Resource Allocation for Wireless Communications with Distributed Reconfigurable Intelligent Surfaces

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***Abstract*—This paper investigates the problem of resource allocation for a wireless communication network with distributed reconfigurable intelligent surfaces (RISs). In this network, multiple RISs are spatially distributed to serve wireless users and the energy efficiency of the network is maximized by dynamically controlling the on-off status of each RIS as well as optimizing the reflection coefficient matrix of the RISs. This problem is posed as a joint optimization problem of transmit power and RIS control, whose goal is to maximize the energy efficiency under minimum rate constraints of the users. To solve this problem, an alternating algorithm is proposed by solving two sub-problems iteratively. The phase optimization sub-problem is solved by using a successive convex approximation method, which admits a closed-form solution at each step. Moreover, the RIS on-off optimization sub-problem is solved by using the dual method. Simulation results show that the proposed scheme achieves up to 27% and 68% gains in terms of the energy efficiency compared to the conventional RIS scheme and amplify-and-forward relay scheme, respectively.**

I. Introduction

Driven by the rapid development of advanced multimedia applications, next-generation wireless networks must support high spectral efficiency and massive connectivity [1]–[3]. Due to high data rate demand and massive numbers of users, energy consumption has become a challenging problem in the design of future wireless networks. In consequence, energy efficiency, defined as the ratio of spectral efficiency over power consumption, has emerged as an important performance index for deploying green and sustainable wireless networks.

Recently, reconfigurable intelligent surface (RIS)-assisted wireless communication has been proposed as a potential solution for enhancing the energy efficiency of wireless networks [4]–[7]. In RIS-assisted wireless communication networks, a base station (BS) sends control signals to an RIS controller so as to optimize the properties of incident waves and improve the communication quality of users. A number of existing works such as in [8]–[13] has studied the deployment of RISs in wireless networks. In [8], the downlink sum-rate of an RIS assisted wireless communication system was characterized. An asymptotic analysis of the uplink transmission rate in an RIS-based large antenna-array system was presented in [9]. Then, in [10], the authors investigated the asymptotic optimality of the achievable rate in a downlink RIS system. Considering energy harvesting, an RIS was invoked for enhancing the sum-rate performance of a simultaneous wireless information and power transfer aided system [11]. Taking the secrecy into consideration, the work in [12] investigated the problem of secrecy rate maximization of an RIS assisted multi-antenna system. Further by considering imperfect CSI, the RIS was considered to enhance the physical layer security of a wireless channel in [13]. In [14], the authors proposed a new approach to maximize the energy efficiency of a multi-user multiple-input single-output (MISO) system by jointly controlling the transmit power of the BS and the phase shifts of the RIS. However, only a single RIS was considered for simplicity in [14]. Deploying a number of low-cost power-efficient RISs in future networks can cooperatively enhance the coverage of the networks. In particular, deploying multiple RISs in wireless networks has several advantages. First, distributed RISs can provide robust data-transmission since different RISs can be deployed geometrically apart from each other. Meanwhile, multiple RISs can provide multiple paths of received signals, which increases the received signal strength. To our best knowledge, this is the first work that optimizes the energy efficiency for a wireless network with multiple RISs.

The main contribution of this paper is a novel energy efficient resource allocation scheme for wireless communication networks with distributed RISs. Our key contributions include:

- → We investigate a downlink wireless communication system with distributed RISs that can be dynamically turned on or off depending on the network requirements. To maximize the energy efficiency of the system, we jointly optimize the phase shifts of all RISs, the transmit power of the transmitter, and the RIS on-off status vector.
- → To maximize the energy efficiency, a suboptimal solution

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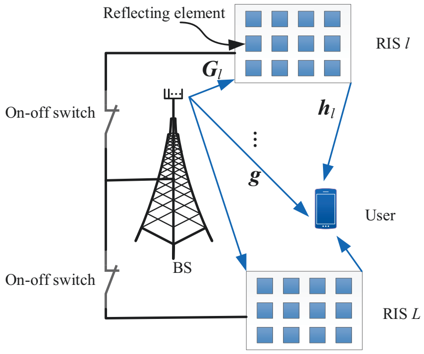

Fig. 1. A downlink MISO system with multiple RISs.

is obtained by using a low-complexity algorithm that iteratively solves two subproblems. For the joint phase and power optimization subproblem, a suboptimal phase is obtained by using the successive convex approximation (SCA) method with low complexity, and the optimal power is subsequently obtained in closed form. For the RIS on-off optimization subproblem, the dual method is used to obtain the optimal solution.

