An Adaptive Electromigration Assessment Algorithm for Full-chip Power/Ground Networks

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Abstract— In this paper, an adaptive algorithm is proposed to perform electromigration (EM) assessment for full-chip power/ground networks. Based on the eigenfunction solutions, the proposed method improves the efficiency by properly selecting the eigenfunction terms and utilizing the closed-form eigenfunctions for commonly seen interconnect wires such as T-shaped or cross-shaped wires. It is demonstrated that the proposed method can trad-off well among the accuracy, efficiency and applicability of the eigenfunction based methods. The experimental results show that the proposed method is about three times faster than the finite difference method and other eigenfunction based methods.

I. Introduction

Electromigration (EM) becomes one of the most critical issue for the nanometer VLSI design. Due to the EM caused by increasing current density and shrinking wire sizes, it is predicted that the life time will decrease by half for each new technology nodes. As the technology node scaling down to 10 nanometers and below, it is crucial to verify the full-chip EM reliability before sign-off.

Traditionally, the statistical models such as Black's equation [1] and Blech limit [2] are adopted to calculate the mean time to failure (MTTF) and the immortality for wires with given current densities and temperatures. However, these methods are under growing criticism for their empirical nature and the lack of consideration of residual stress [3]. Moreover, electromigration on wires of on-chip networks are considered independently in these models, which is improper because the metal atom could migrate across the wire boundaries and the EM actually takes place in the entire interconnect tree [3].

In order to describe EM more precisely, the physics-based EM model was proposed and applied to EM reliability assessment for on-chip power/ground (P/G) networks [3]–[6] and through silicon via (TSV) [7]. The physics-based EM model is based on a couple of diffusion-like partial differential equations (PDEs) proposed by Korhonen [8], which describes the kinetics of the hydrostatic stress on wires. According to the physics-based model, EM-induced voids nucleate when the hydrostatic stress exceeds the critical value.

This work is funded by National Natural Science Foundation of China (NSFC) grants under No. 61602016.

For physics-based EM assessment, accuracy and efficiency are conflicting requirements because the Korhonen's equation is much more computationally expensive to solve than Black's equation. Therefore, previous works do trad-off between accuracy and efficiency when assess EM reliability by physics-based model. For instance, finite element analysis (FEA) based method [7] can only solve small structures such as one TSV because of the expensive computational cost. Another example is that only the steady-state hydrostatic stress was adopted in the compact physics-based EM model proposed for the EM assessment for large multi-branch interconnect trees [3], [4]. Although being fast, these methods sacrifice the accuracy of the prediction of the void nucleation time and MTTF because the transient hydrostatic stress was ommitted. To improve the accuracy, the finite difference method (FDM) was utilized in [5], [9] to provide numerical solution of the transient hydrostatic stress, which could also accommodate the non-uniform residual stress as well as the non-uniform transient thermal and current effects. While accuracy could be improved by sufficient discretizing in FDM, fine discretization results in large computational cost. To mitigate this problem, model order reduction (MOR) technique was applied in [10] to reduce the original system of FDM, which could then facilitate the efficient time-domain solution. But spatial and temporal discretizations are still necessary for this method.

Another effort made to settle the accuracy and efficiency confliction is to seek the analytical solution to Korhonen's equation. The first analytical solution was given by Korhonen for describing the hydrostatic stress of a single wire [8]. Laplace transform was utilized in [11] to develop the analytical solutions to several typical interconnect trees with specific structures. An integral transform based method was proposed in [12] to solve the Korhonen's equations analytically for one-dimentional (1D) multi-segment wires. Although the accuracy and efficiency are balanced well by these analytical methods, all of these methods could only be applied to limited structure of interconnect tree. In order to handle the case of general two-dimentional (2D) interconnect tree, an eigenfunction based method was porposed in [13] to provide semi-analytical solution to coupled Korhonen's equation on 2D interconnect tree. The solution given by this method is in fact semi-analytical because the eigenvalues and eigenfunctions are calculated numerically. Then, an analytic formula for eigenvalues of the

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straight line multisegment and star-structured multiterminal interconnects was proposed by [14]. Because the accuracy of the solution is affected by the number of terms of the eigenfunctions, a quite conservative eigenfunction number was used in [12], [13], which degraded the efficiency.

