

INFRASTRUCTURE TO SUPPORT STUDENTS EXERCISING CONCEPTUAL AGENCY

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Generative activities have been shown to support students to engage in space-creating play and exercise their conceptual agency to generate a mathematical space (e.g. Stroup et al. 2004), yet these studies implement generative activities only with their resonating counterpart, classroom networks, technological infrastructures that connect multiple, co-present students into a shared, digital representation. Because these technologies are in continuous redesign and still inaccessible to many classrooms, we need to understand the crucial features their infrastructure provides to the classroom system. By analyzing the strains on the classroom without classroom networks and how they relieved that pressure and revive the system, we found that the collective public displays provided students with a collective orientation and a sense of connection and individualism.

Keywords: Design Experiments, Technology, Rational Numbers

Introduction

Generative activities are activities operating at the individual, small group, and whole class within which students are actively constructing connections and relations of mathematical ideas in both prepared and emergent participation structures that reflect and build on the mathematical ideas that the group creates (Stroup, Kaput, and Ares 2002; Stroup et al. 2004; Stroup, Ares, and Hurford 2005; Ares, Stroup, and Schademan 2009). In these types of activities, the class's social group functions to explore mathematical structures together and uses their social dynamics as a purposeful resource to support their exploration. A common means for designing and developing such activities take a standard, closed-form question as a starting point, and "inverts" it, making the *answer* of the standard question into the prompt for the generative activity. For example, instead of asking students to "simplify $4(x-3)+12$ " (a closed-form question, with correct answer " $4x$ "), one might ask them each to create several expressions that are "the same as $4x$ " (Stroup, Kaput, and Ares 2002). By inverting the traditional one-correct-answer task, generative activities provide ways for students to construct or apply mathematical principles (e.g., exploring additive inverses by repeatedly adding " $+x-x$ " to an expression known to be equivalent to $4x$). When this kind of construction is occurring in parallel across the classroom, students are able to use the diversity of their group and their ideas for experimentation to generate a mathematical *space*.

Stroup et al. (2002; Stroup, Ares, and Hurford 2005) describe the resonance of generative activities with classroom network technologies to provoke new theoretical, methodological, and design frameworks. They articulate two main principles in the flow of a generative activity: (a) space-creating play and (b) dynamic structure. Space-creating play is the idea of students generating a mathematical space via experimentation, exploration, and playfulness. Dynamic structure refers to the emergent set of connections and meanings that appear as the students produce mathematical creations and respond to each other's work, both by commenting and by imitating, expanding on, or combining work to make new creations. Dynamic structure makes use of a functional sense of activity structure that is brought into being through students' playful

actions and characterizes the unfolding space students are generating. Stroup et al. use these two ideas to argue that the relationship between mathematical/scientific structures and social structures is dialectical, with each mutually building off of the other. Essential to this process is the collective, public display of students' mathematical space, either in some physical/digital inscription or through social display.

As a complementary perspective of these public displays, we can consider them a space for *conocimiento* (Anzaldúa 1987 cited by Gutiérrez 2012), or sense of becoming familiar, connecting, and receptive of others. Through students' shared solidarity in generating the mathematical space, they develop their *conocimiento* of both the unfolding mathematical structures and the persons engaged in the display. Additionally, public displays of their work at the whole-class level may support students' sense of *nos/otras* (Anzaldúa 1987 cited by Gutiérrez 2012), or the juxtaposition of the collective and the individual. Further connections of this perspective with generative activities and classroom networks is unexplored and possibly very fruitful because of their differences in framing knowledge but similarities in positioning participants as generators of that knowledge.

Though Stroup et al. further describe the *resonance* between generative activities and classroom networks, arguing that the networked classroom is particularly suited to support a dialectic relationship between space-creating play and dynamic structure, few studies have explored these constructs in mathematics classrooms without the technology¹. Substantial research has shown the impact of these new networking technologies and their resonance with generative activities (e.g. Ares, Stroup, and Schademan 2009; Ares 2013; Stroup, Carmona, & Davis, 2011), but these technologies are both largely unattainable for most classrooms and still going through continuous redesign. Thus, we need to understand the specific features of the classroom network critical to fostering collective mathematics inquiry through space-creating play and dynamic structure and which are optative. Furthermore, understanding which of the features should be customizable and which are fairly generic to collaboration will both support continued technology design and strengthen the underlying theory of collective mathematics. To investigate these features of classroom networks, we investigate 1) Do generative activities and collective mathematical exploration put strain on normal classroom infrastructure? (and, how?) and 2) Which aspects of classroom networks alleviate that pressure? (and, how?).

