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Noise Reduction in the Inverse Solution for One-dimensional Cardiac Active Stress Reconstruction

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ABSTRACT

Visualizing action potentials within the cardiac tissue enables the identification of abnormal action potential wave propagation patterns for use in both clinical and cardiac research settings. Otani et al.[1] have been investigating the possibility of using 4-D (three spatial dimensions and one time dimension) mechanical deformation data, obtained either from MRI or ultrasound images to reverse-calculate these action potential patterns.[2] However, the inverse system is extremely sensitive to noise; that is, a small amount of perturbation from the signal can lead to a substantial perturbation in the solution if the perturbation has a high-frequency component.[3] Here, we explore three noise reduction methods, in an attempt to reduce the effect of noise in the input data and to regularize the calculated solution.

Keywords: Tikhnov Regularization, Cardiac Imaging, Cardiac Modeling

1. INTRODUCTION

Several cardiac arrhythmias are known to be caused by the emergence of chaotic patterns of action potential (AP) wave propagation in the cardiac tissue. An understanding of the mechanisms responsible for the onset and propagation of these waves is essential in the development of electrical and pharmacological treatments for these conditions. Currently, we rely on electrocardiograms (ECG) or electrophysiology (EP) studies to provide information on the electrical state of the heart. Though this information is valuable, it does not provide sufficient insights into the fundamental dynamics governing cardiac rhythm. We speculate that multiple re-entrant action potential waves cause ventricular fibrillation (VF). There currently exist methods that can be used to detect AP patterns during VF and other abnormal rhythms; however, some of these methods lack sufficient spatial resolution, while others cannot be implemented *in vivo* due to the toxic dyes used in the procedure.

Previously, we proposed a method[4] that could use mechanical tissue displacement data from magnetic resonance imaging (MRI), computed tomography (CT) or ultrasound to non-invasively track the active mechanical stresses, which are the result of the electrical activity in the heart. This would give us the ability to look deep within the heart walls, to "see" the propagation of the AP waves.

In the absence of noise in the displacement data, we demonstrated that this method is quite effective in solving the inverse problem, yielding a nearly exact solution for the active stresses. A problem arises, however, when there is noise in the MRI or CT data, standard inverse methods amplify the noise, generally to levels that render the signal-to-noise ratio no longer viable.

Here, we explore three noise reduction methods — namely reducing the finite-element node number, stretching the node spacing, and Tikhonov regularization. We have successfully applied the above methods to a spatially one-dimensional version of our model and explored their effectiveness in producing practical solutions to the inverse problem when uncertainty is present in the displacement measurements.

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2. THEORY

The one-dimensional version of the system we used was defined as

$$\frac{\partial}{\partial x}\left(k\frac{\partial \delta x}{\partial x}+P+T\right)=0 \tag{1}$$

where k is Hooke's constant, $\delta x(x)$ is the tissue displacement, $P(x)$ is the hydrostatic pressure, and $T(x)$ is the active stress created by the presence of an action potential. A one-dimensional system is necessarily compressible, so we remove P, whose sole purpose is to enforce incompressibility in higher dimensional systems.

In the forward problem, the active stress T is given at each point in the system, and we compute the displacement δx at each point. We define the inverse problem as: given the displacement data (from MRI or ultrasound images) at each point, we compute the active stresses that caused the displacement.

Similar to our previous work, we employed a simple finite element model. The spacing Δx between the nodes was chosen to be uniform. For the displacement field, the basis functions were defined as triangular functions having slopes $\pm 1/\Delta x$. Since the active stress has one less derivative associated with it, the basis functions used to describe it were defined on the elements rather than the nodes, where there they were defined as 1 in the particular element and 0 elsewhere. When the finite element version of Eq. (1) is multiplied by the various weighting functions (also defined to be the triangular functions) and integrated over x , we obtained the following:

$$\sum_{i=1}^{n+1}\left(kc_i\left[N_l\frac{\partial N_i}{\partial x}\Big|_0^L+\int_0^L\frac{\partial N_l}{\partial x}\ \frac{\partial N_i}{\partial x}\ dx\right]\right)+\sum_{e=1}^{n}\left(T_j\left[N_l\tilde{N}_e\Big|_0^L+\int_0^L\frac{\partial N_l}{\partial x}\tilde{N}_e dx\right]\right)=0 \tag{2}$$

where, c_i and T_j are constants based on the displacements and active stresses of the system, N_l and N_i are the weighting and basis functions representing the displacement at the i-th node respectively, and $\tilde{N}_e$ is the basis function representing the active stress on the element e, for a system of length L with n nodes. By setting the boundary terms to zero (for Neumann boundary conditions) and substituting the known constants, we get an equation of the form

$$D\delta x = BT \tag{3}$$

This is the general form of the system we work with, where we define $A = D^+B$, which helps us solve the forward problem, namely $AT = \delta x$, and A^+ is the pseudo-inverse of A, helps us solve the inverse problem, $A^+\delta x = T$.

