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INVESTIGATING TRAJECTORY BASED COMBUSTION CONTROL USING A CONTROLLED TRAJECTORY RAPID COMPRESSION AND EXPANSION MACHINE (CT-RCEM)

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ABSTRACT

This work presents a systematic framework for the real time implementation of a new combustion control strategy – trajectory based combustion control – for the operation of free piston engines. The free piston engine is an alternative architecture of IC engines which does not have a mechanical crankshaft and hence allows extreme operational flexibility in terms of piston trajectory. The key idea of trajectory based combustion control is to modulate the autoignition dynamics by tailoring the pressure and temperature history of the fuel-air mixture inside the combustion chamber, using the piston trajectory as the control input, for the optimal operation of the free piston engine.

Here, we present the experimental investigation of trajectory based combustion control using a novel instrument called controlled trajectory rapid compression and expansion machine (CT-RCEM) that can be used for studying a single combustion cycle of an internal combustion engine with precisely controlled initial and boundary conditions. The effect of the shape of the piston trajectory on the combustion phasing, combustion efficiency and the indicated thermal efficiency has been found to be significant. The experimental results indicate that the trajectory based combustion control is an effective strategy for combustion phasing control for FPE operation.

Keywords: CT-RCEM, trajectory based combustion control, autoignition, free piston engine, iterative learning control

INTRODUCTION

The globally growing concern for reducing emissions and improving fuel efficiency has provided a significant push to the research effort in the transportation sector. In the US alone, the transportation sector contributes to roughly 30% of the national energy consumption and 28% of the national greenhouse gas emissions [1]. A significant portion of this comes from highway transportation – passenger cars to light and heavy duty trucks – which is significantly dominated by internal combustion engines operating on fossil fuels. The current research efforts to address energy security and minimize emissions for the internal combustion engines takes a two-pronged approach. On one hand, efforts are being made to improve the engine efficiency by technological improvements for the conventional engines as well as investigation of advanced low temperature combustion modes such as HCCI, SPCCI, etc. On the other hand, green fuels derived from bio based feedstock are being developed to reduce the carbon footprint. Hence, the end goal is to develop high efficiency low emissions engines that provide the flexibility of operation using renewable fuels.

Free piston engine (FPE) is an alternative architecture of IC engines that does not have a mechanical crankshaft and the combustion energy is either directly converted into compressed fluid or electricity. By eliminating the constraint imposed on the piston motion by the mechanical crankshaft and connecting rod setup, the free piston engine provides several advantages over the conventional IC engines such as variable compression ratio, reduced friction losses, modularity, multi-fuel operation, etc. and hence shows a promising potential for efficiency improvement and emissions reduction [2]–[6].

The key challenge for the FPE operation lies in the control of the piston motion in the absence of a mechanical crankshaft. Several control strategies have been used in the past to achieve sustained operation of the FPE, however, many of these strategies rely on calibration to be effective [3], [4], [7], [8]. The inherent complexity of the combustion and the gas exchange processes in

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the engine together with the dynamic coupling between the gas dynamics and the piston trajectory makes such calibration a tedious task. An active motion controller has been previously developed in our group for the robust operation of the FPE [9]. By precisely tracking any desired piston trajectory and rejecting external disturbances, the motion controller, called the virtual crankshaft mechanism, has been shown to provide reliable starting and stable operation of the FPE as well as the ability to recover from misfires or variability in combustion.

Hence, the FPE when operated using a virtual crankshaft mechanism allows the piston trajectory to be used as an extra control means to control the combustion process inside the combustion chamber. The piston motion is used to modulate the pressure and temperature history of the air-fuel mixture inside the combustion chamber and thereby modulate the chemical kinetics. This is the core idea of the trajectory based combustion control for the FPE. Previously, the authors have investigated the effects of various trajectories on the engine performance and shown that the trajectory-based combustion control enabled by FPE is able to adjust ignition timing, reduce the heat loss and therefore increase the indicated thermal efficiency. Extensive simulations have shown the efficiency benefits and emissions reduction achieved using this control scheme for the FPE by implementing optimal piston trajectories for a wide variety of fuels derived from conventional as well as renewable sources [10], [11].

