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Editorial

Improving the Impact of Research on Practice: Capitalizing on Technological Advances for Research

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November 2020 marks the last issue of the *Journal for Research in Mathematics Education* (*JRME*) under our editorship. When we started this journey 4 years ago, our first editorial called for the mathematics education community to look 25 years ahead and envision a future where the gap between research and practice would be a problem of the past (Cai et al., 2017). In our editorials since then, we have discussed a variety of issues concerning our field, always with a focus on closing the gap between research and practice. Our first set of editorials (Cai et al., 2017, 2017a, 2017b, 2017c, 2017d, 2018a, 2018b, 2018c, 2018d) explored the problem of the impact, seeking to engage the field in understanding and clarifying the nature of research that could have a greater impact on practice. We hypothesized what conditions would need to be in place in a world where teachers and researchers regularly and persistently collaborate within a new research paradigm to address problems of practice and to share knowledge with the profession.[1] In our third set of editorials (Cai, Morris, Hohensee, Hwang, Robison, Cirillo, Kramer, & Hiebert, 2020a, 2020b; Cai, Morris, Hohensee, Hwang, Robison, Cirillo, Kramer, Hiebert, & Bakker, 2020a, 2020b), we took the opportunity to celebrate *JRME*'s 50th anniversary by looking across the field and identifying a set of overarching problems that we believe must be addressed if research in the next 50 years is to fully connect with practice. Now, in this final editorial of our tenure, we look retrospectively at the ideas that we have shared so that we may suggest some actions the field can take to truly make the vision a reality. We do so through the lens of technology.

Technology has become virtually ubiquitous, both in schools and in everyday life, and there is already a significant body of work exploring aspects of technology and mathematics education. For example, researchers have investigated software or hardware tools designed to help students explore and learn concepts (e.g., dynamic geometry software, computer algebra systems, calculators, sensors, and robotics), and they have studied technology-supported innovations in practice, such as flipped classroom models or remote instruction (many such platforms have become much more commonly used during the COVID-19 pandemic-related school closures of this year). However, the actions that are the focus of this editorial involve the rapidly evolving technologies, both in education and more broadly,

[1] Our second set of editorials (Cai et al., 2019, 2019a, 2019b, 2019c, 2019d) focused on the process of conducting and communicating research that could have a greater impact on practice. In these, we drew on our experience as an editorial team to explain how justification, coherence, and significance are fundamental characteristics of research that can have an impact on practice.

that can be used to bring research closer to practice and thus help the field address the overarching problems that we have discussed this year. We hope that our discussion of questions about how technology could support teacher–researcher partnerships working in a new research paradigm will stimulate innovations in our field to address these overarching problems. The questions that we pose will ask, in part, how technology can support and enhance the feedback loops that drive continuous improvement (e.g., feedback from teachers and fine-grained feedback about student thinking during instruction and problem solving).

We suspect that many researchers in mathematics education share our concern that keeping up with rapid technological advances and their implications for mathematics education research can be challenging. Indeed, technology advances so quickly that addressing research problems may not depend so much on developing a new technological capability as on helping, researchers and practitioners learn about new technologies and imagine effective ways to use them. Therefore, in this editorial, we also include questions that explore the best ways to inform the research community about potential capabilities of technology and how these could enhance researchers' questions and answers.

Bringing Research Closer to Practice: Technology as a Research Tool in Our Wonderland