Classroom Networks and the Group-based Cloud Computing System (GbCC).

Classroom networks have been an area active, but uneven, research and development for over 20 years (or much longer, depending on one's definition (see Abrahamson 2006; Abrahamson and Brady 2014; Roschelle, Penuel, and Abrahamson 2004)), with a varied history of research and commercialization efforts. For the purposes of this paper, a classroom network (c.f. Brady et al. 2013) is a representation and communications infrastructure (Hegedus and Moreno-Armella 2009) consisting of hardware, software, and curricular/activity components. The hardware includes a set of devices (laptops, smartphones, or other custom communications-enabled "computers"), with each student (or, less commonly, each small group), having a device. These devices are networked to communicate directly or indirectly with each other and with a teacher computer, which is connected to a public display (usually a digital projector). Software, running on the classroom computers and/or on a networked server, provides aspects of communications infrastructure by routing messages among the participating devices in configurable, activity-specific ways. Software also provides a representation infrastructure, offering students and teachers views of the activity and tools to contribute that are appropriate

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for the discipline, the activity, and the participants' roles. Finally, at the curricular/activity level, "documents" or other specifications of roles or goals can be sent to participants to configure their devices and displays, and to facilitate the activity in real time.

GbCC (Brady et al. 2018) is a system of this kind, emphasizing flexible programmability and rich discipline-specific representations for mathematics, science, and the social sciences. It leverages browser-based open-source tools, building upon the NetLogo Web agent-based modeling environment (Wilensky 2015), married with GeoGebra Web (<https://www.geogebra.org/>) as a dynamic mathematics platform for geometry and algebra in Euclidean and Cartesian representations; and several other extensions to support mapping (Leaflet, <https://leafletjs.com/>) and 2d physics (Box2d, <https://box2d.org/>). As a platform for design-based research environment, its programmability supports an open-ended array of activity structures, and it can be run on any browser-enabled device (phones, tablets, or laptops). Its flexibility, configurability, and programmability make it ideally suited to exploring our research questions.

Data and Methods

The current study was a single four-week cycle from a larger design-based research (DBR) project. The 20 participants came from a 5th grade classroom at a public middle school serving a racially (39% Black, 6% Hispanic, 4% Asian) and economically (41% free or reduced lunch) diverse population within a large metropolitan district in a midsize southern city in the USA. The class period of the DBR study was not students' normal mathematics class but a time when students were tracked based on standardized tests in order to provide individualized attention (called Personal Learning Time, PLT, in the school). The participants from the current study were considered math tier 2 students (i.e., on target but needing some extra time for mathematics). Because of the nature of standardized testing and the flexibility of this class period, students moved from tier to tier or subject to subject depending on the most current testing. Thus about half of the students in the current study had participated in a prior implementation of a design cycle with generative activities without technologyⁱ. The first author facilitated about 2 class sessions each week over a four week period totaling of 8 sessions, each 30-45 minutes in length, and the classroom teacher either co-facilitated or pulled specific students for individual work.

The primary data source for the current study was design and field notes taken by the first author. Audio and video recordings of each lesson were also collected and used to triangulate findings. Analysis was ongoing and continuous throughout the design where the humble theories of the class's mathematical thinking and engagement were revised after each lesson (Cobb et al. 2003), in conversations among the researchers and with the teacher. Posterior analysis took the form of reviewing the progression of the lessons contrasted with the predicted learning trajectory. We paid special attention to anticipated and unanticipated challenges and strains on the classroom system prior to introducing network technology and the nature of how those challenges and strains changed when using it.