Notations: In this paper, the imaginary unit of a complex number is denoted by $j = \sqrt{-1}$. Matrices and vectors are denoted by boldface capital and lower-case letters, respectively. Matrix $\mathrm{diag}(x_1, \cdots, x_N)$ denotes a diagonal matrix whose diagonal components are $x_1, \cdots, x_N$. The real and imaginary parts of a complex number x are denoted by $\Re(x)$ and $\Im(x)$, respectively. $\boldsymbol{x}^*$, $\boldsymbol{x}^T$, and $\boldsymbol{x}^H$ respectively denote the conjugate, transpose, and conjugate transpose of vector $\boldsymbol{x}$. $[\boldsymbol{x}]_n$ and $[\boldsymbol{X}]_{kn}$ denote the n-th and (k,n)-th elements of the respective vector $\boldsymbol{x}$ and matrix $\boldsymbol{X}$. $\|\boldsymbol{x}\|$ denotes the ℓ_2-norm of vector $\boldsymbol{x}$. Gaussian variable with mean x and covariance σ is denoted by $\mathcal{CN}(x, \sigma)$.

II. System Model and Problem Formulation

A. System Model

Consider an RIS-assisted MISO downlink channel that consists of one BS, one user, and a set $\mathcal{L}$ of L RISs, as shown in Fig. 1. The number of transmit antennas at the BS is M, while the user is equipped with one antenna. Such a setting has been used in many practical scenarios such as in Internet-of-Things networks [8]–[10]. Each RIS, $l \in \mathcal{L}$, has N_l reflecting elements. The RISs are configured to assist the communication between the BS and the user. In particular, the RISs will be installed on the walls of the surrounding high-rise buildings.

The transmitted signal at the BS is $\boldsymbol{w}s$, where s is unit-power information symbol and $\boldsymbol{w} \in \mathbb{C}^M$ is the beamforming vector for the user. The power consumption of an RIS depends on both the type and the resolution of the reflecting elements that effectively perform phase shifting on the impinging signal [14]. Considering the power consumption of RISs due to controlling the phase shift values of the reflecting elements [14], it is often not energy efficient to turn on all the RISs. We now introduce a binary variable $x_l \in \{0,1\}$, where $x_l = 1$ indicates that RIS l is on. When $x_l = 1$, the phase shift matrix of RIS l can be optimized through a diagonal matrix $\boldsymbol{\Theta}_l = \mathrm{diag}(e^{j\theta_{l1}}, \cdots, e^{j\theta_{lN_l}}) \in \mathbb{C}^{N_l \times N_l}$ with $\theta_{ln} \in [0, 2\pi]$, $l \in \mathcal{L}$, and $n \in \mathcal{N}_l = \{1, \cdots, N_l\}$, where $\boldsymbol{\Theta}_l$ captures the effective phase shifts applied by all reflecting elements of RIS l. In contrast, when $x_l = 0$, RIS l is off and does not consume any power. Then, with the multiple RISs, the received signal at the user can be given by

$$y = \left(\boldsymbol{g}^H + \sum_{l=1}^{L} x_l \boldsymbol{h}_l^H \boldsymbol{\Theta}_l \boldsymbol{G}_l\right) \boldsymbol{w}s + n, \tag{1}$$

where $\boldsymbol{g} \in \mathbb{C}^M$, $\boldsymbol{G}_l \in \mathbb{C}^{N_l \times M}$, and $\boldsymbol{h}_l \in \mathbb{C}^{N_l}$, respectively, denote the channel responses from the BS to the user, from the BS to RIS l, and from RIS l to the user, and $n \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise.

Based on (1), the received signal-to-interference-plus-noise ratio (SINR) at the user is

$$\gamma = \frac{\left|\left(\boldsymbol{g}^H + \sum_{l=1}^{L} x_l \boldsymbol{h}_l^H \boldsymbol{\Theta}_l \boldsymbol{G}_l\right)\boldsymbol{w}\right|^2}{\sigma^2}. \tag{2}$$

Since there is no multi-user interference, it is well-known that beaming as the maximum ratio transmission (MRT) at the BS is optimal [15]. That is

$$\boldsymbol{w} = \sqrt{p}\frac{\boldsymbol{g} + \sum_{l=1}^{L} x_l \boldsymbol{G}_l^H \boldsymbol{\Theta}_l^H \boldsymbol{h}_l}{\left\|\boldsymbol{g} + \sum_{l=1}^{L} x_l \boldsymbol{G}_l^H \boldsymbol{\Theta}_l^H \boldsymbol{h}_l\right\|}, \tag{3}$$

where p is the transmit power at the BS.

The total power consumption of the considered RIS-assisted system includes the transmit power of the BS, the circuit power consumption of both the BS and all users, and the power consumption of all RISs. Consequently, the total power of the system will be given by

$$P_{\mathrm{t}} = \mu p + P_{\mathrm{B}} + P_{\mathrm{U}} + \sum_{l=1}^{L} x_l N_l P_{\mathrm{R}}, \tag{4}$$

where $\mu = \nu^{-1}$ with ν being the power amplifier efficiency of the BS, P_{B} is the circuit power consumption of the BS, P_{U} is the circuit power consumption of the user, and P_{R} is the power consumption of each reflecting element in the RIS. In (4), $x_l N_l P_{\mathrm{R}}$ is the power consumption of RIS l.