This work is a natural extension of the previous work and focuses on the experimental investigation of the feasibility and real-time implementation of trajectory based combustion control. Since combustion dynamics is a highly nonlinear system with a complex interplay between the thermodynamic states, piston trajectory and the chemical kinetics, a well-controlled experimental apparatus is required for such an investigation to minimize the uncertainty in the analysis. For this reason, we use a controlled trajectory rapid compression and expansion machine (CT-RCEM) recently developed at the University of Minnesota – Twin Cities. It essentially consists of an electrohydraulic actuator driving the piston inside a combustion chamber, for the detailed study of a single combustion event. The high fidelity control algorithm ensures precise tracking of the reference trajectory to create the desired piston motion inside the combustion chamber. With the ability to track any desired reference trajectory in the combustion chamber, the CT-RCEM provides the ability for an in-depth analysis of a single combustion cycle of an IC engine.

The unique advantage of using the CT-RCEM for such an analysis comes from the fact that with only a single cycle being investigated, it is possible to precisely regulate the initial and boundary conditions, such as the initial wall temperature, initial charge temperature, the boost pressure, charge composition, etc. with accuracies that are difficult to achieve on a running engine. Moreover, several operating conditions in terms of compression ratio, operating speed and even shapes of piston trajectories can be investigated with a small turnaround time since any changes to the piston trajectory are made electronically. Additionally, the in-situ and ex-situ capabilities for chemical species diagnostics coupled with the quenching capability provide a unique opportunity for a deeper investigation of the chemical kinetics and especially the emissions formation such as soot, NOx, etc. [12].

Here, using the CT-RCEM we experimentally demonstrate that the shape of the piston trajectory can be used as a control input to control the combustion phasing, characterized by the location of peak pressure (LPP). An iterative learning control (ILC) has been implemented to obtain the desired combustion phasing. We also validate a trend consistently observed in our previous simulation studies that piston trajectories with a sharper motion profile around the TDC (less time near TDC) lead to a higher indicated thermal efficiency [11], [13], [14].

The rest of the paper is organized as follows. First, a brief description of the trajectory based combustion control framework is provided followed by a description of the hardware setup. Next, the implementation of the combustion phasing control is described, followed by experimental results demonstrating successful control of the combustion phasing using piston trajectory shape. Finally, a heat release analysis is presented to show that a higher indicated thermal efficiency is achieved by attaining desired combustion phasing through the implementation of trajectory based combustion control, followed by conclusions.

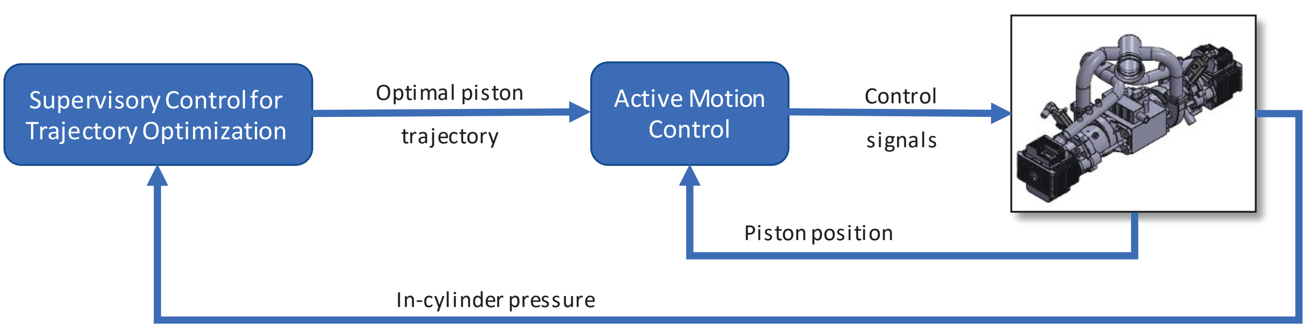

Figure 1: Trajectory based combustion control framework for free piston engine operation

