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Modeling Active Layer Depth of Permafrost under Changing Surface Boundary Conditions

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Summary

A physically based model is formulated for the active layer depth of permafrost under changing boundary condition instead of constant boundary condition considered in the traditional Stefan problem. Time-varying ground heat flux is obtained from net radiation and surface temperature using the Maximum Entropy Production (MEP) model as the driver of the active layer melting process. Conductive heat flux at the melting front is approximated in terms of an analytical function of ground heat flux. The simulated active layer depth is in good agreement with the field observations.

Classical Two-Phase Stefan Problem

Assumptions

1. Semi-infinite slab $0 \le x < \infty$
2. Initial condition $T(x,0) = T_s < T_m$ (melting temperature)
3. Boundary condition $T(0, t>0) = T_L > T_m$, $\lim_{x\to\infty} T(x,t) = T_s$
4. Melting front $T(S(t),t) = T_m, S(0) = 0$

   $\rho_s \lambda_f \dot{S}(t) = -k_L T_x(S(t),t)$ (**Stefan condition**)

ρ_s ice density of bulk active layer
C_s heat capacity of bulk active layer material
k_L thermal conductivity of bulk active layer material
α_L thermal diffusivity of bulk active layer material
λ_f latent heat of fusion (3.3×10^5 J kg^{-1})
"s" – solid phase, "L" – liquid phase

Constant surface boundary condition $T(0,t) = T_L$

Heat Transfer Equations

$$\frac{\partial T(x,t)}{\partial t} = \alpha_L \frac{\partial^2 T(x,t)}{\partial x^2}, \qquad 0 < x < S(t) \text{ active layer depth}$$

Neumann Solution [*Alexiades & Solomon*, 1993]

$$S(t) = 2\lambda\sqrt{\alpha_L t}$$

$$\sqrt{\pi}\lambda \exp(\lambda^2)\operatorname{erf}(\lambda) = St_L \quad \text{(Stefan number)}$$

Permafrost Active Layer Melting Problem

Conditions

1. Semi-infinite slab $0 \le x < \infty$
2. Initial condition $T(x,0) = T_m$
3. Boundary condition $T(0,t>0) = T_L(t) > T_m$, $\lim_{x\to\infty} T(x,t) = T_m$
4. Melting front $T(S(t),t) = T_m, S(0) = 0$

   $\rho_s \lambda_f \dot{S}(t) = -k_L T_x(S(t),t)$

Changing surface boundary condition $T(0,t) = T_L(t)$

Ground Heat Flux *G* Based Model of Active Layer Depth

Energy balance of active layer

$$\int_0^t G(\tau)d\tau = \int_0^{S(t)} C_s[T(x,t) - T_m]dx + \rho_s\lambda_f \int_0^{S(t)} \theta(x,t)dx$$

Approximate analytical solution of $T(x,t)$ [*Yang et al.*, 2017]

$$T(x,t) = T_m + \frac{1}{\sqrt{\pi C_s k_L}} \int_0^t \exp\left[-\frac{x^2}{4\alpha_L(t-\tau)}\right] \frac{G(\tau)d\tau}{\sqrt{t-\tau}}$$

Model Formulation

$$\boxed{\int_0^t \operatorname{erfc}\left[\frac{S(t)}{2\sqrt{\alpha_L(t-\tau)}}\right] G(\tau)d\tau = \rho_s\lambda_f\bar{\theta}S(t)}$$

$\theta(x,t)$ water content of active layer
$\bar{\theta}$ depth averaged water content of active layer

MEP Model of ground heat flux [*Wang and Bras*, 2011]