In this paper, an adaptive algorithm is proposed for the full-chip EM assessment. The proposed method is also eigenfunction-based but choose the eigenfunction terms wisely to mitigate the efficiency problem in [13]. The main contributions of this paper are as follows.

- A spectrum energy based criteria is proposed in this paper to adaptively decide the number of the eigenfunction terms. By showing the connection between the eigenfunction series and the discrete cosine transformation (DCT), it is demonstrated that the frequency spectrum determines the accuracy of the eigenfunction based solution. The illustrations of frequency spectrum for practical on-chip networks show that small number of eigenfunction terms are enough for sufficient accuracy for most of the interconnect tree. As a consequence, the appropriate eigenfunction number is determined so that the coefficients in transient stress solution represent most of the spectrum energy.
- The closed-form eigenvalues and eigenfunctions for some common interconnect tree structures are utilized to speedup the transient hydrostatic stress calculation. Based on the insight that eigenvalues and eigenfunctions are irrelevant to current densities, it is shown that the eigenvalues and eigenfunctions could be calculated at much lower cost for most interconnect trees of practical power/ground networks.
- For power/ground networks, an adaptive algorithm is proposed to trad-off the computational cost with the feasibility of the analytical hydrostatic stress solutions. Majority of the interconnect trees with simple topology are assessed by simple yet fast method, while complex interconnect trees are handled by the more general yet time consuming method.

This paper is organized as follows. Section II reviews the recently proposed physics-based EM model and existing eigenfunction-based solution. Section III explained the adaptive accuracy guarantee for eigenfunction-based solution. Section IV presented an adaptive algorithm accelerated by closed-form formula of eigenvalues and eigenfunctions. The accuracy and efficiency of the proposed method are demonstrated by the experimental results in Section V. Section VI concludes this paper.

II. Review of Physics-based EM Models and Analysis Methods

The physics-based EM model proposed by Korhonen [8] describes hydrostatic stress to model the void nucleation and kinetics of void size evolution. In this model, the hydrostatic stress evolution $\sigma(x,t)$ for a branch of the interconnect tree could be described as the diffusion equation (1).

$$\frac{\partial \sigma(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[\kappa\left(\frac{\partial \sigma(x,t)}{\partial x} + \mathcal{G}\right)\right] \tag{1}$$

where $\kappa = \frac{D_a B \Omega}{\kappa_B T}$ is the diffusivity of stress, $\mathcal{G} = \frac{Eq^*}{\Omega}$ is the EM driving force and t is time.

The hydrostatic stress on branches interact with each other, which is confirmed by the fact that diffusion equations of each branches are coupled by boundary conditions (BCs). For junction nodes on the tree, the BCs (2) represent the facts that the hydrostatic stress should be continuous and the atom flux should be conserved to 0.

$$\begin{gathered}\sigma_{ij_1}(x=x_i,t) = \sigma_{ij_2}(x=x_i,t) \\ \sum_i w_{ij} J_{a,ij}(x_i,t) = \sum_i w_{ij}\kappa_{ij}\left(\left.\frac{\partial \sigma_{ij}}{\partial x}\right|_{x=x_i} + \mathcal{G}_{ij}\right) = 0\end{gathered} \tag{2}$$

where ij represents the branches connected to junction node i, w_{ij} is the cross-section area of branch ij, κ_{ij} is the diffusivity of branch ij, $\sigma_{ij}(x,t)$ is stress distribution on branch ij and $J_{a,ij}(x,t)$ is the atom flux on branch ij.