Mathematical Context and Predicted Learning Trajectory

We chose to target 5th grade fractions standards involving equivalence, operations, and comparison for this study. Fractions have been found to be a particularly difficult concept for students, yet they can be readily used as the basis for generative activities because the mathematical space of equivalent fractions is both core to the standards and very rich. We

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created a sequence of generative activities, to explore equivalence for the first two weeks and then operations on fractions for the second two weeks. The activity for both topics followed a similar rough structure. The first day of each of the two weeks focused on “space-creating play” to generate the space of ways to make $\frac{1}{2}$, either with equivalent fractions or with fraction operations, depending on the topic. Students worked in small groups during these times, to foster connections in their space-creating play and reflection on the dynamic structure they were creating. Following this small group work to make $\frac{1}{2}$, a whole-class discussion explored the different kinds of objects in the space (to make $\frac{1}{2}$) and the mathematical principles students used to generate the space. Building off this the following class session (a week later), students returned to small groups to generate ways to make a fraction of their group’s choice followed by another whole-class discussion of the mathematical principles. This trajectory was supported by research both on fractions (Lamon 2012) and generative activities (Stroup, Kaput, and Ares 2002; Stroup, Ares, and Hurford 2005), the key difference from the latter was the lack of networking technology. Beyond the curricular goals, we predicted the generative activities would support students to take conceptual agency (Boaler and Greeno 2000) in the classroom to create mathematical principles of equivalence and operation and to voice their conceptual perceptions even without technology. We remained open to the question of whether these technologies would be needed, by observing the classroom system, students’ engagement in the tasks, and the degree to which they exercised conceptual agency.

Results

Through our design and analysis of generative activities to support students’ conceptual agency in exploring fractions *without* technology, we found that these activities put multiple strains on the classroom system for students to engage and participate. Without the technological infrastructure and additional ways to participate in the activity, the whole-class discussions led by the first author were not able to support students to have a platform to show the work they did in small groups, or to have much of a “voice” at the whole-class level. This central strain reduced students’ engagement over time, and following the second whole-class discussion (week 2), the necessity of additional infrastructural support was apparent, both to the authors and to the classroom teacher. Upon the introduction of technology, students’ re-engagement in the generative-activity process was visible, as usual with the introduction of any new technology. Yet more meaningfully, students’ engagement was sustained through the last two weeks, and their conceptual agency increased in that time. This process contrasted significantly with the time without technology when their engagement and utilization of conceptual agency decreased over the course of the same time period. By comparing the strains on the classroom system during generative activities without technology and how the infrastructure provided by the technology relieved those strains, we can begin to identify some of the crucial features of classroom networks.

Generative Activities’ Strains on the Classroom System

Progressively throughout the first two weeks of equivalent-fraction generative activities, we documented how students became less and less engaged and utilized their conceptual agency less and less. This process came to a climax when the classroom teacher requested a change in the activity in order to re-engage students at the end of week two. Upon analysis of the design, students’ disengagement was progressive. Students engaged readily in the initial generative activity convening the space-creating play in almost all the small groups. Some groups even utilized their conceptual agency to recognize patterns and methods in their generation of

equivalent fractions. Yet, during the whole-class discussion, students struggled to know how to participate in productive ways and see their hard work validated. Multiple students made various bids to read aloud their list of fractions in its entirety, but with upwards of over 30 fractions, this was not logistically possible. Moreover, without a means to organize or represent these contributions visibly, a reading would not have contributed to the dynamic structure. Instead, the first author focused on having students share out their methods of generating fractions and patterns they observed in their set of equivalences. While students did engage in the discussion and built multiplicative conceptual resources for fractions, field notes capture a number of students' feelings of discontent.

The following week, the first author launched another generative activity to build on students' work with $\frac{1}{2}$ by generating fractions the same as a fraction of their choice. Unlike the start of the previous activity, the teacher and the first author struggled to support students to begin the activity (even to choose a fraction), and to convene space-creating play in their small groups. In the students' eyes, the small group work had lost its importance and meaning after the previous week's whole-class discussion when they perceived their work was left unchecked, ungraded, and unshared with the class. While either adult was present, students would work together to generate equivalent fractions, but their motivation reflected a perceived lack of importance of their work at the whole-class level. Thus, students' patterns and methods were much less robust during the whole-class discussion the following day, and fewer students participated. Additionally, one of the students from the previous week made another bid to read all of her fractions aloud, demonstrating a continued desire to showcase her work at the whole class level, to hear her voice as part of the group, and receive validation for the effort she had put in. Because of students' steeply declining engagement, we decided to introduce technology to re-engage students and support their sustained participation in generative activities. Our prediction was that the introduction of technology would quickly re-engage students with the task of generative activities, and that comparison in students' sustained engagement would reveal the some of the crucial features of classroom networks to support students' collective mathematics in generative activities.