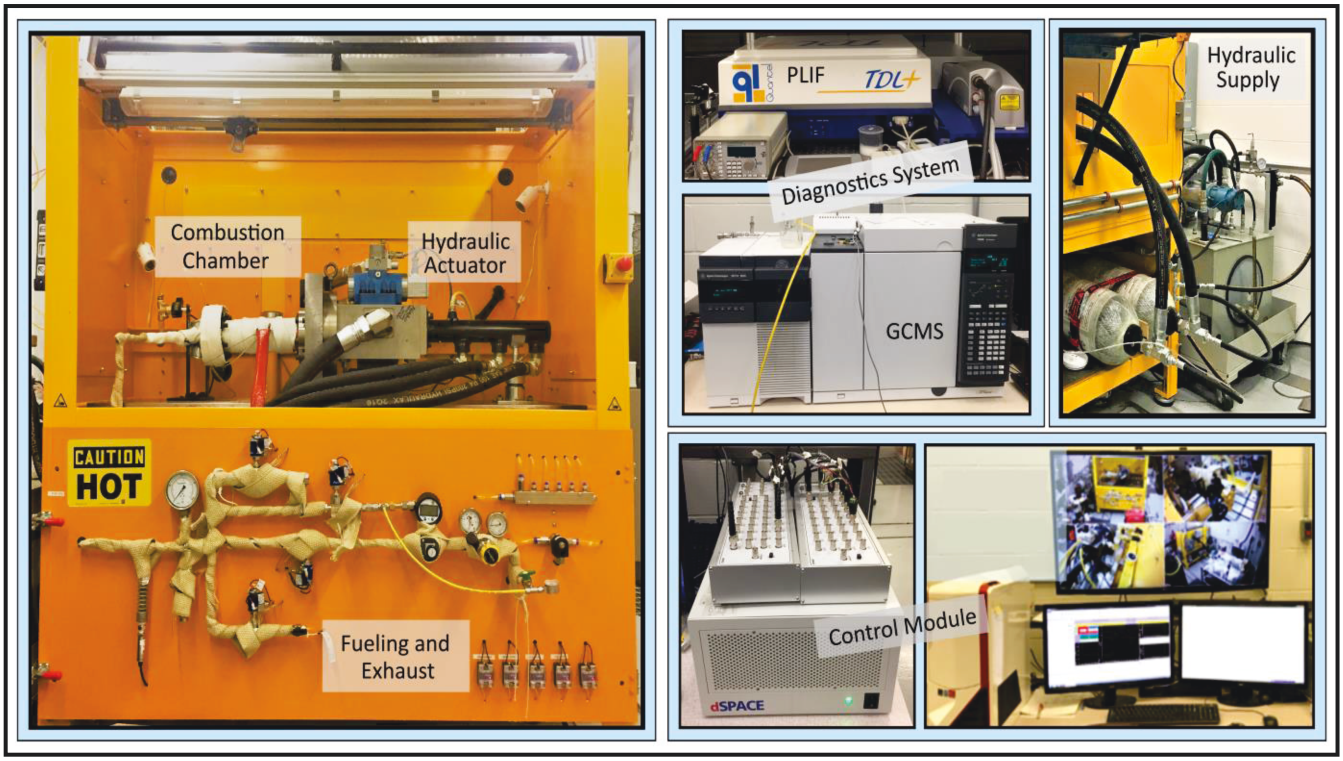

Figure 2: Controlled Trajectory Rapid Compression and Expansion Machine (CT-RCEM) University of Minnesota -Twin Cities

TRAJECTORY BASED COMBUSTION CONTROL FRAMEWORK

The central idea of the trajectory based combustion control is to use the controllable piston trajectory in the FPE as an extra control input to actively regulate the combustion chamber volume in real-time. This, in turn, provides the capability to tailor the thermodynamic path of the fuel air mixture inside the combustion chamber (essentially the pressure-temperature history) and hence modulate the chemical kinetics prior, during, and after the combustion event.

The true potential of the trajectory based combustion control strategy, however, is realized when the FPE is operated in advanced combustion modes such as homogeneous charge compression ignition (HCCI) combustion mode. HCCI is primarily governed by the chemical kinetics of the fuel-air mixture, mainly the autoignition dynamics. Hence with the ability to shape the instantaneous volume profile inside the combustion chamber in real time, the trajectory-based combustion control enables the optimization of its chemical reactivity and heat transfer processes by removing the traditional constraints on the thermodynamic path of the air-fuel mixture imposed by the mechanical crankshaft.

