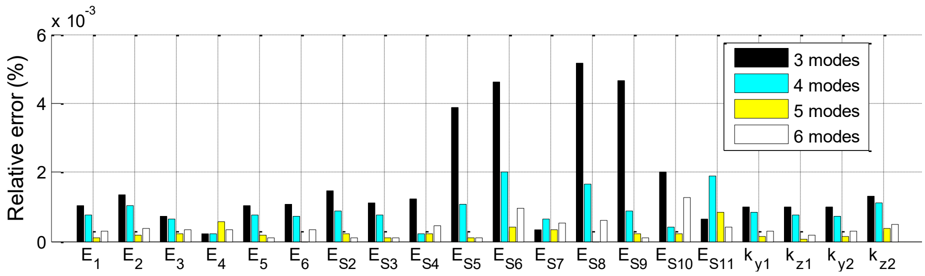

Figure 3. Relative errors of the updated parameters by minimizing modal dynamic residuals (Case 2(a) MATLAB `lsqnonlin` and Case 2(b) MATLAB `MultiStart`)

30 starting points converged to the optimal solution in the end. This indicates that the optimization problem in the modal dynamic residual approach has much better convexity than the modal property difference approach.

Finally, in Case 2(c), the iterative linearization procedure is applied on the modal dynamic residual approach. The model updating results are shown in Table 5. For every available number of modes, most of the optimal results are close to the ideal correct values. Subsequently, Figure 4 plots the relative errors of the updating parameters, with different numbers of available modes. Although compared to results from MATLAB toolboxes, the error is relative larger, especially for E_4, the overall accuracy of the iterative linearization approach is still acceptable.

3.2 Sensitivity analysis

The results in Section 3.1 indicate that the convexity of the modal dynamic residual approach appears better than that of the modal property difference approach. In this section, sensitivity analysis of the objective functions against an updating parameter is conducted to validate the hypothesis.

To perform the sensitivity analysis of an objective function (Eq. 2 or Eq. 3) against an updating parameter, the objective function is evaluated by changing the parameter value while holding all other parameters constant. If equality constraints exist, the relevant parameters are also updated to satisfy the constraints. In this paper, parameter E_1, Young's modulus of longitudinal top chord of the space frame, is chosen for the sensitivity study. As listed in Table 1, the ideal correct value of E_1 is 5%. In the sensitivity study for each model updating approach, the objective function value is calculated while changing E_1 around its correct value, from -3% 8%, while keeping all other stiffness parameters at their optimal values. As for the modal

Table 5 Updated stiffness parameter changes (%) by minimizing modal dynamic residuals (Case 2(c) iterative linearization procedure)

Available modes	**Quantities throughout the entire bridge**						**Spring**			
	E_1	E_2	E_3	E_4	E_5	E_6	k_{y1}	k_{z1}	k_{y1}	k_{z2}
3 modes	5.01	5.01	-5.40	-9.07	5.00	-5.00	-30.00	59.97	-30.00	60.01
4 modes	5.00	5.01	-5.53	-8.69	5.00	-5.06	-29.99	59.88	-30.00	60.01
5 modes	5.00	5.00	-5.32	-9.09	5.01	-5.01	-30.00	59.94	-30.00	60.03
6 modes	5.00	5.00	-5.26	-9.35	5.01	-5.00	-30.00	59.97	-30.00	60.01
Available modes	**Segmental quantities**									
	E_{S2}	E_{S3}	E_{S4}	E_{S5}	E_{S6}	E_{S7}	E_{S8}	E_{S9}	E_{S10}	E_{S11}
3 modes	-9.27	-10.01	-9.95	-5.12	-5.07	-5.02	-14.95	-10.05	-4.98	-4.95
4 modes	-9.18	-9.98	-9.96	-5.08	-4.92	-5.05	-14.88	-10.15	-4.99	-4.94
5 modes	-9.35	-9.99	-10.07	-4.97	-4.90	-5.08	-14.97	-10.01	-4.93	-5.03
6 modes	-9.45	-10.00	-10.04	-4.98	-4.97	-5.01	-14.99	-9.98	-4.96	-5.01

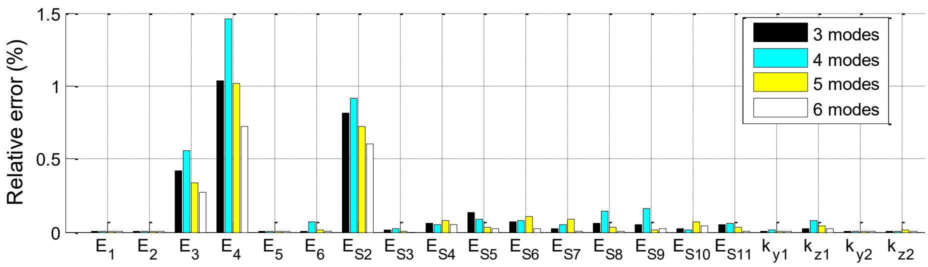

Figure 4. Relative errors of the updated parameters by minimizing modal dynamic residuals (Case 2(c) iterative linearization procedure)

property difference approach, the simulated modal properties ($\boldsymbol{\lambda}^{\mathrm{FE}}$, $\boldsymbol{\psi}^{\mathrm{FE,m}}$ and $\boldsymbol{\psi}^{\mathrm{FE,u}}$) are updated with respect to the equality constrains in Eq. 2(d).