B. Problem Formulation

Our objective is to jointly optimize the reflection coefficient matrix, transmit power, and RIS on-off vector so as to maximize the energy efficiency under the minimum rate requirements and total power constraint. Mathematically, the problem for the distributed RISs can be given by

$$\max_{\boldsymbol{\theta}, p, \boldsymbol{x}} \quad \frac{B\log_2\left(1 + \frac{p\left\|\boldsymbol{g}^H + \sum_{l=1}^{L} x_l \boldsymbol{h}_l^H \boldsymbol{\Theta}_l \boldsymbol{G}_l\right\|^2}{\sigma^2}\right)}{\mu p + P_{\mathrm{U}} + P_{\mathrm{B}} + \sum_{l=1}^{L} x_l N_l P_{\mathrm{R}}} \tag{5}$$

$$\text{s.t.} \quad B\log_2\left(1 + \frac{p\left\|\boldsymbol{g}^H + \sum_{l=1}^{L} x_l \boldsymbol{h}_l^H \boldsymbol{\Theta}_l \boldsymbol{G}_l\right\|^2}{\sigma^2}\right) \geq R, \tag{5a}$$

$$0 \leq p \leq P_{\max}, \tag{5b}$$

$$\theta_{ln} \in [0, 2\pi], \quad \forall l \in \mathcal{L}, n \in \mathcal{N}_l, \tag{5c}$$

$$x_l \in \{0,1\}, \quad \forall l \in \mathcal{L}, \tag{5d}$$

where $\boldsymbol{\theta} = [\theta_{11}, \cdots, \theta_{1N_1}, \cdots, \theta_{LN_L}]^T$, $\boldsymbol{x} = [x_1, \cdots, x_L]^T$, B is the bandwidth of the channel, R is the minimum data rate requirement of the user, and $P_{\max}$ is the maximum transmit power of the BS. Constraint (5a) follows from (2) and (3). The minimum rate constraint for the user is given in (5a) and (5b) represents the maximum power constraint. The phase shift constraint for each reflecting element is provided in (5c), which can also be seen as the unit-modulus constraint since $|e^{j\theta_{ln}}| = 1$.

III. Energy Efficiency Optimization

Due to the involvement of integer variable $\boldsymbol{x}$, it is difficult to obtain the globally optimal solution of (5). As such, we propose an iterative algorithm to solve problem (5) sub-optimally with low complexity. The proposed iterative algorithm contains two major steps. In the first step, we jointly optimize phase and power $(\boldsymbol{\theta}, p)$ with given $\boldsymbol{x}$. Then, in the second step, we update RIS on-off vector $\boldsymbol{x}$ with the optimized $(\boldsymbol{\theta}, p)$ in the previous step.

A. Joint Phase and Power Optimization

For a fixed integer variable $\boldsymbol{x}$, problem (5) becomes

$$\max_{\boldsymbol{\theta},p} \quad \frac{B\log_2\left(1+\frac{p\left|\boldsymbol{g}^H+\sum_{l=1}^{L}x_l\boldsymbol{h}_l^H\boldsymbol{\Theta}_l\boldsymbol{G}_l\right|^2}{\sigma^2}\right)}{\mu p+P_{\mathrm{U}}+P_{\mathrm{B}}+\sum_{l=1}^{L}x_lN_lP_{\mathrm{R}}} \tag{6}$$

$$\text{s.t.} \quad \text{(5a)-(5c)}. \tag{6a}$$

From the objective function (6) and the constraint in (6a), we observe that the optimal $\boldsymbol{\theta}$ is the one that maximizes the channel gain, i.e., $\left\|\boldsymbol{g}^H+\sum_{l=1}^{L}x_l\boldsymbol{h}_l^H\boldsymbol{\Theta}_l\boldsymbol{G}_l\right\|^2$. With this in mind, the optimal solution of problem (6) can be obtained in two stages, i.e., obtain the value of $\boldsymbol{\theta}$ that maximizes the channel gain in the first stage and, then, calculate the optimal p in the second stage with the obtained $\boldsymbol{\theta}$ in the first stage.

1) First stage: We first optimize the phase shift vector $\boldsymbol{\theta}$ of problem (6). Before optimizing $\boldsymbol{\theta}$, we show that $\boldsymbol{h}_l^H\boldsymbol{\Theta}_l\boldsymbol{G}_l = \boldsymbol{\theta}_l^T\boldsymbol{U}_l$, where $\boldsymbol{U}_l = \mathrm{diag}(\boldsymbol{h}_l^H)\boldsymbol{G}_l \in \mathbb{C}^{N_l\times M}$ and $\boldsymbol{\theta}_l = [e^{j\theta_{l1}}, \cdots, e^{j\theta_{lN_l}}]^T$. According to problem (6), the optimal $\boldsymbol{\theta}$ can be calculated by solving the following problem:

$$\max_{\boldsymbol{\theta}} \quad \left\|\boldsymbol{g}^H+\sum_{l=1}^{L}x_l\boldsymbol{\theta}_l^T\boldsymbol{U}_l\right\|^2 \tag{7}$$

$$\text{s.t.} \quad \theta_{ln}\in[0,2\pi], \quad \forall l\in\mathcal{N}, n\in\mathcal{O}_l. \tag{7a}$$