Figure 1 shows the implementation scheme of the trajectory based combustion control for the FPE. The outer control loop acts as the supervisory control which calculates the optimal piston trajectory for a given operating condition based on the in-cylinder gas pressure measurement. The inner loop acts as the regulatory control, in other words the virtual crankshaft mechanism, which ensures that the piston precisely tracks the optimal piston trajectory determined by the supervisory control. However, the implementation of the outer loop supervisory control is non-trivial due to the extreme computational complexity. The dynamic models to be used for trajectory optimization tend to be highly non-linear with a large number of system states due to heat and mass transfer effects as well as the chemical kinetics. While a detailed chemical mechanism can easily have hundreds of chemical species, even simplified mechanisms can have about 50 chemical species for reasonable prediction accuracy. While authors have previously proposed trajectory optimization scheme based on control oriented model of combustion dynamics to reduce the computational burden [15], [16], modeling uncertainty still remains an issue wherein the optimal combustion pressure profile may not be attained when the calculated optimal piston trajectory is implemented on the FPE.

This work is the first stage for experimental evaluation with real-time implementation of the trajectory based combustion control framework, demonstrating for the first time successful combustion phasing control using piston trajectory as control input. For a given operating point, the optimal combustion phasing and the corresponding piston trajectory is determined from a map, created offline, through the control oriented model based optimization [15], [16]. The task of the supervisory control implemented in real-time control is thus simplified to providing robustness against modeling uncertainties and fine-tune the piston trajectory so as to achieve the desired combustion phasing by using a cycle-to-cycle iterative learning scheme.

HARDWARE SETUP: CT-RCEM

The hardware, design and performance details of the CT-RCEM have been described in our previous works and hence only a brief overview is presented here [12], [17]–[19].

Figure 2 shows the hardware setup of the CT-RCEM at the University of Minnesota – Twin Cities. The architecture of the CT-RCEM consists of five major functional sub-units – hydraulic actuator unit, control module, combustion chamber unit, fueling and exhaust purging system, and diagnostics system. The actuation unit uses a high bandwidth servovalve to drive a hydraulic actuator which, in turn, drives the piston in the combustion chamber. The piston position signal is measured using a Linear Variable Differential Transformer (LVDT) and is used for the feedback control of the servovalve.

The control module consists of centralized data logging and motion control unit. A proportional controller is used to stabilize the hydraulic actuator and a frequency domain iterative learning control is implemented over the stabilized actuator for precise trajectory tracking. With this control scheme, the tracking error has been reduced to about 0.6 mm tested for various strokes up to 140 mm and compression time of 20 ms. The accuracy of the position sensor and the pressure sensor is within ±0.1 mm and ±0.2 bar, respectively. Separate PID temperature control modules are used to regulate the fueling system and combustion chamber temperature using heating elements to within $\pm 1°C$. High accuracy pressure regulators are used to control the initial pressure during the fueling process to within $\pm$ 5 millibar. This enables the user to set the initial and boundary conditions for a given combustion cycle with higher accuracy compared to a running IC engine [12], [17].

The combustion chamber unit consists of the combustion cylinder with the piston driven inside it. The piston features an interchangeable crown design to facilitate the use of different types of piston crowns – such as creviced, flat or bowl shaped – depending on the requirement. The fueling and exhaust purging system is used for fueling, extracting exhaust gases, and purging the combustion chamber by operating different sets of check valves. The diagnostic unit consists of a gas chromatography - mass spectroscopy (GCMS) system and a planar laser induced fluorescence (PLIF) system.

SUPERVISORY CONTROL IMPLEMENTATION

In this work, the objective of the supervisory control is to achieve the desired combustion phasing, which is determined by the location of peak pressure (LPP) in terms of equivalent crank angle degrees (Λ), and the control signal generated is the shape of the piston trajectory. The desired combustion phasing and the initial piston trajectory for a given operating point are determined through offline optimization. The control objective is to minimize the combustion phasing error which is defined for an iteration i as the difference between the desired LPP (Λ_{des}) and the measured LPP (Λ_i)

$$e_i = \Lambda_{des} - \Lambda_i \tag{1}$$

To reduce the computational burden of the supervisory control, the piston trajectory is parameterized in terms of a shape parameter Ω, such that the output of the supervisory control becomes a single variable as opposed to an entire trajectory.