Adjusted Learning Trajectory and Use of Classroom Networks

Because of the strains of the classroom system for students to see their work as meaningful at the whole-class level, we adjusted the research plan to incorporate GbCC support for the activities in the final two weeks. Since the activities designed with the technology did not strictly align with the original learning plans, we adjusted the curricular goals to target fraction comparison instead of fraction operations. We planned to use GbCC's public display to create a joint representation for students to see a reflection of themselves and their classmates as they engage with mathematics. The classroom network assembled students' fraction input as a character moving on a vertical line between a teacher-defined maximum and minimum value, with its y-coordinate corresponding to the fraction value. The class appeared as a collection of these characters moving between the max and min values. If a student's fraction input was outside of this range, their character was shown into a gray area above or below. The goal of the first week was for students to make connections from their work with equivalence within the technology as a way to begin to understand the representational forms it used and then for the class to quickly transition into comparing 'easy' fractions. We wanted students to have the chance to explore within a technologically enhanced representational world and for the class to see each other's explorations to discuss our methods and strategies. In this way, the classroom network would provide additional communicative pathways for students to feel their work and

their classmates' work were meaningful at the whole-class level. We planned to end the activity sequence with supporting students to see the *density* of fractions (i.e. that between any two fractions there is another fraction). We conceptualized this as a 'zooming in' effect with the technology where the teacher could make the range a subset of the previous defined range and fractions could still be found.

The first two days of implementing GbCC went as predicted. The technology served to revive students' engagement and enthusiasm while also providing additional tools and representations to the work they were doing as a whole class. The public, anonymous display provoked a collective responsibility to fill it, positioning students to hold each other accountable during the activity, and during whole-class discussions, this public representation was a collective object for us to reference. During this space-creating play, students exercised their conceptual agency by choosing personally relevant numbers (not something seen the previous week). For example, one student found the fraction equivalent to 1/7th where the numerator was her birthday (mmddyy). Students patterns and methods extended the ideas from previous weeks using multiplicative relationships to generate equivalent fractions.

The final week of the study focused on comparing fractions, with the goal of students' having insight into the density of fractions. We started with a whole-class discussion of the previous weeks' work and asking if students had ways to know if one fraction is bigger than another (no technology). Even without technology, the class sustained a meaningful discussion, leveraging the collective perspective provided by the classroom network activities the previous week. In the following two days of activities, students sustained engagement and motivation, unlike the second week without classroom networks. Furthermore, students' utilization of conceptual agency grew as their fluency with the technology grew, compared to declining as their engagement declined, in the first two weeks. As students interacted with and in the mathematical space, a few began to use the public display as a dynamic representation - moving their characters across the screen by manipulating their fraction input successively. This type of play showcased how the classroom network became an embedded infrastructure for students to represent movement and communicate their actions to me and to others. Additionally, while these playful actions were unexpected and in fact went against the underlying goal of the activity for students to develop insight into the density of fractions, students were developing individual and share-able fluency with manipulating and comparing fractions in service of the personally-meaningful goal to predict the movement of their character up, down, and into the middle. Such spontaneous, and unpredicted, utilization of conceptual agency was not present without the classroom network's representational and communication infrastructure.

Crucial Features of the Classroom Network

The above analysis explored how generative activities strained a classroom system without adequate representational and communicational infrastructure and identified features of classroom networks that were crucial to relieving those strains and supporting students in utilizing their conceptual agency. The collective, public representation of students' work with fractions was the focal point of two such crucial features that supported collective mathematics and that were very difficult to provide without technological support. First, as demonstrated in the first week and the follow-up discussion without technology, the *public display of an aggregate representation of students' contributions* provided an essential means of discussing the activity, referring to students' work in context, facilitating activity flow, and sustaining students' attention. Leveraging this feature, we were able to facilitate whole-class discussions where students engaged in illuminating the underlying multiplicative structure of equivalent

fractions and continue the conversation even when the technology was temporarily removed. These types of whole-class discussions were very different prior to implementing the technology when students did not have such a collective orientation, and they made multiple bids to reorient the discussion towards what they felt was important (e.g., their personal lists of equivalent fractions).