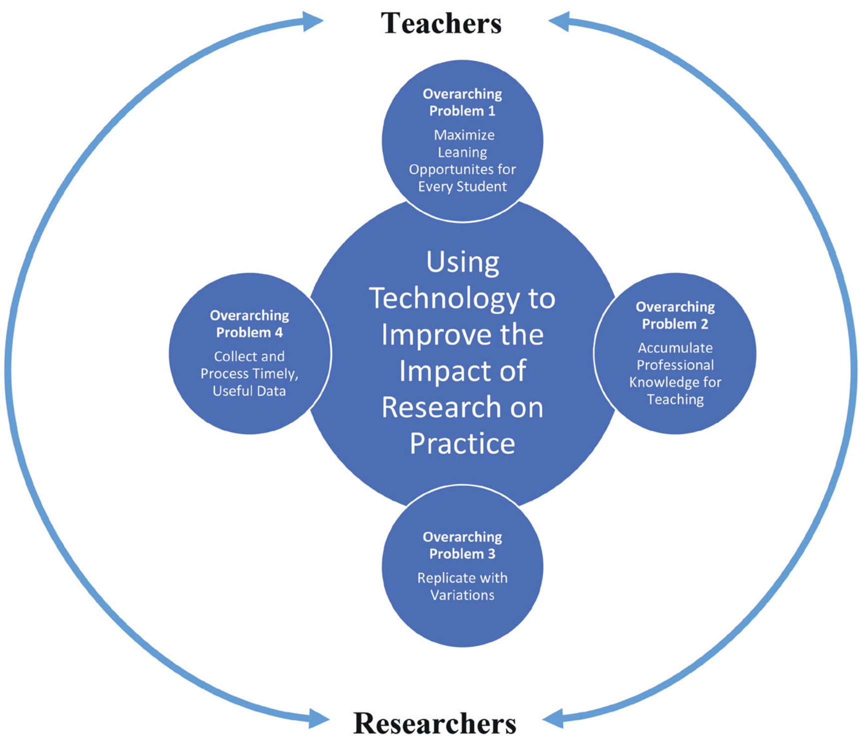
A key idea that we have built on in our editorials is that the divide between research and practice is, in part, due to the fact that the products of research are typically not in a form that can readily be used to guide the instructional decisions that teachers make. This means that extensive translation is needed to fit research findings into local contexts. Unfortunately, translation, even when we know how to do it, is not sufficient to solve the problem. This is because translations cannot provide information about how teachers in particular settings can adapt the findings to improve learning outcomes for students in their classrooms (Burkhardt & Schoenfeld, 2003; Cai et al., 2017; Sabatini, 2009; Sabelli & Harris, 2015). This kind of connection is made through fundamentally different kinds of research, which are designed to bridge the gap between the original findings and the contexts in which they are to be implemented.

As a response to this situation, we have advocated for alternative research pathways (Cai et al., 2019) built on a paradigm of partnerships between researchers and practitioners working to continuously improve mathematics teaching and learning. This paradigm would include incentives for educational researchers and teachers to identify and solve important problems of practice through sustained, iterative cycles of hypothesis formation, instructional design, testing in real classrooms, and analysis of data. As shown in Figure 1, the partnerships between teachers and researchers in this paradigm are bidirectional, with each group contributing to the cycles of improvement, which will ultimately maximize the quality of the learning opportunities offered to every student.

AQ1 In the subsections below, we consider questions about how well-designed technological tools could amplify efforts to make research immediately applicable to practice without the translation barrier. In the context of this future paradigm—our potential Wonderland of research and practice—we reconsider key themes from the four overarching problems that we discussed this year, and we pose questions about how technology might be used to address these problems. We

Figure 1

Capitalizing on Technological Advances to Improve the Impact of Research on Practice by Addressing Overarching Problems of Mathematics Education



Note. To achieve the goal of maximizing the quality of learning opportunities for every student, researchers and teachers must partner to conduct cycles of improvement that implement innovations with variation; collect and process timely and useful data to inform instruction and revision; and accumulate, in broadly sharable artifacts, the professional knowledge that is developed.

recognize that technologies can be applied in many innovative ways to enhance classroom instruction, but we are restricting this editorial to discussions of how technology might be used to address the four big research problems and help to close the gap between research and practice.

Problem 1: Maximize Learning Opportunities for Every Student

In our January 2020 editorial (Cai, Morris, Hohensee, Hwang, Robison, Cirillo, Kramer, Hiebert, & Bakker, 2020b), we suggested that a useful focus for teacher–researcher partnerships could be the quality of learning opportunities that students experience. We posed an overarching question about the kinds of measures and research designs that could reveal the nature of learning opportunities. Specifically, the problem we highlighted was defining and measuring learning opportunities precisely enough to empirically study how to maximize the quality of the opportunities experienced by every student. Because learning opportunities involve the interactions among tasks, teaching, and students, researchers studying learning opportunities will have to develop and employ tools that can provide teachers with timely and useful data about these interactions in the classroom. This is one place where we see technology playing a supportive role in maximizing student learning opportunities.