Let $\boldsymbol{\theta}_l^*$ be the conjugate vector of $\boldsymbol{\theta}_l$. The total number of elements for all RISs is denoted by $Q=\sum_{l=1}^{L}N_l$. Denote $\boldsymbol{v}=[\boldsymbol{\theta}_1^*;\cdots;\boldsymbol{\theta}_L^*]\in\mathbb{C}^Q$ and $\boldsymbol{U}=[x_1\boldsymbol{U}_1;\cdots;x_L\boldsymbol{U}_L]\in\mathbb{C}^{Q\times M}$. Problem (7) can be rewritten as

$$\max_{\boldsymbol{v}} \quad \left\|\boldsymbol{g}+\boldsymbol{U}^H\boldsymbol{v}\right\|^2 \tag{8}$$

$$\text{s.t.} \quad |v_q|=1, \quad \forall q\in\mathcal{R}, \tag{8a}$$

where $\mathcal{R}=\{1,\cdots,Q\}$.

To handle the nonconvexity of objective function (8), we adopt the SCA method and, consequently, objective function (8) can be approximated by

$$2\mathrm{Re}\{((\boldsymbol{g}+\boldsymbol{U}^H\boldsymbol{v}^{(n-1)})^H\boldsymbol{U}^H\boldsymbol{v})\}-\left\|\boldsymbol{g}+\boldsymbol{U}^H\boldsymbol{v}^{(n-1)}\right\|^2, \tag{9}$$

and the superscript $(n-1)$ represents the value of the variable at the $(n-1)$-th iteration. With approximation (9), problem (8) can be approximated by

Algorithm 1 SCA Method for Phase Optimization

1: Initialize $\boldsymbol{v}^{(0)}$. Set iteration number $n=1$.
2: **repeat**
3: Set $\boldsymbol{v}^{(n)}=e^{-j\angle(\boldsymbol{U}(\boldsymbol{g}+\boldsymbol{U}^H\boldsymbol{v}^{(n-1)}))}$, and $n=n+1$.
4: **until** the objective value (8) converges.
5: Output $\boldsymbol{\theta}=(\boldsymbol{v}^{(n)})^*$.

$$\max_{\boldsymbol{v}} \; 2\mathrm{Re}\{((\boldsymbol{g}+\boldsymbol{U}^H\boldsymbol{v}^{(n-1)})^H\boldsymbol{U}^H\boldsymbol{v})\}-\left\|\boldsymbol{g}+\boldsymbol{U}^H\boldsymbol{v}^{(n-1)}\right\|^2 \tag{10}$$

$$\text{s.t.} \quad |v_q|\le 1, \quad \forall q\in\mathcal{R}, \tag{10a}$$

where constraint (8a) is temporarily relaxed as (10a). In the following lemma, we show that (10a) always holds with equality for the optimal solution of problem (10).

Lemma 1: The optimal solution of problem (10) is

$$\boldsymbol{v}=e^{-j\angle(\boldsymbol{U}(\boldsymbol{g}+\boldsymbol{U}^H\boldsymbol{v}^{(n-1)}))}, \tag{11}$$

where $\angle(\cdot)$ represents the angle vector of a vector, i.e, for $q\in\mathcal{R}$, $[\angle(\boldsymbol{y})]_q=\arctan\frac{\mathcal{R}([\boldsymbol{y}]_q)}{([\boldsymbol{y}]_q)}$.

Proof: To maximize $(\boldsymbol{g}+\boldsymbol{U}^H\boldsymbol{v}^{(n-1)})^H\boldsymbol{U}^H\boldsymbol{v}$ in (10), the optimal $\boldsymbol{v}$ should be chosen such that $[(\boldsymbol{g}+\boldsymbol{U}^H\boldsymbol{v}^{(n-1)})^H\boldsymbol{U}^H]_q[\boldsymbol{v}]_q$ is a real number and $|[\boldsymbol{v}]_q|=1$ for any q, i.e., the optimal $\boldsymbol{v}$ should be given as (11). □

From (11) and Lemma 1, we can see that the optimal phase vector $\boldsymbol{v}$ should be adjusted such that the signal that goes through all RISs is aligned to be a signal vector with equal phase at each element. The SCA algorithm for solving problem (8) is summarized in Algorithm 1.