This is a relatively easy problem to solve when there is no error associated with the displacement. Since both forward and inverse systems have full rank matrix representation, information is conserved while converting from one to the other.

It is known for the forward problem, when noise is present in $T(x)$, the mapping from $T(x)$ to $\delta x(x)$ has a smoothing effect, so there is a damping effect on the higher frequency of the input signal. Thus the noise in $T(x)$ will not drastically influence the $\delta x(x)$ we obtain from the forward problem.

However, in the inverse problem, when $\delta x(x)$ is mapped to $T(x)$, there is an amplifying effect, such that the higher spatial frequencies (shorter wavelengths) in the signal are magnified. So the complication in the inverse problem is that, along with the signal, the noise as a small perturbation in $\delta x(x)$ will be amplified significantly.

In practice, we do not have the information about the signal-to-noise ratio. Thus the argument about the effectiveness of the noise reduction in the following sections might be relative. In general, we would assume the noise is small, but not insignificant compared to the signal, approximately one order of magnitude lower.

Here, we have explored three potential methods to reduce the noise in the one-dimensional system, namely, reducing the number of nodes, stretching the node space, and Tikhonov regularization.

3. RESULTS

In the current implementation, we use the model proposed by Otani *et al.*[4] to produce a noiseless displacement from a given active stress. Then, we add pseudo-random noise into the displacement data and use the inverse model to calculate the active stress comparing the result with the pure input active stress. In the following, we describe results from the three methods.

3.1 Reducing the Number of Nodes

Since the higher frequencies are most significantly amplified in the inverse problem, sacrificing the spatial resolution to gain a higher signal-to-noise ratio could provide a consistent result. In this case, we take the displacement data with noise and consider only every n^{th} node, ignoring all nodes between every n^{th} node, effectively increases node spacing by a factor of n. This reduces the information density making the application of this method limited. This reduced data set is used to obtain the active stress. This active stress is then compared to the original active stress signal and the active stress obtained from the full data set.

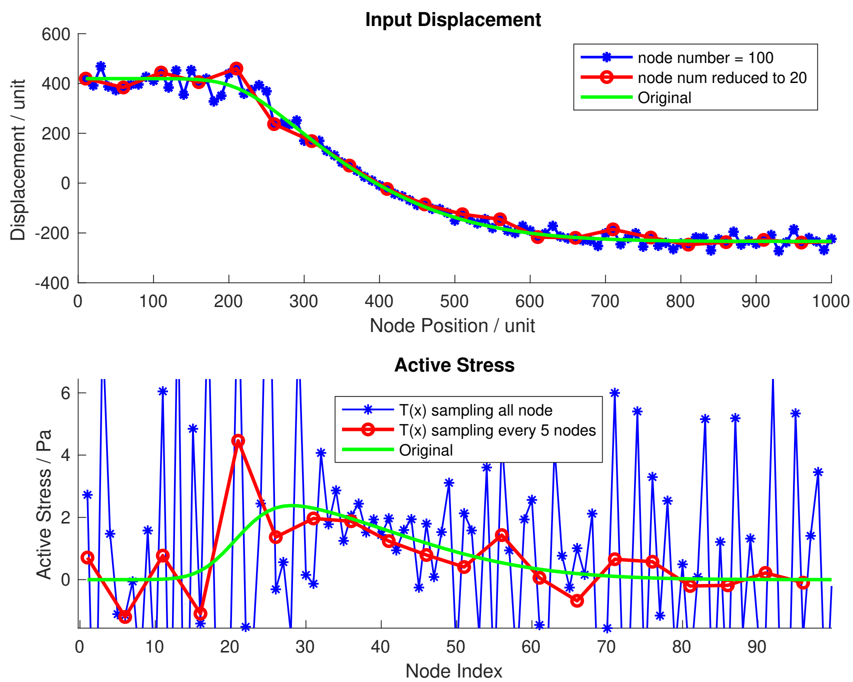

Figure 1. Above: All displacement data (blue) and reduced data (red) as a function of x. Below: Original active stress (green), and active stresses calculated from all the data (blue) and from the reduced data (red), as functions of x.