For example, consider the scenario involving Mr. Lovemath, a fourth-grade teacher, and Ms. Research, a mathematics education researcher, which we described in our first series of editorials. A fraction-ordering task used by Mr. Lovemath did not produce the desired learning opportunities for his students. In response, these partners engaged in an iterative design and hypothesis-testing process through which they refined the lesson to create greater learning opportunities. In this scenario, we can pose specific questions about how technology could support and enhance the work of this teacher–researcher partnership in a way that brings research closer to practice.

First, how could Mr. Lovemath and Ms. Research best make use of technological tools to help them gather data on how the students thought and acted in response to the teaching and the task? How could these tools be used to preserve a record of the students' explorations and work, allowing Mr. Lovemath to see, both in the moment and retrospectively, how individual students or groups of students are thinking about this and other fraction tasks and better understand their learning trajectories over a longer period time? For example, could Mr. Lovemath design online problems that can be solved using tools that would keep a record of students' work, including their missteps and their solution strategies? Might some tools that are currently used to help students solve particular kinds of problems be appropriated for this purpose? For example, could virtual manipulatives designed to help students coordinate representations of number and operation concepts be adapted to gather such data?

The data on students' thinking become especially useful for improving learning opportunities if they can be indexed to particular tasks and instructional moves. This would require gathering data on the teaching and the tasks presented to students, at an appropriate level of detail, alongside data on students' responses. Pairing data on instructional tasks with students' responses would allow Mr. Lovemath and Ms. Research to reflect on tasks and responses from a cause-and-effect perspective. This kind of reflection is essential for developing and testing hypotheses about how revisions in tasks could improve learning opportunities for students. Could the same tools used to gather data on students' responses be expanded to gather data on instructional moves and tasks and keep these data linked for further analyses? What is the appropriate grain size of these data to enable cause-and-effect reasoning?

Problems 2 and 3: Accumulate Professional Knowledge for Teaching and Replicate with Variations

Two of the problems that we have highlighted in this year's editorials have had to do with how to accumulate professional knowledge and how to share it. In addition to accumulating information on students' responses and linking them to instructional tasks, technology seems well suited for enhancing the sharing of information among teachers and researchers. Technology for communication and collaboration could support the formation of virtual teacher–researcher partnership networks that share what is learned across the network. How could technological tools best be used to help Mr. Lovemath and Ms. Research share the knowledge that they acquire with other teacher–researcher partnerships that are tackling similar problems of practice? How could technological tools that support communication and sharing of data and resources allow multiple teacher–researcher partnerships to generalize knowledge from multiple cycles of feedback, tweaking artifacts and instructional strategies, and testing them with teachers and students in other contexts? Answers to these questions are essential to understanding how technology can be used wisely and effectively to realize the iterative cycle of implementing, testing, assessing, revising, and sharing that is at the heart of our proposals to improve learning opportunities for all students (Cai, Morris, Hohensee, Hwang, Robison, Cirillo, Kramer, Hiebert, & Bakker, 2020b).

One opportunity to build up professional knowledge through sharing across contexts is through studying multiple implementations of innovations with variation. Over time and across multiple sites, the results of intentionally varying parts of an instructional lesson could be contrasted with an eye toward generalizing the aspects that worked well in different settings with different students. How could technology support this practice, a practice we termed *replimentation* (Cai, Morris, Hohensee, Hwang, Robison, Cirillo, Kramer, & Hiebert, 2020b)? How can technology enable the detailed analysis of data produced through multiple implementations with planned variations? A key challenge is to create a data-processing system that can assemble and interpret similar data from multiple classrooms. Disaggregated data from multiple classrooms are needed to sort out what changes to instruction yielded improved learning within classrooms versus those that yielded improvements across classrooms and contexts. Systematic and sustained analyses of large datasets would be needed to learn as much as possible about how innovations interact with contexts and why they work (or fail to work). As Nuthall (2004) stated, "Part of the professional knowledge of teachers must be the distinction between what is unique to a specific context and what is generalizable across all students and/or contexts" (p. 294). Clearly, technology would need to play a major role if these distinctions are to be accumulated into useable knowledge by teachers. What, then, are the technologies that will be useful for this purpose, and how should they be employed? And, given that methodological innovations often percolate slowly through the research community, how can the field more quickly become aware of newly developed technological supports for these kinds of analyses?