2) Second stage: We now obtain the optimal power allocation p. With the obtained $\boldsymbol{\theta}$ in Algorithm 1 and defining $\bar{g}=\frac{\left|\boldsymbol{g}^H+\sum_{l=1}^{L}x_l\boldsymbol{h}_l^H\boldsymbol{\Theta}_l\boldsymbol{G}_l\right|^2}{\sigma^2}$, problem (6) reduces to

$$\max_{p} \quad \frac{B\log_2(1+\bar{g}p)}{\mu p+P_0} \tag{12}$$

$$\text{s.t.} \quad P_{\min}\le p\le P_{\max}, \tag{12a}$$

where $P_0=P_{\mathrm{U}}+P_{\mathrm{B}}+\sum_{l=1}^{L}x_lN_lP_{\mathrm{R}}$, and $P_{\min}=(2^{\frac{R}{B}}-1)/\bar{g}$. In (12a), $P_{\min}$ is used to guarantee the minimum rate requirement for the user.

For problem (12), the Dinkelbach method from [16] can be used. The Dinkelbach method involves solving a series of convex subproblems, which increases the computational complexity. However, the optimal solution of (12) can be derived in closed form using the following theorem.

Theorem 1: The optimal transmit power of the energy efficiency maximization problem in (12) is

$$p=\left[\frac{\bar{g}P_0-\mu}{\mu\bar{g}W\left(\frac{\bar{g}P_0-\mu}{\mu e}\right)}-\frac{1}{\bar{g}}\right]_{P_{\min}}^{P_{\max}}, \tag{13}$$

where $W(\cdot)$ is the Lambert-W function and $[a]_b^c=\min\{\max\{a,b\},c\}$.

Proof: The first-order derivative of the objective function (12) with respect to power p is

$$\frac{\partial\frac{B\log_2(1+\bar{g}p)}{\mu p+P_0}}{\partial p}=\frac{B(\bar{g}(\mu p+P_0)-\mu(1+\bar{g}p)\ln(1+\bar{g}p))}{(1+\bar{g}p)(\mu p+P_0)^2\ln 2}. \tag{14}$$

To show the increasing trend of the objective function (12), we further denote

$$f(p) = \bar{g}(\mu p + P_0) - \mu(1+\bar{g}p)\ln(1+\bar{g}p), \quad \forall p > 0. \tag{15}$$

The first-order derivative of function $f(p)$ is:

$$f'(p) = -\mu\bar{g}\ln(1+\bar{g}p) < 0, \tag{16}$$

which indicates that $f(p)$ is a monotonically decreasing function. Since $f(0) = \bar{g}P_0 > 0$ and $\lim_{p\to\infty} f(p) < 0$, there must exist a unique $\bar{p}$ such that $f(\bar{p}) = 0$, where

$$\bar{p} = \frac{\bar{g}P_0 - \mu}{\mu\bar{g}W\left(\frac{(\bar{g}P_0-\mu)}{\mu e}\right)} - \frac{1}{\bar{g}}. \tag{17}$$

Hence, the objective function (12) first increases in interval $(0, \bar{p}]$ and then decreases in interval $(\bar{p}, \infty)$, which indicates that the optimal solution can be presented as in (13). □

From Theorem 1, the optimal power control of problem (12) is obtained in closed-form, as shown in (13). According to (13), it is shown that the optimal power control lies in one of three values, i.e., the minimum transmit power, the power with zero first-order derivative, and the maximum transmit power.

B. RIS On-Off Optimization

We introduce an auxiliary variable y and problem (5) with fixed phase and power variables $(\boldsymbol{\theta}, p)$ is equivalent to

$$\max_{\boldsymbol{x},y} \quad \frac{B\log_2\left(1+\frac{py}{\sigma^2}\right)}{\mu p + P_{\rm U} + P_{\rm B} + \sum_{l=1}^{L} x_l N_l P_{\rm R}} \tag{18}$$

$$\text{s.t.} \quad y \leq \left|\boldsymbol{g}^H + \sum_{l=1}^{L} x_l \boldsymbol{h}_l^H \boldsymbol{\Theta}_l \boldsymbol{G}_l\right|^2, \tag{18a}$$

$$y \geq \left(2^{\frac{R}{B}} - 1\right)\frac{\sigma^2}{p}, \tag{18b}$$

$$x_l \in \{0,1\}, \quad \forall l \in \mathcal{N}, \tag{18c}$$

where constraint (18b) is used to ensure the minimum rate demand. For the optimal solution of problem (18), constraint (18a) will always hold with equality since the objective function monotonically increases with y. There are two difficulties in solving problem (18). The first difficulty is that objective function (18) has a fractional form, which is difficult to solve. The second difficulty is that constraint (18a) is nonconvex.

To handle the first difficulty, we use the parametric approach in [16] and consider the following problem:

$$H(\lambda) = \max_{(\boldsymbol{x},y)\in\mathcal{I}} B\log_2\left(1+\frac{py}{\sigma^2}\right) - \lambda\left(P_{\rm C} + \sum_{l=1}^{L} x_l N_l P_{\rm R}\right), \tag{19}$$

where $\mathcal{I}$ is the feasible set of $(\boldsymbol{x}, y)$ satisfying constraints (18a)-(18c) and $P_{\rm C} = \mu p + P_{\rm U} + P_{\rm B}$. It was proved in [16] that solving (18) is equivalent to finding the root of the nonlinear function $H(\lambda)$, which can be obtained by using the Dinkelbach method. By introducing parameter λ, the objective function of problem (18) can be simplified, as shown in (19).