Figure 1 shows the result of using every fifth displacement data point as input data into the inverse model. When compared to solving the inverse problem using all data points, the result from the reduced data is improved, with less fluctuation in the active stress signal. Since the presence of strain tensors and passive stresses in the three-dimensional model complicates the equations, we have yet to determine whether this method would reduce noise in all directions, and how we would reduce the sampling in a three-dimensional mesh. Moreover, the existence of a fiber direction may add more complications.

3.2 Stretching the Node Space

By increasing the node spacing, we are dampening the short wavelengths from the input signal. This reduces the magnitudes of the partial derivatives in the model equations. Consequently, while the noise is reduced, so is the signal after inverting the system. We leave the displacement data and noise unchanged and keep the number of data points the same, but increase the node spacing. This increases the system length, so we scale the system length back by decreasing the node spacing to its initial value after obtaining the active stress. So, the elongated data set is used to obtain the active stress, scaled back to the same initial system length, and compared to the original active stress signal and the active stress obtained from the initial data set.

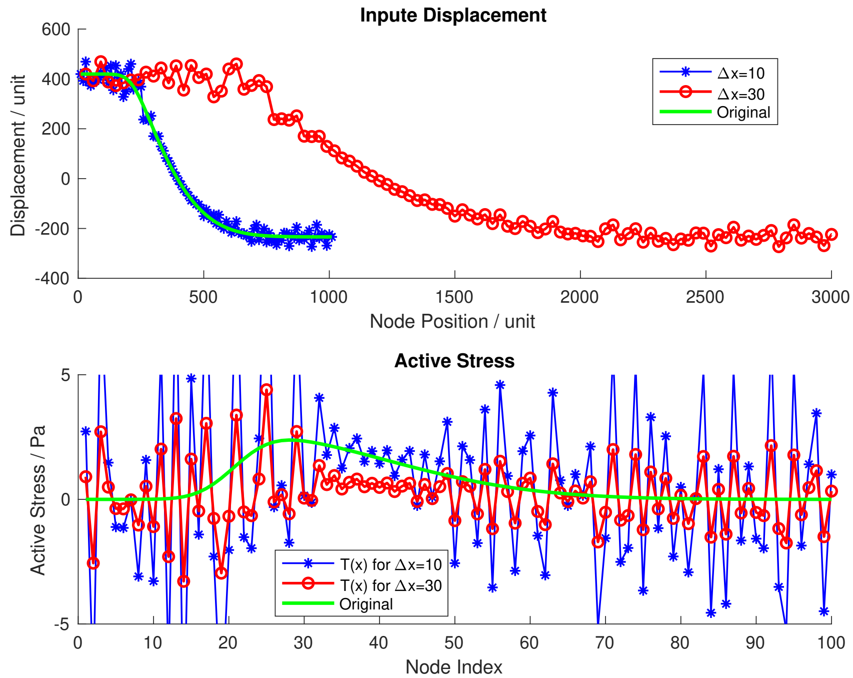

Figure 2. Above: Original (blue) and stretched displacement data (red) as functions of x. Below: Original active stresses (green), and active stresses calculated from the original (blue) and stretched data (red), as functions of node number.

In Fig 2, as the node spacing increased by a factor of 3, the active stress is dampened accordingly. At first, the results may seem to have less noise than those from the unaltered data set. However, closer examination shows that the signal is reduced by the same amount as the noise. Though the information density remains the same, this method is not as effective. So this method, while instructive, is probably not of practical use.

This result is not unexpected, since the relationship between the active stress and strain are not influenced by the scale of the system. Thus, the node spacing, Δx, does not affect the calculation when using this method. This characteristic was also seen in the mathematical derivation.

3.3 Tikhonov Regularization

Finally, we considered the inverse problem as an optimization problem[3] in which we seek the function $T(x)$ such that

$$\|AT(x) - \delta x\|_2^2 + \lambda^2 \|T(x)\|_2^2 \tag{4}$$

is minimized. Here, λ is the regularization parameter, controlling the relative weight of the two terms in the criterion function:

i.) The term $\|AT(x) - \delta x\|_2^2$ represents how good the fit is, similar to the usual least-square type problem.

ii.) The second term $\|T(x)\|_2^2$ represents the regularity of the solution. Since the naive inverse solution is dominated by the high-frequency components with large amplitude in the input displacement signal, we anticipate that the large noise components will be suppressed if we control the norm of $T(x)$.