Finally, as a field, we have often failed to sufficiently attend to the expertise and observations of teachers. We have undervalued the professional knowledge of teachers by not developing a system to gather and preserve that knowledge. Furthermore, because we have not accumulated teachers' knowledge in a systematic and accessible way, we have had no means to vet that knowledge by sharing it with other teachers and researchers and determining whether and how it warrants application on a wider basis (Cai et al., 2018a). In what ways could technology facilitate the recording, processing, vetting, and sharing of teachers' knowledge? How can technology help to reduce the demand on teachers' time needed to do this kind of work? Respecting teachers' expertise is essential, but, unless we can find ways that reduce the time usually involved in this kind of work, most teachers will be unable to participate.

Problem 4: Gathering and Using Data

Several of the overarching problems that our field needs to address involve gathering and processing data. Many of the approaches that we have considered demand that we carefully think about what data we need to collect to inform efforts to improve teaching and to raise the quality of the learning opportunities that students encounter. In this section, we consider how technology could inform partnerships about how much data to collect. We could imagine several ways that one might gather data to inform instructional decisions, ranging from a collect-everything approach to a much more targeted system of collecting small amounts of data to address specific, focused questions about instruction. Which approaches could best help teachers and researchers make better, evidenced-based instructional decisions and solve instructional problems?

Given the rapid development of imaging, data-processing, and storage technologies, it is possible to imagine a future in which every relevant element of every classroom lesson could be captured and saved for analysis. Consider how mapping tools like Google Earth make use of comprehensive geographical data to allow users to easily visualize any location on Earth at scales ranging from an entire hemisphere to a street-level view of a single house. In a similar way, data-gathering technology could unobtrusively collect video and audio recordings of the teacher and every student throughout a lesson, images of all student work while it is being produced, automatic transcriptions of all discourse, and so forth, allowing teachers and researchers to easily visualize any relevant element of the lesson. Even now, computer-assisted qualitative data analysis software can be used to analyze digital video or audio data and allow researchers to produce multiple "transcripts" for a single video that can be used to document and code not only the spoken discourse but also the visual images or gestures of teachers and students that supported communication during the lesson. For example, one such program, Transana, allows the researcher to sync together multiple videos of the same lesson shot from different camera angles to allow analyses to occur from different perspectives, such as the teacher versus student perspective. We highlight capabilities like these because, as Nuthall (2004) argued, to truly connect teaching with student learning, researchers must collect "*Complete, continuous data on individual student experience*" and "*Complete, continuous data on classroom activities*" (p. 296). Analyses would then have a chance to connect the records of teaching and student activity with individual student learning and to identify places in a lesson where student thinking needs to be elicited to understand the effects of an instructional move on learning opportunities. Technology thus has the potential to provide us with a fuller picture than ever before of what occurred during a classroom lesson.

Large, complete datasets of classroom activity also have the advantage of allowing teachers and researchers to address any instructional problems that emerge during the recorded lessons by revisiting the relevant data (Derry et al., 2010; Stigler et al., 2000). In other words, they would not need to anticipate all research questions before the data are collected, and they would not need to guess what data will be most relevant (cf. Ing & Samkian, 2018, for a discussion of possible methodological challenges with this approach). For example, teachers and researchers could intensify their search for a connection between a particular instructional activity and individual student learning by returning to examine all exchanges among students while they worked on the activity in small groups.