For this reason, we parameterize the piston trajectories as the x-axis displacement of a point moving on an ellipse in the cartesian coordinate system, given as

$$X = \frac{A\Omega\cos(\omega t)}{\sqrt{(\Omega^2\cos^2(\omega t) + \sin^2(\omega t))}} + B \tag{2}$$

$$A = X_{bdc} - B; B = \frac{X_{bdc} + X_{tdc}}{2}; \omega = \frac{2\pi}{T}$$

where A and B are major and minor axis of the ellipse respectively, X_{tdc} and X_{bdc} are the position of the TDC and BDC respectively with $X_{bdc} > X_{tdc}$, T is the total time of piston travel for the compression and expansion stroke, Ω is the eccentricity of the ellipse (ratio of major and minor axis), and ω is the angular velocity of the point moving on the ellipse with respect to the center of the ellipse [14]. The position of the TDC is fixed at $X_{tdc} = 0$. X_{bdc} is determined by the stroke, which, in turn, is

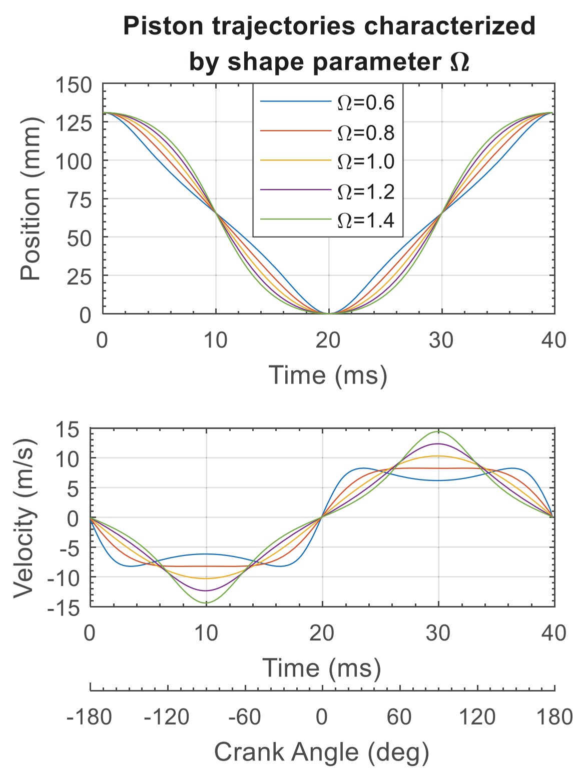

Figure 3: Piston position and velocity profiles for different trajectories characterized by shape parameter Ω

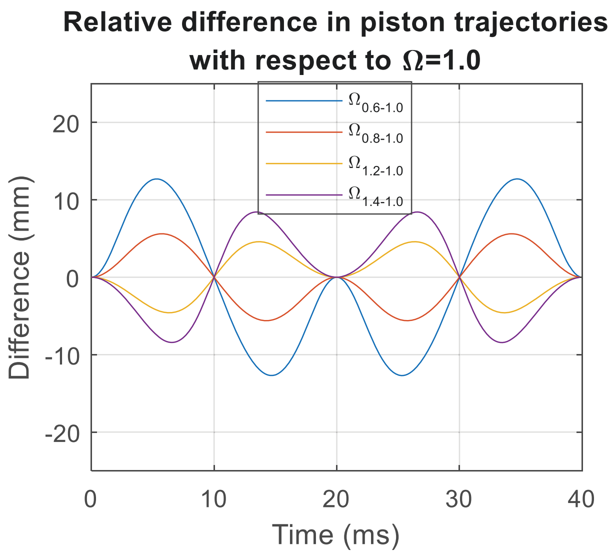

Figure 4: Relative difference of piston trajectories with $\Omega = 0.8$ and $\Omega = 1.2$, with respect to $\Omega = 1.0$

determined by the compression ratio, and the operating speed determines T. Hence, once the operating point is defined by the CR and speed, the piston trajectory can be completely characterized by the eccentricity Ω. For the special case of $\Omega = 1.0$, the ellipse turns into a circle and the piston trajectory is purely sinusoidal.