As for the implementation, we use singular value decomposition (SVD) to decompose the matrix A, which allows us to convert the optimization problem into the following equation:

$$T(x) = (A^T A + \lambda^2 \mathbf{I})^{-1} A^T \delta x = \left[(\Sigma^T \Sigma + \lambda^2 \mathbf{I}) V^T \right]^+ \Sigma^T U^T \delta x \tag{5}$$

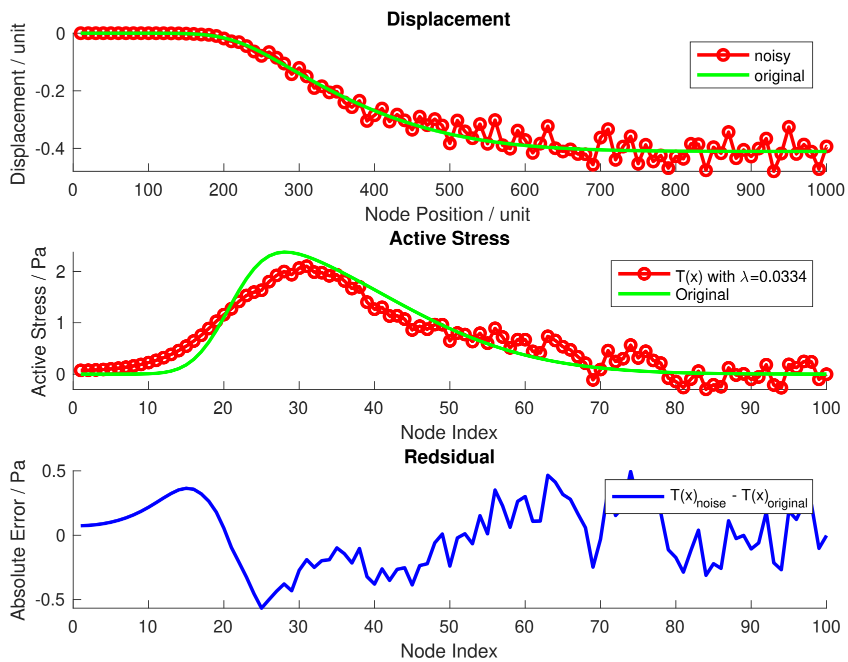

Figure 3. Results using Tikhonov regularization. Top panel: Original (blue) and noisy (red) displacement data. Middle panel: Original active stress data (green) and active stress calculated using Tikhonov regularization (red). Bottom panel: Error in the calculated solution.

Currently, we use a brute-force method to find an optimal solution, by solving the entire inverse problem repeatedly with different values of the regularization parameter λ. Our solution also has an overall vertical shift where the noise level is low. This is because the Neumann boundary conditions dictate that if $T(x)$ is a solution, then so is $T(x) + C$ for any constant C.

We see that this method has the potential to be a powerful tool for resolving our problem. Since the problem is one-dimensional and we have the original active stress to compare with, the complexity of the problem is minuscule, so we can get some useful results with a manageable run-time and RAM usage.

4. DISCUSSION, SUMMARY AND FUTURE WORK

The inverse problem solution for reconstructing active stresses from recorded tissue displacement data leads to noisy patterns, which may impede the accurate detection and visualization of the action potential wave that elicits the active stress. Here we propose and test three different methods for reducing noise and improving the signal to noise ratio, and we show their performance in the context of a 1D active stress- displacement model.

In the three methods we have experimented with above, the Tikhonov regularization provides a promising result in one-dimension, and reducing the sample rate may also help to filter the noise from the input. On the contrary, stretching the node spacing may not improve the noise reduction result.

However, the cost using SVD to solve Tikhonov will be considerably more expensive in higher spatial dimensions, so finding a better algorithm to solve the system is another goal we will want to achieve with future exploration.

A feasible solution is to avoid using SVD when solving the regularization problem. Instead, we could consider the inverse problem to be an optimization problem and solve it using the least-square-type method. The trade-off is between accuracy and efficiency. This is observable even in the one-dimensional model. In an example of a one-dimensional system with 100 nodes, the run-time of Tikhonov regularization with SVD and least square are 0.3076 second and 0.1858 seconds, respectively. In the one-dimensional case, we did not observe a difference in accuracy as we obtained identical residuals in both cases. This may not hold for higher dimensions.

The future complication in the three-dimension case may include considering the fiber angle and rotation, the passive stress, and realistic heart mesh modeling.

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