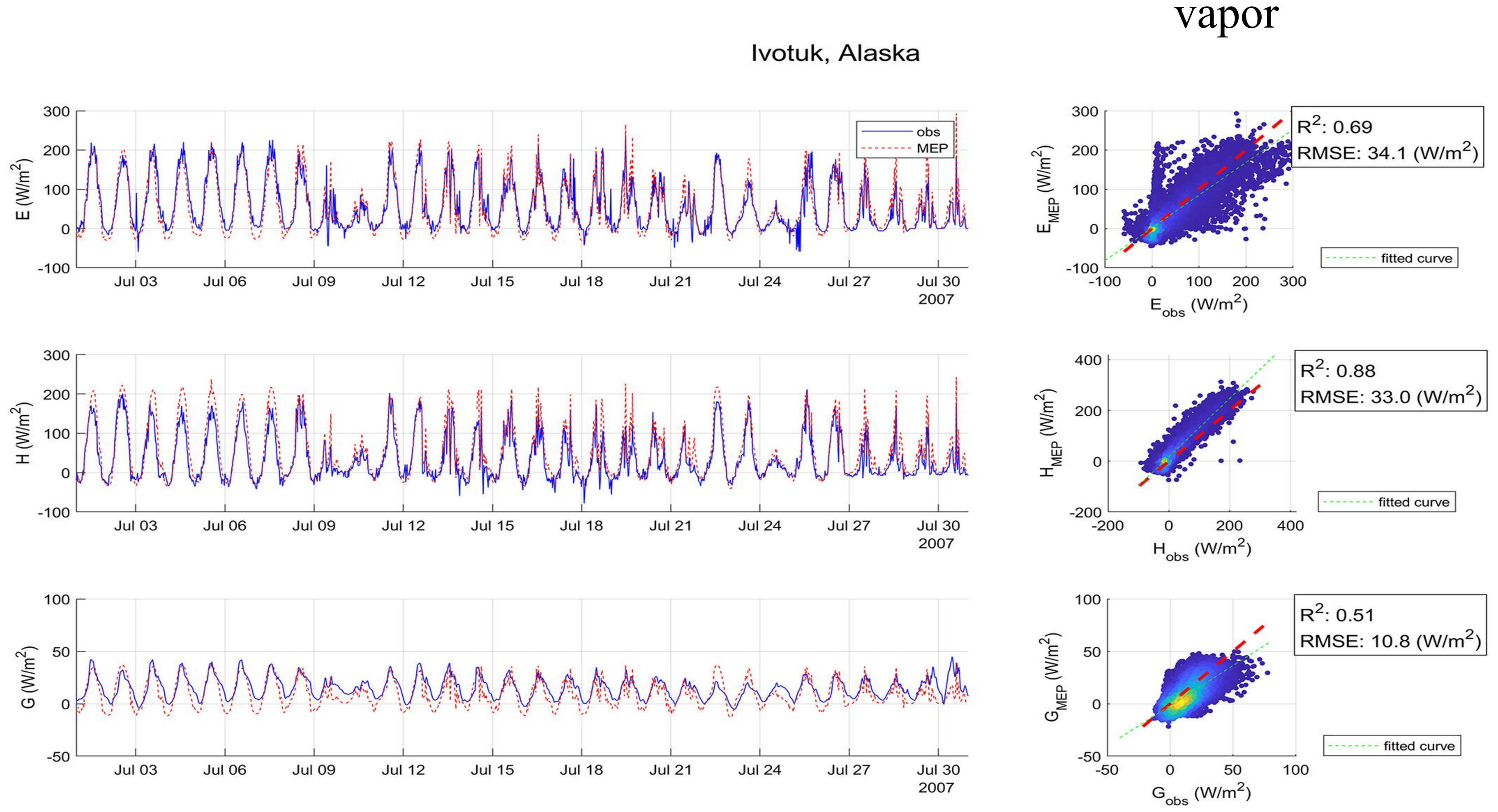
G over permafrost is parameterized based on the Maximum Entropy Production (MEP) Model of surface (latent *E* and sensible *H*) heat fluxes using meteorological data:

$$G = R_n - E - H$$

$$\left[1 + B(\sigma) + \frac{B(\sigma)}{\sigma}\frac{\sqrt{C_s k_L}}{I_0 |H|^{1/6}}\right] H = R_n;\ E = B(\sigma)H$$

$$\sigma = \frac{\lambda_v^2}{c_p R_v}\frac{q_s}{T_L^2};\ B(\sigma) = 6\left(\sqrt{1 + \frac{11}{36}\sigma} - 1\right)$$

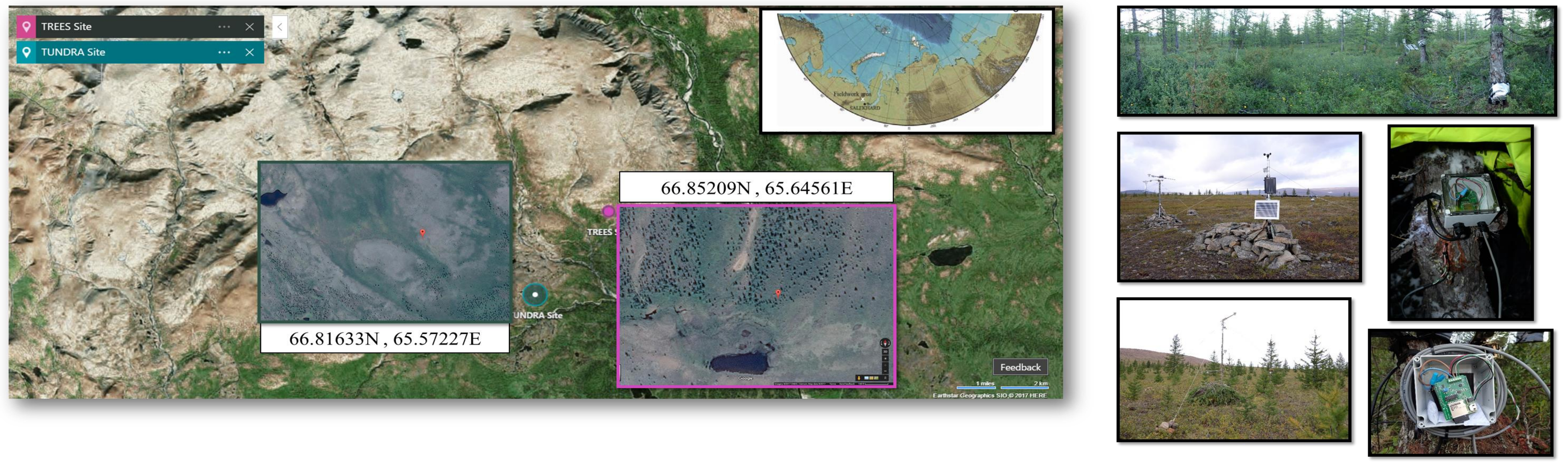
R_n net radiation
q_s surface specific humidity
λ_v latent heat of vaporization
c_p specific heat of air
R_v gas constant of vapor