Figure 3 shows the piston position and the corresponding piston velocity for five different piston trajectories characterized by the shape parameter Ω. It is seen that the piston motion is the fastest around TDC for $\Omega = 0.6$ and consequently the fuel-air mixture spends least amount of time at the high temperature and pressure condition near the TDC for this trajectory. Also, despite the absence of a crankshaft, the piston motion can still be presented as the conventional crank-angle resolved measurement. Figure 4 shows the relative difference in the piston position for trajectories with $\Omega = \{0.6, 0.8, 1.2, 1.4\}$ with respect to the trajectory with $\Omega = 1.0$, i.e. pure sinusoidal trajectory. Based on this, and accounting for the fact that the inner loop motion control can ensure trajectory tracking up to 0.6 mm accuracy, the resolution of the change in the shape parameter is set at $\delta\Omega = \pm 0.2$ to clearly differentiate between the trajectories, especially near the TDC.

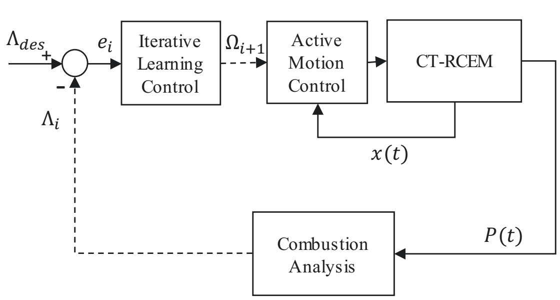

Figure 5: Iterative learning control implementation for obtaining desired LPP

The supervisory control consists of a P-type iterative learning scheme [20], [21]. The control implementation schematic is shown in Figure 5 with the following control law

$$\Omega_{i+1} = \Omega_i + \boldsymbol{K}_{ILC} e_i; \qquad (3)$$

where i is the current iteration, Ω_i and Ω_{i+1} are the trajectory shape parameters sent to the active motion controller for the current and next iteration, respectively; $\boldsymbol{K}_{ILC}$ is the control gain adjusted through trial and error about an initial estimate from the simulation, e_i is the error in the LPP for the current iteration as defined in (1). The stopping criterion for the ILC action is that the change in the control output Ω is less than the allowable resolution for $\delta\Omega$, i.e.

$$\Omega_{i+1} - \Omega_i < 0.1 \qquad (4)$$

It may be noted that for a well calibrated offline optimization model, the expected change in Ω attained by the supervisory control is not very large.

EXPERIMENTAL RESULTS AND ANALYSIS

For this study, we consider the lean HCCI combustion of dimethyl-ether (DME) as a fuel with the fuel-air equivalence ratio (ϕ) of 0.6. The compression ratio is 12.1 and equivalent operating speed is 1500 rpm. The desired set point for the LPP is $\Lambda_{des} = 5°$, after TDC, which was found to provide the best thermal efficiency in simulation studies. For all the iterations, the initial fuel-air mixture temperature is set to $45°C$. Table 1 summarizes the change in the piston trajectory shape parameter Ω and the corresponding effect on the LPP.

Figure 6 shows the change in the combustion chamber pressure profile corresponding to the change in the piston trajectories for the successive iterations of the controller. The shape parameter Ω for the starting run is set at $\Omega_1 = 1.2$, as predicted by simulation studies, and for this run the measured LPP was $\Lambda_1 = 0.4°$ CA, i.e. almost at TDC. Based on the ILC law, the required change in the shape parameter is $\delta\Omega_1 = -0.27$. However, since the resolution of change in Ω is ± 0.2, the shape parameter implemented for the second iteration is $\Omega_2 = 1.0$ which shifts the LPP to $\Lambda_2 = 3.1°$ after TDC. Again, based on the ILC control law and accounting for the resolution in $\delta\Omega$, the shape parameter for the third iteration is $\Omega_3 = 0.8$ which shifts the LPP almost to the Λ_{des} at $\Lambda_3 = 5.4°$, at which point, further iterations are no longer required, since the stopping criterion for the ILC specified in (4) has been met.