Figure 5 (a) ~ (c) show how the objective function value of modal property difference approach changes with respect to E_1. As shown in Figure 5 (a), the objective function achieves minimum when the selected parameter E_1 is at the ideal value of 5%, as expected. Figure 5 (b) and (c) are the close-up views of Figure 5 (a), when E_1 is at -0.03% ~ 0.03% and 4.95 ~ 5.05%, respectively. Although Figure 5 (a) appears to show a satisfactory convexity of the modal property difference approach, Figure 5 (b) and (c) reveal that the objective function of the modal property difference approach does not decrease monotonically with the increase of the updating parameter. Many local minima exist within very small ranges of E_1. This explains why in both Case 1(a) and 1(b) the modal property difference approach stops at a local minimum around the starting point. Figure 5 (d) ~ (f) show the sensitivity analysis results of the modal dynamic residual approach. As seen in Figure 5 (d), the objective function achieves minimum when E_1 is at the correct value of 5%, which is also as expected. In addition, Figure 5 (e) and (f) show that in both close-up views, the objective function of the modal dynamic residual approach does not have any zig-zag pattern with many local minima (as with the modal property difference approach). Therefore, the figures demonstrate that the modal dynamic residual approach can have better convexity than the modal property difference approach within the assigned bounds. This concurs with the better updating performance of Cases 2(a) through 2(c), as compared with Cases 1(a) and 1(b).

4 CONCLUSIONS

This study investigates two frequency-domain FE model updating approaches and three optimization procedures. The modal property difference approach aims to minimize the difference in natural frequencies and mode shapes obtained from the experimental data and analytical model. The modal dynamic residual approach intends to minimize the residuals of the generalized eigenvalue equation. MATLAB `lsqnonlin` and `MultiStart` optimization

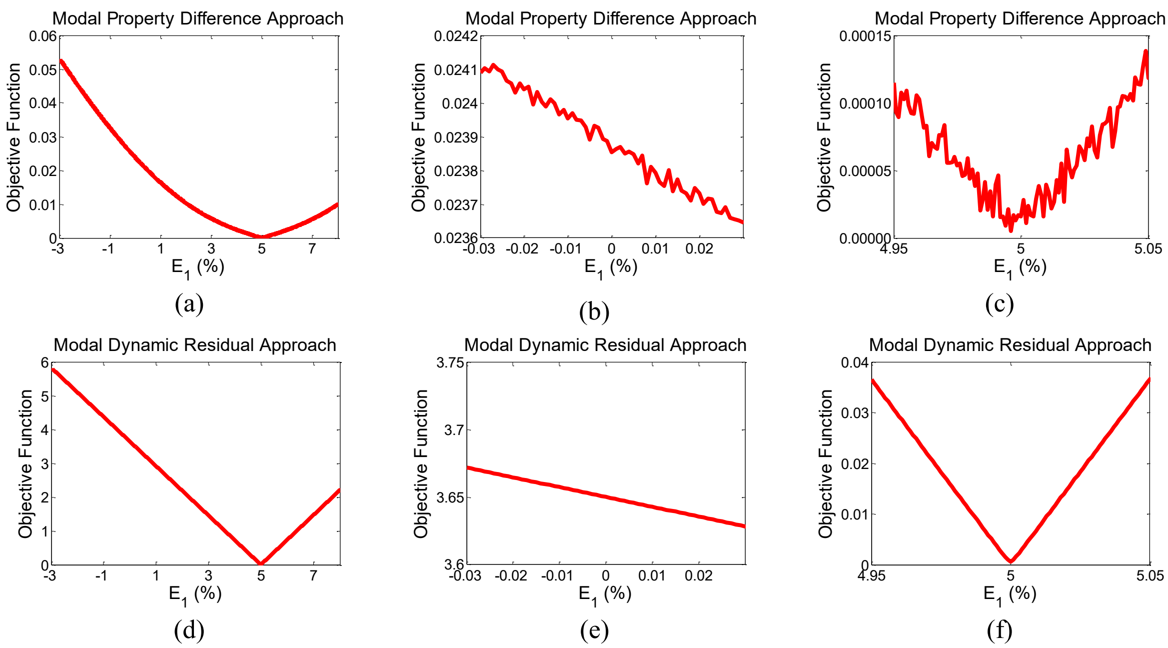

Figure 5. Sensitivities of the objective functions to the updating parameter E_1

solvers can be applied on both model updating approaches, and the iterative linearization optimization procedure is also adopted for solving the modal dynamic residual approach.

The presented model updating approaches and optimization procedures are evaluated using a numerical example of a space frame bridge. The modal property difference approach does not achieve reasonable model updating accuracy with either MATLAB `lsqnonlin` or `MultiStart` solver. The modal dynamic residual approach can accurately identify all updating parameters using both MATLAB optimization solvers. Although the results from the iterative linearization approach is worse than those from the MATLAB toolboxes in this example, the overall updating accuracy is still acceptable.

Furthermore, the sensitivity analysis of the updating variables is conducted to study the convexity of both model updating approaches within the assigned bounds. The results shows that the objective function of the modal property difference approach has many local minima within the assigned bounds of an updating parameter, while the modal dynamic residual approach only has one local minimum that is also the global minimum. The sensitivity analysis demonstrates that the modal dynamic residual approach has better convexity than the modal property difference approach. Future research will continue to investigate the model updating approaches on more complicated structural models, through both simulations and experiments. In the meantime, other optimization procedures will be investigated for accurately and efficiently solving the model updating problems.

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