To handle the second difficulty, due to the fact that $x_l \in \{0,1\}$, we can rewrite the right hand side of (18a) as

$$\left|\boldsymbol{g}^H + \sum_{l=1}^{L} x_l \boldsymbol{h}_l^H \boldsymbol{\Theta}_l \boldsymbol{G}_l\right|^2 = D_0 + \sum_{l=1}^{L} D_l x_l + \sum_{l=2}^{L}\sum_{m=1}^{l-1} D_{lm} x_l x_m, \tag{20}$$

where we set $D_0 = \boldsymbol{g}^H\boldsymbol{g}$, $D_l = \boldsymbol{h}_l^H\boldsymbol{\Theta}_l\boldsymbol{G}_l\boldsymbol{G}_l^H\boldsymbol{\Theta}_l^H\boldsymbol{h}_l + \boldsymbol{g}^H\boldsymbol{G}_l^H\boldsymbol{\Theta}_l^H\boldsymbol{h}_l + \boldsymbol{h}_l^H\boldsymbol{\Theta}_l\boldsymbol{G}_l\boldsymbol{g}$, and $D_{lm} = \boldsymbol{h}_l^H\boldsymbol{\Theta}_l\boldsymbol{G}_l\boldsymbol{G}_m^H\boldsymbol{\Theta}_m^H\boldsymbol{h}_m + \boldsymbol{h}_m^H\boldsymbol{\Theta}_m\boldsymbol{G}_m\boldsymbol{G}_l^H\boldsymbol{\Theta}_l^H\boldsymbol{h}_l$. To solve problem (18), we introduce new variable $z_{lm} = x_l x_m$. Since $x_l \in \{0,1\}$, constraint $z_{lm} = x_l x_m$ is equivalent to

$$z_{lm} \geq x_l + x_m - 1, \quad 0 \leq z_{lm} \leq 1, \tag{21}$$

$$z_{lm} \leq x_l, \quad z_{lm} \leq x_m, \tag{22}$$

for all $l = 2, \cdots, L$, $m = 1, \cdots, l-1$. According to (19)-(22), problem (18) can be reformulated as

$$\max_{\boldsymbol{x},y,\boldsymbol{z}} \quad B\log_2\left(1+\frac{py}{\sigma^2}\right) - \lambda\left(P_{\rm C} + \sum_{l=1}^{L} x_l N_l P_{\rm R}\right) \tag{23}$$

$$\text{s.t.} \quad y \leq D_0 + \sum_{l=1}^{L} D_l x_l + \sum_{l=2}^{L}\sum_{m=1}^{l-1} D_{lm} z_{lm}, \tag{23a}$$

$$(18b), (18c), (21), (22), \tag{23b}$$

where $\boldsymbol{z} = [z_{21}, z_{31}, z_{32}, \cdots, z_{L(L-1)}]^T$.

Due to constraints (23e), it is difficult to handle problem (23). By relaxing the integer constraints (23b) with $x_l \in [0,1]$, problem (23) becomes a convex problem. For problem (23) with relaxed constraints, the optimal solution can be obtained through the dual method. We show that the dual method obtains the integer solution, which guarantees both optimality and feasibility of the original problem. To obtain the optimal solution of problem (23), we have the following theorem.

Theorem 2: For problem (23), the optimal RIS on-off vector $\boldsymbol{x}$ and auxiliary variables $(y, \boldsymbol{z})$ can be respectively expressed as

$$x_l = \begin{cases} 1, & \text{if } C_l > 0, \\ 0, & \text{otherwise}, \end{cases} \tag{24}$$

$$y = \left[\frac{B}{(\ln 2)\alpha} - \frac{\sigma^2}{p}\right]^{+}_{\left(2^{\frac{R}{B}}-1\right)\frac{\sigma^2}{p}}, \tag{25}$$

and

$$z_{lm} = \begin{cases} 1, & \text{if } \alpha D_{lm} + \kappa_{1lm} - \kappa_{2lm} - \kappa_{3lm} > 0, \\ 0, & \text{otherwise}, \end{cases} \tag{26}$$

where

$$C_l = \begin{cases} -\lambda N_1 P_{\rm R} + \alpha D_1 + \sum_{m=2}^{L}(\kappa_{3ml} - \kappa_{1ml}), & \text{if } l = 1, \\ -\lambda N_l P_{\rm R} + \alpha D_l + \sum_{m=1}^{l-1}(\kappa_{2lm} - \kappa_{1lm}) \\ \quad + \sum_{m=l+1}^{L}(\kappa_{3ml} - \kappa_{1ml}), & \text{if } 2 \leq l \leq L-1, \\ -\lambda N_L P_{\rm R} + \alpha D_L + \sum_{m=1}^{L-1}(\kappa_{2lm} - \kappa_{1lm}), & \text{if } l = L, \end{cases} \tag{27}$$

$\{\alpha, \kappa_{1lm}, \kappa_{2lm}, \kappa_{3lm}\}$ are the Lagrange multipliers associated with corresponding constraints of (23), and $a_b = \max\{a, b\}$.