Table 1: Shape parameter and corresponding location of peak pressure over successive iterations

Iteration	Ω_i	Λ_i	Req $\delta\Omega_i$	Actual Ω_{i+1}
$i = 1$	1.2	0.4°	-0.27	1.0
$i = 2$	1.0	3.1°	-0.12	0.8
$i = 3$	0.8	5.4°	-	-

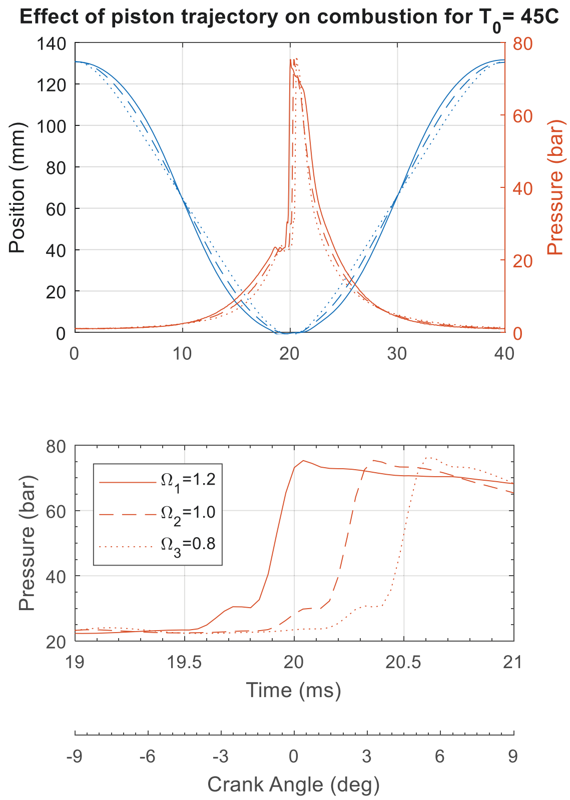

Figure 6: Change in piston trajectory and corresponding combustion phasing over three successive iterations. Top plot shows the overall piston trajectory and gas pressure profile while bottom plot shows zoomed-in view.

The tracking performance provided by the active motion control of the CT-RCEM is shown in Figure 7. Based on Figure 4 and Figure 7, we see that such a tracking performance justifies the use of $\delta\Omega = \pm 0.2$ as the resolution for the change in the shape of the piston trajectory imposed on the ILC law to reasonably differentiate between various trajectories near the TDC.

Next, to evaluate the impact of the piston trajectory on the combustion performance, a heat release analysis has been performed using the pressure data where the apparent rate of heat release (dQ_{app}) is defined as

$$dQ_{app} = dQ_{comb} - dQ_{loss} = dU + PdV \qquad (5)$$

where dQ_{comb} is the actual heat released during combustion, dQ_{loss} is the heat loss, dU is the change in internal energy, P is the gas pressure inside the combustion chamber and V is the chamber volume. Using ideal gas law and the relationship between the specific heats (C_p and C_v), ratio of specific heats (γ) and the universal gas constant (R), neglecting blowby and crevice enthalpy transfer [22], (5) can be rewritten as

$$dQ_{app} = \frac{\gamma}{\gamma - 1} PdV + \frac{1}{\gamma - 1} VdP - \frac{PV}{(\gamma - 1)^2} d\gamma \qquad (6)$$

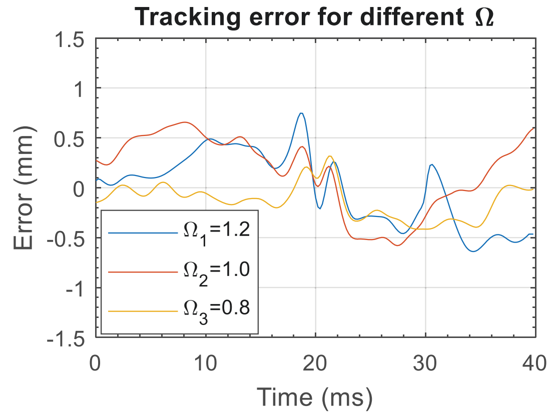

Figure 7: Tracking performance for different iterations

The start of combustion (SOC) is defined as the time when the rate of apparent heat release becomes positive for the first time and the end of combustion (EOC) is defined as the time when the heat release rate finally drops below zero after major heat release [23], [24]. Hence, the apparent heat release is defined as

$$Q_{app} = \int_{t_{soc}}^{t_{eoc}} dQ_{app} \qquad (7)$$

The chemical energy of the fuel in a combustion cycle is given as

$$Q_f = m_f LHV_f \qquad (8)$$

where LHV_f is the lower heating value of the fuel and m_f is the mass of the fuel.