For nodes at blocking boundaries of the interconnect tree, the atom diffusion is blocked because the metal lines are confined. Therefore, the atom flux at the block boundary is 0,reflected by the BCs (3).

$$J_{a,\ell j}(x_\ell,t) = \kappa_{\ell j}\left(\left.\frac{\partial \sigma}{\partial x}\right|_{x=x_\ell} + \mathcal{G}_{\ell j}\right) = 0 \tag{3}$$

where ℓ is the node at blocking boundaries.

Under the effect of EM-induced driving force, the hydrostatic stress will build up as tensile stress (i.e. positive stress) or compressive stress (i.e. negative stress). As long as the tensile stress exceeds the critical stress σ_{crit}, the void nucleates. After the void nucleation, the wire comes to the void growth phase, in which the void would enlarge in size as a result of the atom depletion caused by current density and the tensile stress built before will be effectively released.

To calculate the transient hydrostatic stress accurately, the eigenfunction based methods have been proposed in [12] and [13] for one-dimension (1D) multi-segment wire and two-dimension (2D) general multi-branch interconnect tree, respectively. The essential steps of these eigenfunction-based methods consist of : 1) calculating the eigenvalues and eigenfunctions. 2) computing the coefficients of transient stress solution in terms of eigenfunctions.

The eigenvalues λ_m and eigenfunctions $\psi_m(x)$ are solely decided by equation (4) and the boundary conditions (2) and (3), which means that the λ_m and $\psi_m(x)$ have nothing to do with the current density but are intrinsically related to the property of interconnect tree.

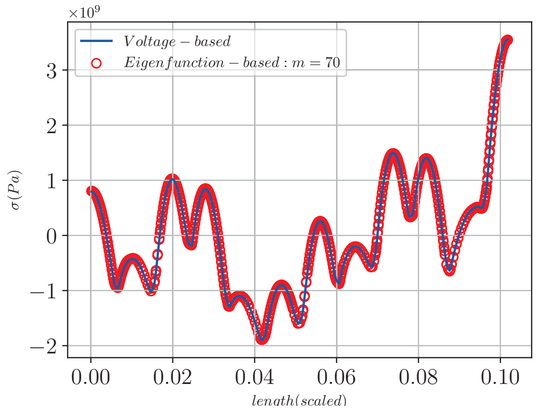
$$\frac{\partial^2 \psi_{ij,m}(x)}{\partial x^2} + \frac{\lambda_m^2}{\kappa_{ij}}\psi_{ij,m}(x) = 0 \tag{4}$$

The transient hydrostatic stress could be represented as the linear combination of eigenfunctions as shown by equation (5).

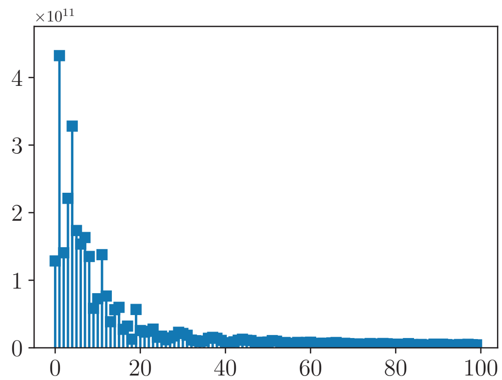
$$\begin{gathered}\hat{\sigma}(x,t) = \sum_{m=1}^{\infty} C_m e^{-\lambda_m^2 t}\psi_m(x) \\ \sigma(x,t) = \sigma(x,\infty) - \hat{\sigma}(x,t)\end{gathered} \tag{5}$$

The coefficients C_m are decided to satisfy the initial conditions as following equation (6).

$$\sum_{m=1}^{\infty} C_m \psi_m(x) = \hat{\sigma}_0(x) \tag{6}$$



(a)



(b)

Fig. 1. The steady state stress $\sigma(x, \infty)$ (a) and its frequency spectrum (b) on a interconnect of IBMPG6 benchmark.

where $\hat{\sigma}_0(x) = \sigma(x, \infty) - \sigma_T$ and σ_T is the residual stress which is usually a constant.