The second crucial feature relating to the public display was *the communal, real-time dynamic nature of the public representation*. Students displayed a sense of both collective effort and individual publicity, or *nos/otras* (Gutiérrez 2012). Simultaneously feeling both connected to the community and represented as an individual was essential for collective mathematics. The importance of this feature was demonstrated first when the classroom network was first introduced as students began to hold each other accountable to participate in the activity, and it grew further when students began utilizing their conceptual agency and publicizing their new skill of predicting the movement of their character, showing their abilities to others and sharing how they did it.

Discussion: Students Utilizing Conceptual Agency with the Technology

Understanding how the representational and communicational infrastructure of classroom networks support students' space-creating play and utilization of their conceptual agency can provide insight into these technologies' functionality and support their ongoing design. At the same time, it also can inform efforts to enact generative activities without classroom networks, identifying needs and resources for alternative supports in such classrooms. Based on our comparison here of a classroom with and without technology, two crucial features of the dynamic infrastructure emerged, in the collective orientation provided by the public representation and the simultaneous communicative avenues of collective and individual voice developing a sense of *nos/otras*. These aspects are vital to keep in mind as we continue to design classroom networks, infrastructure, and activities to further support students exercising their conceptual agency.

Additionally, generative activities need to be flexible enough to support students' adaptation of the task as they exercise their conceptual agency. Similar to work in microworlds (Edwards 1998), generative activities supported by classroom networks are not capsules of disciplinary learning and conceptual agency. Rather, we need to design for and encourage students to make expressive and unpredicted conceptual moves as they interact with the representations and concepts of the activities. On the other hand when the classroom system does not have the infrastructure of classroom networks, traditional infrastructures must be adjusted to foster collective orientation and *nos/otras*. Specifically, students need some form of collective representation of the concept to orient their individual or small-group work towards each other. Furthermore, social infrastructure must support students as they make their work public to both hear their own voice and, metaphorically, hear the voice of the choir. Over time, classroom systems can develop these types of social infrastructures through socio-mathematical norms, but classroom networks may foster more rapid development of them or a lower threshold of effort for sustaining them over time.

Implications for Further Research

Classroom networks provide a flexible space for students to interact, both with mathematical ideas and with each other, and a dynamic, public display of their work as it unfolds. This space quickly creates infrastructure in the class to foster students' prolonged engagement and

utilization of their conceptual agency. Yet pragmatically, teachers, administrators, and researchers may question the necessity of this technology when compared to its cost and disruption. By observing and documenting first how a classroom group experienced strain without the technology and then was supported by it, we understand better the value of the technology, what types of additional activities may supplement it, and ideas on how we might support the classrooms without it. Additional work should compare other types of representational and communication infrastructures (Hegedus and Moreno-Armella 2009) and curriculum activity systems (Roschelle, Knudsen, & Hegedus, 2010) to better understand how students participate in collective inquiry and the necessary of these infrastructures to support students in exercising their conceptual agency. Specifically, previous studies have shown collective inquiry is possible without technology (e.g. Ball 1993; Lehrer, Kobiela, and Weinberg 2013; Fiori and Selling 2016), and exploring the infrastructure imbedded in these types of classrooms will provide insight into both the dynamics of group mathematics learning and into the design of networking technology.

Endnotes

Stroup's introduction of the construct of generative activities clarifies that their roots lie outside of mathematics, connecting to work in reading comprehension by Wittrock and in shared identity building by Freire (the identification of a community's "generative words").

² A disruption of losing half the participants and gaining the same number of new students caused analysis of the two design cycles to lose much of its meaning, but the class during the analyzed cycle remained intact.

Acknowledgments

We gratefully acknowledge the support of NSF grants #1652372 and #1614745.

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