The indicated thermal efficiency is defined as the ratio of net-work ($\int PdV$) obtained during the cycle and the chemical energy of the fuel (Q_f). It may be noted that this efficiency calculation does not include pumping losses since the CT-RCEM only includes compression and expansion stroke.

An interesting trend in terms of the effect of the piston trajectory on the indicated thermal efficiency can be observed from Table 2. A trajectory with a lower Ω is more efficient due to less time

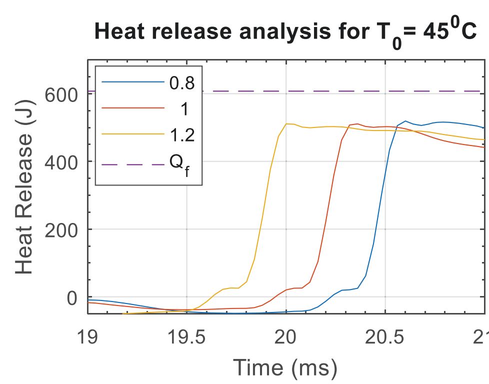

Figure 8: Cumulative heat release profile for the trajectories

Table 2: Effect of piston trajectory shape on combustion characteristics at two different initial temperatures

Initial temperature	**$45°C$**		
Omega	**1.2**	**1.0**	**0.8**
LPP ($°CA$)	0.37°	3.24°	5.41°
Comb. efficiency (Q_{app}/Q_f)	90.9%	89.4%	90.6%
Indicated thermal efficiency	19.5%	21.4%	23.6%

spent at the TDC leading to lower heat loss. This is essentially a validation of the simulation predictions previously reported in [11], [16], [25]. However, the results also indicate that the combustion efficiency first decreases with the Ω and then increases again. The reason for this trend is that the apparent heat release (Q_{app}), defined in (7), depends mainly on two competing effects – (i) the degree of completion of the combustion reactions, and (ii) heat loss during the combustion heat release (between t_{soc} and t_{eoc}). While for the higher Ω the piston spends more time near the TDC, i.e. the fuel-air mixture is at higher pressure and temperature conditions for longer, leading to a more complete combustion, at the same time, the heat loss is also higher. The combined effect of these two factors manifests as the cumulative heat release profile as shown in Figure 8.

While the bulk heat loss, at a given operating point is mainly determined by the piston trajectory, several factors such as the effect of fluid dynamics inside the combustion chamber, boundary quenching due to wall heat transfer, crevice effects, etc., affect both – the heat loss and the chemical kinetics - contributing to the combustion efficiency trend seen above, which are difficult to account in a zero-D combustion model. Future work will involve further experimentation over a larger operating region to better understand such non-linear trends, testing the effectiveness and robustness of the supervisory control, and, further investigation of such nonlinearities.

CONCLUSION

This work lays the foundation of the experimental investigation of the trajectory based combustion control strategy for free piston engine operation in the advanced, kinetically modulated, combustion modes. While extensive simulation work had been shown the potential of this control strategy for efficiency benefit and emissions reduction, experimental validation is essential to establish the feasibility and implementation details. A newly developed combustion research instrument called the CT-RCEM has been used for the detailed analysis of the combustion performance. The CT-RCEM allows for the mimicking a single combustion cycle of the free-piston engine under well controlled environment with advanced diagnostic capabilities. This work presents the first experimental validation of the concept of trajectory based combustion control. It has been shown that the piston trajectory shape can be used as an effective control input to achieve combustion phasing control in a real-time implementation scheme. Moreover, the effect of the piston trajectory on combustion phasing and efficiency have been demonstrated to follow similar trends as predicted in the previous simulation works.

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