III. Adaptive Accuracy Guarantee for Eigenfunction Based Solutions

For eigenfunction based solutions [12] and [13], it is essential to choose proper number of eigenfunction terms because too many terms cost too much time to compute eigenvalues and eigenfunctions while too few terms lose accuracy of solution.

In eigenfunction based solutions, the transient hydrostatic stress $\sigma(x, t)$ is usually represented by an infinite series of eigenfunctions $\psi_m(x)$ with corresponding coefficients C_m, as shown in equation (5). Practically, only top M terms of eigenfunction are kept to provide the transient stress solution as long as enough accuracy is guaranteed. The basic idea to guarantee the accuracy is to keep the eigenfunctions with significant coefficients C_m and drop out those with trivial coefficients.

In order to figure out the significance of the coefficients C_m, we establish a connection between the eigenfunction series and the discrete cosine transformation. By investigating the case of one-dimentional multi-segment wires, it is found that the eigenfunctions are exactly the same as the basis function of DCT, which are $\psi_m(x) = \cos\left(\frac{m\pi}{L}x\right)$, and the coefficients C_m are essentially the amplitude spectrum of DCT. For eigenfunctions ψ_m, the coefficients C_m are solved as equation (7) to satisfy the initial conditions.

$$C_m = \frac{\langle \psi_m(x) \cdot \hat{\sigma}_0(x) \rangle}{\langle \psi_m(x) \cdot \psi_m(x) \rangle} = \frac{\int_0^L \hat{\sigma}_0(x)\psi_m(x)dx}{\int_0^L \psi_m^2(x)dx} \tag{7}$$

where $\langle \cdot \rangle$ is the inner product. Notice that $\hat{\sigma}_0(x)$ is exactly the same as steady state stress $\sigma(x, \infty)$ except its average value is 0. Therefore, steady state stress is used interchangeably with $\hat{\sigma}_0(x)$ below. At the meantime, if the DCT would be applied to discretized stress $\hat{\sigma}_0(x_i)$, the amplitude spectrum could be obtained as equation (8).

$$Y_m = w_m \sum_{i=1}^{N} \hat{\sigma}_0(x_i) \cos\left[\frac{m\pi}{N}(i - \frac{1}{2})\right] \tag{8}$$

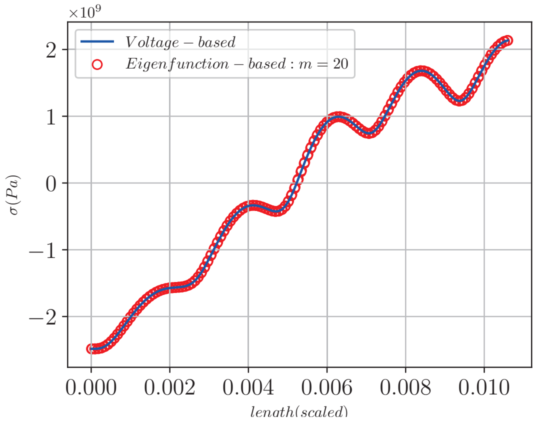
where Y_m is the result of DCT for $m = 1, 2, \cdots$, N is the discretization number, $\hat{\sigma}_0(x_i), i \in [1, N]$ is the sequence of the discretized stress on position x_i and w_m is the weight parameter. The connection between eigenfunction coefficients C_m and DCT could be established as equation (9).

$$\begin{aligned}
&\langle \psi_m(x) \cdot \hat{\sigma}_0(x) \rangle = \int_0^L \hat{\sigma}_0(x) \cos\left(\frac{m\pi}{L}x\right) dx \\
&\longleftrightarrow \sum_{i=1}^{N} \hat{\sigma}_0(x_i) \cos\left[\frac{m\pi}{N}(i - \frac{1}{2})\right] \\
&\langle \psi_m(x) \cdot \psi_m(x) \rangle = \frac{L}{2} \longleftrightarrow \frac{1}{w_m} = \frac{N}{2} \\
&C_m \longleftrightarrow Y_m
\end{aligned} \tag{9}$$