However, the collect-everything approach brings with it some significant questions. Although it might obviate the need to design a specific data-collection strategy to address each new instructional problem, how can researchers and teachers deal with the sheer scale of the data? Is this kind of massive data collection, processing, and indexing manageable given limited time and human resources? Can an indexing and search process be developed that allows the advantages to be realized, or will teacher–researcher partnerships be overwhelmed with an overabundance of data? In addition, how can researchers deal with the practical and ethical issues of privacy and consent when seeking to gather such a comprehensive dataset? Clearly, although this kind of data-gathering effort is inconceivable without technology, a major question is whether technological tools are sufficiently intelligent, and whether they can be used cleverly enough, to make this effort feasible.

On the other end of the continuum is an approach that focuses on quickly gathering small amounts of data to test very local, targeted hypotheses about instruction and learning opportunities—in other words, just enough data to tell whether something is working. The goal of this alternative approach to data collection is to inform the continuous, iterative development of a gradually accumulating set of small improvements in instruction (Cai et al., 2018d, 2019, 2019a; Cai, Morris, Hohensee, Hwang, Robison, Cirillo, Kramer, & Hiebert, 2020a). Because classroom time is a highly limited resource, such data collection will require what Improvement Science researchers have called "practical measures": measures to inform improvement that can be embedded in the regular work of teaching and learning (Bryk et al., 2015). Depending on the kind of question being asked, teachers might not need all the data on every student but, instead, could focus on a subset of data from a small set of indicator students in a class. Relevant types of data might include instantaneous data gathered in the moment during instruction through the use of clickers, after-the-lesson data collected through one or two targeted questions on an exit ticket, survey questions about student interest or mindsets added to the end of an online homework assignment, or data from online or in-class tasks specifically designed to make students' thinking visible. Some of the data-gathering techniques from this approach fall under the umbrella of formative assessment (e.g., Black & Wiliam, 2005; Petit & Zawojewski, 2010; Silver & Mills, 2018).

Falling Down the Rabbit Hole: Cautions and Crises

Although the development of new technologies often fuels enthusiasm for finding useful applications, some caution is warranted. Poorly designed or implemented technological tools and tools that are mismatched to the context might conflict with or even be fundamentally incompatible with solutions to the problems that we have addressed. For example, there are many new technologies that allow teachers to rapidly collect and display all student responses to a task (e.g., Pear Deck). However, if the teacher does not appropriately select and sequence students' responses to organize the discussion (Smith & Stein, 2011), then displaying an array of solutions without facilitating a productive discussion about them would be overwhelming to students and ultimately an ineffective use of class time. Thus, the effectiveness of applying new technologies to longstanding problems such as improving the impact of research on practice is dependent on making careful choices and assessing both the positive and negative effects of using new technologies. Moreover, we can ask how technologies themselves might need to be "continuously improved" to enhance their positive effects and limit their negative effects. This question is equally important whether a new tool is applied or an existing tool is applied to a new setting.

The adoption of new technologies also gives rise to essential questions about equity. One way to conceptualize equitable education is as about maximizing the quality of the learning opportunities that every student experiences.[2] Technology has sometimes been portrayed as a means to achieving this outcome, for example, by providing individually tailored and optimized instruction. However, not all schools and students have the same access to or comfort with new technologies. Some may need additional supports to make use of technology effectively. An important question for the field is how to prevent technology from reproducing or even widening the inequities in learning opportunities across groups of students. Under what conditions do these unwanted effects occur? Even in the context of teacher–researcher partnerships using technology to assist with gathering data and processing it in ways that are useful for making decisions, the issue of inequitable resource distribution arises. If technology is used to support systemic change and widespread continuous improvement, it will be critical to actively plan for the inclusion of all groups of teachers and students, not simply the best-resourced groups. How can useful technologies be distributed and shared across settings so that all students benefit from the resources available to their teachers?