Proof: The dual problem of (23) with relaxed constraints is

$$\min_{\alpha,\boldsymbol{\kappa}} \quad \mathcal{F}(\alpha, \boldsymbol{\kappa}), \tag{28}$$

where

$$\mathcal{F}(\alpha,\boldsymbol{\kappa}) = \begin{cases} \max\limits_{\boldsymbol{x},y,\boldsymbol{z}} & \mathcal{L}(\boldsymbol{x}, y, \boldsymbol{z}, \alpha, \boldsymbol{\kappa}) \\ \text{s.t.} & y \geq \left(2^{\frac{R}{B}}-1\right)\frac{\sigma^2}{p}, \\ & 0 \leq z_{lm} \leq 1, \quad \forall 2 \leq l \leq L, 1 \leq m \leq l-1, \\ & 0 \leq x_l \leq 1, \quad \forall l \in \mathcal{N}, \end{cases} \tag{29}$$

Algorithm 2 Dual Method for Problem (18)

Initialize parameter λ and set the accuracy ϵ.
repeat
  Initialize dual variables $(\alpha, \boldsymbol{\kappa})$.
  repeat
    Update the RIS on-off vector $\boldsymbol{x}$ and the auxiliary variables $(y, \boldsymbol{z})$ according to (24)-(26).
    Update dual variables $(\alpha, \boldsymbol{\kappa})$ based on the sub-gradient method.
  until the objective value (23) converges
  Denote the objective value (23) by $H(\lambda)$.
  Update $\lambda = \frac{B\log_2\left(1+\frac{py}{\sigma^2}\right)}{\mu p + P_{\mathrm{U}} + P_{\mathrm{B}} + \sum_{l=1}^{L} x_l N_l P_{\mathrm{R}}}$.
until $H(\lambda) < \epsilon$.

$$\mathcal{N}(\boldsymbol{x}, y, \boldsymbol{z}, \alpha, \boldsymbol{\kappa}) = B\log_2\left(1+\frac{py}{\sigma^2}\right) - \lambda\left(P_{\mathrm{C}} + \sum_{l=1}^{L} x_l N_l P_{\mathrm{R}}\right) + \alpha\left(D_0 + \sum_{l=1}^{L} D_l x_l + \sum_{l=2}^{L}\sum_{m=1}^{l-1} D_{lm} z_{lm} - y\right) + \sum_{l=2}^{L}\sum_{m=1}^{l-1}\left[\kappa_{1lm}(z_{lm} - x_l - x_m + 1) + \kappa_{2lm}(x_l - z_{lm}) + \kappa_{3lm}(x_m - z_{lm})\right], \tag{30}$$

and $\boldsymbol{\kappa} = \{\kappa_{1lm}, \kappa_{2lm}, \kappa_{3lm} | l = 2, \cdots, L, m = 1, \cdots, l-1\}$.

To maximize the objective function in (29), which is a linear combination of x_l and z_{lm}, we must let the positive coefficients corresponding to the x_l and z_{lm} be 1. Therefore, the optimal x_l and z_{lm} are given as (24) and (26), respectively.

To optimize y from (29), we set the first derivative of objective function to zero, i.e.,

$$\frac{\partial \mathcal{N}(\boldsymbol{x}, y, \boldsymbol{z}, \alpha, \boldsymbol{\kappa})}{\partial y} = \frac{Bp}{(py+\sigma^2)\ln 2} - \alpha = 0, \tag{31}$$

which yields $y = \frac{B}{(\ln 2)\alpha} - \frac{\sigma^2}{p}$. Considering constraint (23c), we obtain the optimal solution to problem (23) as (25). □

Theorem 2 states that RIS l that has a positive coefficient C_l should be on. According to the expression of C_l in (27), the negative term $-\lambda N_l P_{\mathrm{R}}$, is the effect of introducing additional power consumption if RIS l is on, while the remaining term represents the benefit of increasing the transmit rate by keeping RIS l in operation. When $C_l > 0$, the benefit of increasing the transmit rate is larger than the effect of introducing additional power consumption, which means that the energy efficiency can be improved if RIS l is on. The values of $(\alpha, \boldsymbol{\kappa})$ are determined by the sub-gradient method.