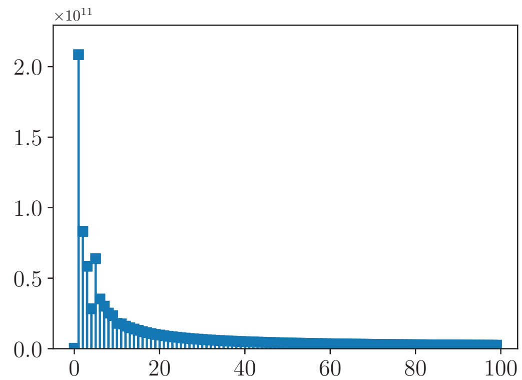
As we can see, the DCT is essentially a discrete version of the calculation of C_m and coefficients C_m is essentially the frequency spectrum of DCT.

Therefore, the truncation of the eigenfunction series could be done in the similar way as the data reduction utilizing DCT. Here, the frequency spectra for some interconnect trees of practical on-chip P/G networks are analyzed by DCT to demonstrate the number of eigenfunction terms for enough accuracy. Fig. 1 shows the steady-state stress and its frequency spectrum on a interconnect tree of IBMPG6 power networks. This interconnect tree has 947 segments and the steady state stress $\sigma(x, \infty)$ is decided according to the voltage distribution as proposed in [15]. It can be seen from the frequency spectrum Fig. 1(b) that 70 eigenfunction terms should be enough for accurate solution to because it covers most of the spectrum energy with these eigenfunctions. This fact is proven by Fig. 1(a), showing the eigenfunction based steady state solution with $M = 70$ agrees quite well with the exact steady state stress. In contrast, top 20 terms of eigenfunction are enough for another interconnect tree from IBMPG5 networks according its frequency spectrum, which is shown by Fig. 2. As a matter of fact, the proper number of eigenfunction terms varies case by case, depending on the frequency spectrum of steady state stress. This observation makes sense because the steady state stress essentially represents the product of length and current density and more variations of current density require finer analysis.

For general interconnect tree, the proper number of eigenfunctions could be similarly determined so that most of the



(a)



(b)

Fig. 2. The steady state stress $\sigma(x,\infty)$ (a) and its frequency spectrum (b) on a interconnect of IBMPG5 benchmark.

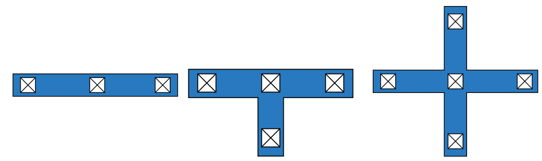
spectrum energy would be covered. The relative error could then be measured by difference of the energy, as shown by equation (10).

$$
\begin{aligned}
err &= \frac{\int_T (\sigma(x,\infty) - \hat{\sigma}(x,0))^2 dx}{\int_T \sigma^2(x,\infty)dx} \\
&= \frac{\int_T \left(\sigma(x,\infty) - \sum\limits_{m=1}^{M} C_m \psi_m(x)\right)^2 dx}{\int_T \sigma^2(x,\infty)dx}
\end{aligned}
\tag{10}
$$

where $\int_T \cdot$ is the integral all over the interconnect tree. Notice that above relative error is calculated in time domain instead of frequency domain because this facilitates the incremental calculation of eigenvalues and eigenfunctions. In practice, the eigenfunction series is increased term by term to improve the solution accuracy until the error is tolerable.

IV. Acceleration Using Closed-form Eigenvalues and Eigenfunctions

While eigenvalues and eigenfunctions have to be determined numerically for general two-dimentional (2D) interconnect trees, there are closed-form eigenvalues and eigenfunctions for some typical interconnect trees with specific structures, which could facilitate the speedup of the eigenfunction based solution.