Finally, we recognize that many students and teachers have had to rapidly adapt to online instruction this year because of the closure of school and university buildings in response to the COVID-19 pandemic. This has posed many challenges, both technological and practical. One consequence of the move online is that the nature of teachers' communication with students (e.g., precision in mathematical language) and their access to in-person feedback is different than in classroom settings (Cirillo et al., 2020). Moreover, teachers who are in the same room as their students can monitor engagement and motivation. When students and their teachers are separated, each in their own space with their own screen, what challenges do they face to stay engaged and on task? These concerns lead to empirical questions about how teachers and researchers can plan and develop

[2] This is not to downplay the role of structural inequities and system-level forces that reinforce patterns of inequity in education. Rather, we may conceptualize a quality-maximized learning opportunity for a given student as one that actively works against such forces. Alternatively, we may consider ways to set cognitive and noncognitive learning goals that are informed by equity concerns and, consequently, work to maximize the quality of the learning opportunities associated with those goals.

mechanisms to gather feedback and data on students' learning and engagement effectively in online instruction. How can technology be harnessed to gather more or different feedback in online instruction than in face-to-face instruction?

Making the Vision a Reality: Taking Actions

We close this final editorial in the same way that we began our first editorial—with a story. Some months ago, a few members of our editorial team met with a group of colleagues at the University of Delaware who are data scientists and technology experts. The goal of the meeting was to discuss our vision for bringing research and practice closer together and to better understand the possibilities that new technologies could offer. The conversation was focused on this question: What are the best ways to use technology to improve the impact of research on practice and to improve students' learning opportunities?

We were greatly encouraged by the enthusiasm expressed by the data scientists about solving the general problems that we presented. The technology experts described many recent technological advancements and shared examples of exciting things that technology could do in the domains of geography, urban architecture, medicine, and so forth. Their explanations helped to broaden our horizons and our views about what was possible with technology in education. It was very clear that, from the data scientists' perspective, technology is sufficiently advanced to address the data collection, processing, accumulation, and sharing that we proposed. The limitations came not from the lack of technological capability but from what could be imagined.

Toward the end of our conversation, we found ourselves stymied by a fundamental question. More than once, we were asked what, exactly, we wanted the technology to do. Precisely what kind of data did we want to collect? Did we want to gather data in the moment? From every student? From the teacher? What was our wish list? We were told that, if we could specify exactly what we wanted clearly enough that they would not need to interpret, they could provide the technology. The technology was advanced enough that they had no question that they could do it. However, they needed to know exactly what "it" is.

We came away from our meeting motivated to answer this question. We met several times as a group to generate an appropriately specific wish list. After much deliberation, it became painfully clear that pinning down exactly what we wanted in the precise, specific ways that the technology and data experts needed was itself a big problem. Although we had ideas about what we wanted, and we could express the ideas in language that we understood, we struggled to put our ideas into the kind of language the data scientists required to develop the technology that could make our ideas a reality.

We close our final editorial with this story for three reasons. First, we want to emphasize that the opportunities to address our problems with technology are not unattainable. The problem is not that the technology is unavailable. Rather, a major challenge is to crystalize the vision and articulate it with enough precision to shape the technology into the tools that we need. There is significant work yet to be done to identify the specific actions needed—the concrete plan—to make it all work.

Second, we make the observation that, as mathematics education researchers, we work in an interdisciplinary field. To move research forward, we must always look for ways to expand the circle of our expertise. This includes figuring out how to communicate across disciplinary boundaries—boundaries made up of differences in discourse, theory, and methodology and even differences in basic assumptions. To make progress on the most persistent problems of mathematics education, we will need to be aware that there are relevant perspectives across the usual academic and professional specialties, including data science, improvement science, cognitive science, and neuroscience.

Finally, the vision that we have presented requires action not only by individuals or even by teams of researchers and teachers but by networks of teams joining together in interdisciplinary collaborations that push forward the frontiers of mathematics education research. Although we have shared some ideas for how we think the field can move forward, they only represent a first step. Our field will need to harness the imagination and vision of researchers and teachers to not only change the research paradigms but to create the structures, incentives, and daily work routines needed to make continuing progress in bringing research closer to practice.

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Author Query

AQ1: In the sentence beginning "In the subsections below, we consider questions…." Please specify which section or subsection "subsections below" refers to here.