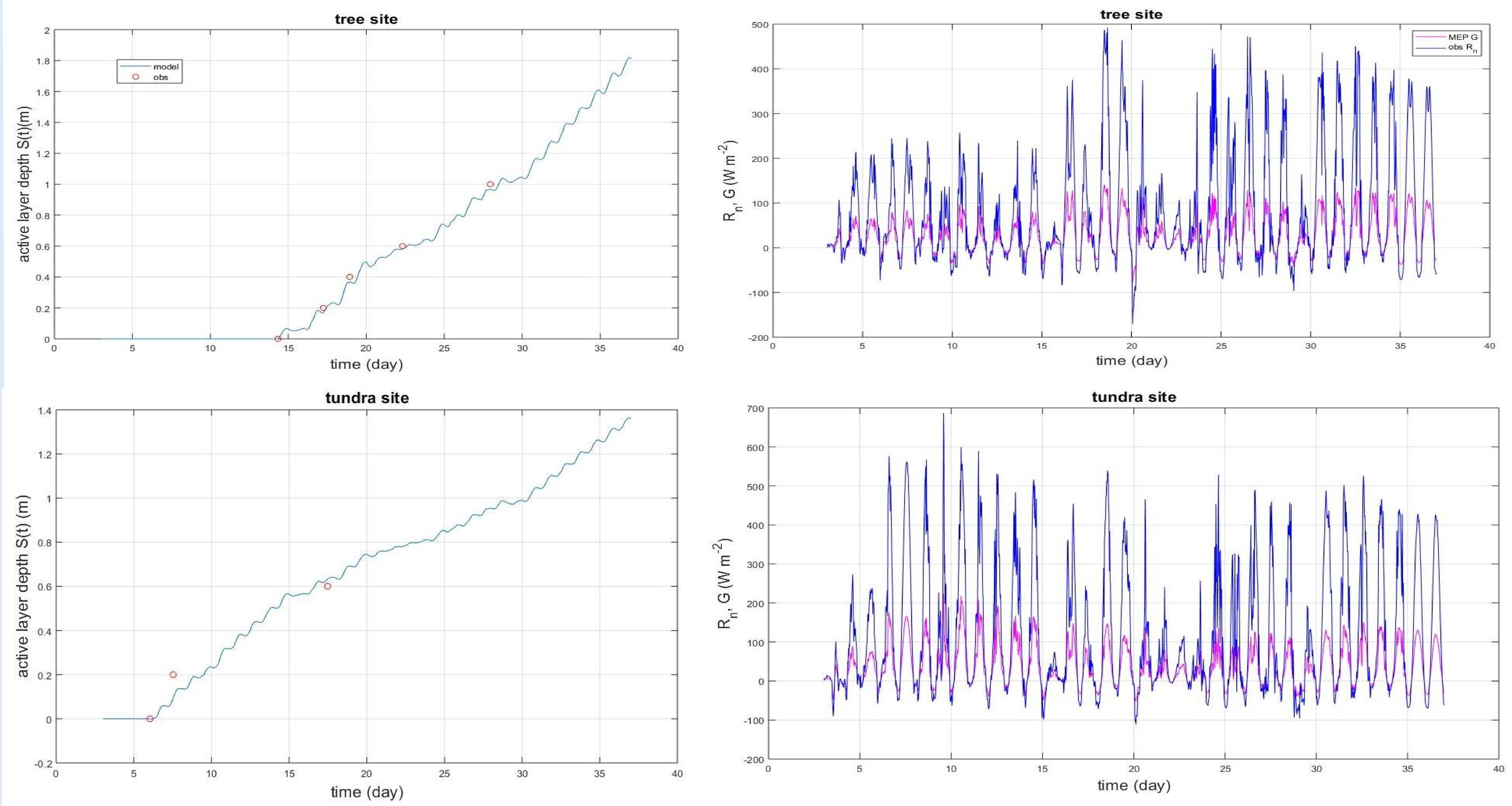


MEP modeled vs. field observed surface heat fluxes at a permafrost tundra site in Alaska [*El Sharif et al.*, 2019].

Model Test

Field observations of soil temperature and ground meteorological data collected at an Arctic tundra (66.8163°, 65.5723°) and a tree (66.8526°, 65.6475°) site in the Polar Urals, Yamal-Nenets Autonomous District, Russia [*Mazepa*, 2005], June-July 2007.





Conclusion

Use of time-varying ground heat flux in modeling active layer depth directly links the permafrost melting process to its energy driver. This proof-of-concept study provides a promising alternative approach of modeling dynamics of permafrost freeze-thaw cycle.

References

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