(a) (b) (c)

Fig. 3. Typical interconnect trees with specific structures : multi-segment 1D wire (a), 4-terminal T-shape tree (b) and 5-terminal cross-shape tree.

These common interconnect trees include multi-segment 1D wire, T-shape tree and cross-shape tree as mentioned in [11], that are shown by Fig 3(a), 3(b) and 3(c), respectively. The multi-segment 1D wire is the most common structure on P/G networks, of which the eigenvalues and eigenfunctions are known as equation

$$
\begin{aligned}
\lambda_m &= \frac{m\pi}{L} \\
\psi_m(x) &= \cos(\lambda_m x) = \cos\left(\frac{m\pi}{L}x\right)
\end{aligned}
\tag{11}
$$

where L is the total length of the wire. For 4-terminal T-shape interconnect tree, it can be verified that the eigenvalues $\lambda_m = \frac{m}{2l}\pi$ satisfy the governing equation and BCs, where l is the length of each branch. There are three eigenfunctions corresponding to λ_m when m is odd, shown as equation (12). Notice there are only two linear independent eigenfunctions since $\psi^{(3)}_{ij,m} = \psi^{(2)}_{ij,m} - \psi^{(1)}_{ij,m}$.

$$
\begin{aligned}
&\psi^{(1)}_{01,m} = \cos\left(\frac{m\pi}{2l}x\right), \psi^{(1)}_{02,m} = -\cos\left(\frac{m\pi}{2l}x\right), \psi^{(1)}_{03,m} = 0 \\
&\psi^{(2)}_{01,m} = \cos\left(\frac{m\pi}{2l}x\right), \psi^{(2)}_{02,m} = 0, \psi^{(2)}_{03,m} = -\cos\left(\frac{m\pi}{2l}x\right) \\
&\psi^{(3)}_{01,m} = 0, \psi^{(3)}_{02,m} = \cos\left(\frac{m\pi}{2l}x\right), \psi^{(3)}_{03,m} = -\cos\left(\frac{m\pi}{2l}x\right)
\end{aligned}
\tag{12}
$$

When m is even, the eigenfunction for T-shape interconnect tree is $\psi_{ij,m} = \cos\left(\frac{m\pi}{2l}x\right)$. For 5-terminal cross-shape interconnect tree, the eigenvalues are the same as those of T-shape tree as $\lambda_m = \frac{m}{2l}\pi$ while there are three linear independent eigenfunctions corresponding to λ_m with odd m built in the similar way as those of T-shape tree and eigenfunction for even m is still $\psi_{ij,m} = \cos\left(\frac{m\pi}{2l}x\right)$.

Since the closed-form eigenvalues and eigenfunctions for the T-shape and cross-shape with arbitrary branch lengths are hardly available,the closed-form eigenvalues and eigenfunctions for 1D wire are the most common-used.One of the most important characteristics of eigenvalues and eigenfunctions is that they are irrelevant to current densities on branch. Therefore, the multi-segment branches could be regarded as one branch on the basis of the cross-section and diffusivity, as illustrated by Fig. 4. Then, the eigenvalues and eigenfunctions could be find according to simplified structure.

It can be seen from above that the cheap closed-form eigenfunctions are only available for interconnect trees with simple structure while general numerical method to decide eigenfunctions are computationally expensive. In order to mitigate this conflict, an adaptive algorithm is proposed to trad-off the computational cost with the feasibility of the analytical hydrostatic stress solutions. For on-chip P/G networks,most of the interconnect trees are multi-segment wires, which will

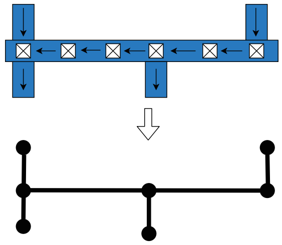


Fig. 4. Multi-segment branches reduced as one.

Algorithm 1: Adaptive algorithm to find t_{nuc} considering immortality and structure of interconnect tree.

Input: P/G networks and the accuracy requirement $error_{critical}$.
Output: Void nucleation time t_{nuc}.
Calculate $\sigma(x, \infty)$ according to [15] ;
for *interconnect:IBMPG* **do**
    if $\max \sigma(x, \infty) < \sigma_{crit}$ **then**
        delete interconnect;
    else
        while $error > error_{critical}$ **do**
            Calculate M eigenvalues and eigenfunctions;
            if *interconnect structure is straight wire* **then**
                Use closed-form λ_m and $\psi_m(x)$;
            else
                Use WW Algorithm;
            end
            $error = 1 - \sum_{m=1}^{M} C_m \psi_m(x)/\sigma(x, \infty)$;
            M++;
        end
    end
end
Use bisection algorithm to find the t_{nuc}

be assessed by analytical stress solution with known eigenfunctions. For other complex interconnect trees, the numerical W-W algorithm are utilized to figure out the eigenfunctions upon which transient hydrostatic stress is built. In addition, the immortality of interconnect tree are checked as proposed in [15] before the transient stress analysis begin. If the highest stress of the tree exceeds the critical stress, the eigenfunction series is increased term by term to improve the solution accuracy until the error is tolerable, otherwise, no further transient stress analysis is required. Since all eigenfunction based methods share the characteristics of analytical solution, the technique of bisection could be adopted when find the void nucleation time. As a result, an adaptive algorithm is proposed to figure out the void nucleation time for P/G networks, as shown by Algorithm 1.

V. Experimental Results

It has been demonstrated by Fig. 1(a) and Fig. 2(a) that the adaptive approach to select proper eigenfunction terms could provide accurate solution for steady state stress, with error less than 1%. To further verify that the adaptive selection of eigenfunctions could also guarantee the accuracy of transient hydrostatic stress, we compare the transient hydrostatic stress for these interconnect trees here. Fig. 6 shows the transient stress on interconnect tree of IBMPG6 at $t = 6.1 \times 10^9 sec$ calculated by eigenfunction based solution with $M = 70$ as well as those calculated by FDM. The discrepancy between two results are 1.6%. Fig. 5 shows the transient stress on

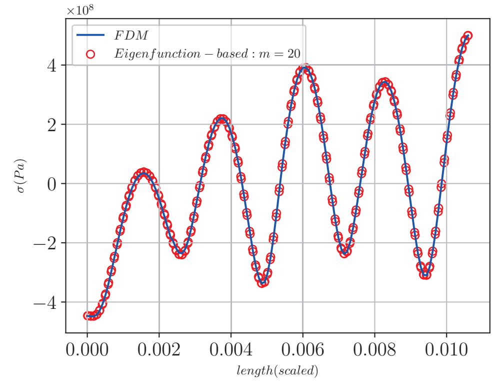


Fig. 5. The transient stress at $1.3 \times 10^9 sec$ for interconnect of IBMPG5.

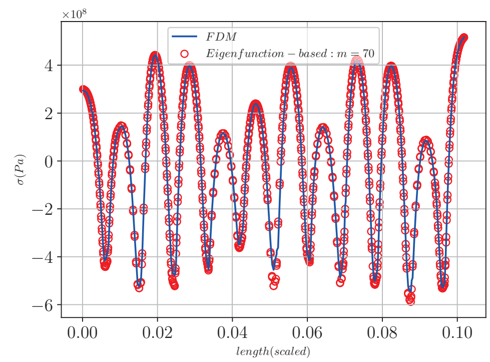


Fig. 6. The transient stress at $6.1 \times 10^9 sec$ for interconnect tree of IBMPG6.

interconnect tree of IBMPG5 at $t = 1.3 \times 10^9 sec$ calculated by eigenfunction based solution with $M = 20$ as well as those calculated by FDM. The discrepancy between two results are less than 0.15%. The number of eigenfunction terms M in both of cases are decided so that 99% of the spectrum energy is represented. These results validate the effectiveness of adaptive approach of eigenfunction selection.

The histogram of the number of eigenfunction terms M is given in Fig. 7. As we can see, the number of interconnect is about 74.3% that the eigenfunction terms between 20 and 100. And the eigenfunction term below 140 more than 90%. A few of interconnect trees with intensively fluctuated current

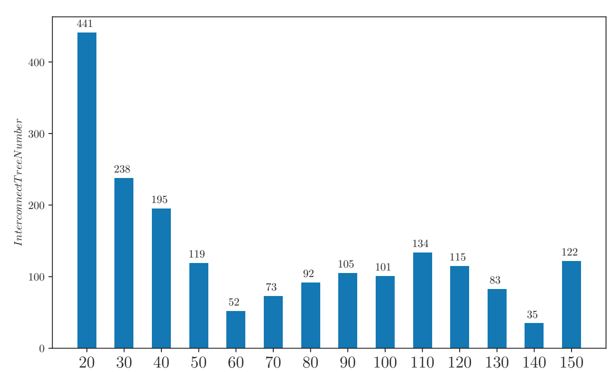


Fig. 7. The histogram of the number of eigenfunction terms for IBMPG5.

TABLE I
RUNTIME COMPARASION OF THE PROPOSED ALGORITHM AND OTHER METHODS.

Power Grid		Trees Number		Void Nucleation Time (yrs)				Runtime (sec)			
Name	#Trees	#Immortal	#1D	FDM	Eigen	Adaptive	Proposed	FDM	Eigen	Adaptive	Proposed
IBMPG2	462	52	410	0.56	0.56	0.56	0.56	614	909	1067	424
IBMPG3	6516	4403	2113	4.31	4.56	4.20	4.20	726	616	837	220
IBMPG4	1347	378	947	2.91	3.02	2.78	2.78	1626	1003	1451	404
IBMPG5	1982	25	1957	1.62	1.60	1.61	1.61	5645	4815	4392	1724
IBMPG6	3964	2040	1924	6.95	7.08	6.96	6.96	9989	6395	10325	2689

desnity require more terms $M > 140$. Nevertheless, this result shows that less terms of eigenfunction are needed than those used in [12], [13] for most cases.

In order to demonstrate the efficiency of the proposed algorithm, The proposed method competes with the original eigenfunction based method and the FDM to find the void nucleation time t_{nuc} for IBMPG benchmarks [16]. The proposed method and FDM are both implemented in C++ and tested on a linux workstation with a 3.6GHz dual-core CPU and 8GB memory. The experimental results are shown in Table I, where column "#Trees" is the number of interconnect trees we tested, column "#Immortal" is the number of immortal interconnect tree, whose $\sigma(x, \infty) < \sigma_{crit}$, column "#1D" is the number of 1D multi-segment wire, and columns "FDM", "Eigen", "Adaptive" and "Proposed" stand for the finite difference method, the eigenfunction based method in [13], the eigenfunction method with adaptive term selection and the proposed Algorithm 1, respectively. The runtime shows that the proposed method is about three times faster than other methods.

The experimental results shown in Table I demonstrate that the proposed method do an excellent trad-off among the accuracy, efficiency and applicability of the eigenfunction based methods, which makes it significantly faster than other methods.

VI. CONCLUSION

In this paper, it is verified that the proper number of eigenfunction terms could be determined by frequency-domain analysis for steady-state stress and small number of eigenfunction terms are enough for sufficient accuracy for most of the interconnect trees. In addition, closed-form eigenvalues and eigenfunctions for interconnect trees with specific structures are utilized to accelerate the computation of transient hydrostatic stress. Based on these, an adaptive algorithm is proposed to carry out fast EM assessment for full-chip P/G networks.

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