By iteratively optimizing primal variables $(\boldsymbol{x}, y, \boldsymbol{z})$ and dual variables $(\alpha, \boldsymbol{\kappa})$, the optimal RIS on-off vector is obtained. The dual method for solving problem (23) and the Dinkelbach method to update parameter λ are given in Algorithm 2. Notice that the optimal x_l is either 0 or 1 according to (24), even though we relax x_l as (29). Consequently, the optimal solution to problem (23) is obtained by using the dual method, i.e., $H(\lambda)$ in (19) is obtained for given λ. Using the Dinkelbach method, we can obtain the root of $H(\lambda) = 0$, which indicates that the optimal solution of fractional energy efficiency optimization problem (18) is obtained.

Algorithm 3 Iterative Optimization for Problem (5)

Initialize $(\boldsymbol{\theta}^{(0)}, p^{(0)}, \boldsymbol{x}^{(0)})$. Set iteration number $n = 1$.
repeat
  Given $\boldsymbol{x}^{(n-1)}$, solve the phase optimization problem (8) by using Algorithm 1 and the solution is denoted by $\boldsymbol{\theta}^{(n)}$.
  Given $\boldsymbol{x}^{(n-1)}$ and the optimized $\boldsymbol{\theta}^{(n)}$, solve the power control problem (12) according to Theorem 1 and the optimal power is denoted by $p^{(n)}$.
  Given $(\boldsymbol{\theta}^{(n)}, p^{(n)})$, solve the RIS on-off optimization problem (18) by using Algorithm 2 and the solution is denoted by $\boldsymbol{x}^{(n)}$.
  Set $n = n + 1$.
until the objective value (5) converges.

C. Complexity Analysis

The iterative algorithm for solving problem (5) is given in Algorithm 3. From Algorithm 3, the main complexity of solving problem (5) lies in solving the phase optimization problem (8) and the RIS on-off optimization problem (18).

According to Algorithm 1, to solve the phase optimization problem (8), the complexity lies in computing $\boldsymbol{v}^{(n)} = e^{j\angle(\boldsymbol{U}_1(\boldsymbol{g}_1 + \boldsymbol{U}_1^H \boldsymbol{v}^{(n-1)}))}$ at each iteration, which involves the complexity of $\mathcal{O}(QM)$. Hence, the total complexity of solving problem (8) with Algorithm 1 is $\mathcal{O}(T_1 QM)$, where T_1 is the total number of the iterations of Algorithm 1.

According to Algorithm 2, the main complexity of solving problem (18) lies in solving RIS on-off vector $\boldsymbol{x}$, which involves the complexity of $\mathcal{O}(L^2)$ based on (24) and (27). Hence, the complexity of solving problem (18) with Algorithm 2 is $\mathcal{O}(T_2 T_3 L^2)$, where T_2 is the number of inner iterations by updating primal variables and dual variables and T_3 is the number of inner iterations by updating the parameter λ.

As a result, the total complexity of solving problem (5) is $\mathcal{O}(T_0 T_1 QM + T_0 T_2 T_3 L^2)$, where T_0 is the total number of iterations for Algorithm 3.

IV. SIMULATION RESULTS AND DISCUSSIONS

There are K users uniformly distributed in a square area of size 300 m $\times$ 300 m with the BS located at its center. There are L RISs and the location of RIS l is given by $(\cos(2l\pi/L), \sin(2l\pi/L)) \times 100$ m. The main system parameters are listed in Table I. For small scale fading, we consider $[\boldsymbol{g}_k]_m, [\boldsymbol{h}_{kl}]_m, [\boldsymbol{G}_l]_{mn} \sim \mathcal{CN}(0,1), \forall k, l, m, n$ [17], [18]. The AF relay is assumed to transmit with the maximum power P_{T}. Unless specified otherwise, we set $P_{\max} = 50$ dBm, $M = 8$, $L = 8$, $N_1 = \cdots = N_L = N = 4$, and $R = 1$ Mbps. We compare the proposed scheme using distributed RISs (labeled 'DRIS') with the following schemes: the conventional scheme with the central deployment of one RIS located at (100, 0) m in [14] (labeled 'CRIS') and the conventional AF relay scheme [19] (labeled 'AFR'). In particular, the number of reflecting elements for one central RIS in CRIS is set as the total number of reflecting elements for all RISs in DRIS. In AFR, we consider the same deployment of DRIS, i.e., there are L AF relays, where AF l with N antennas is located at $(\cos(2l\pi/L), \sin(2l\pi/L)) \times 100$ m.

Fig. 2 shows how the energy efficiency changes as the maximum transmit power of the BS varies. In this figure,

TABLE I
SYSTEM PARAMETERS

Parameters	Values
Bandwidth of the BS B	1 MHz
Noise power σ^2	-104 dBm
Maximum transmit power of the AF relay P_{T}	30 dBm
Large scale fading model at distance d	$\frac{10^{-3.53}}{d^{3.76}}$
Circuit power of the BS P_{B}	39 dBm
Power amplifier efficiency at the BS/ AF relay ν	0.8
Circuit power of the user P_{U}	10 dBm
Circuit power of each RIS element P_{R}	10 dBm
Circuit power of each AF relay transmit-receive antenna P_{A}	10 dBm

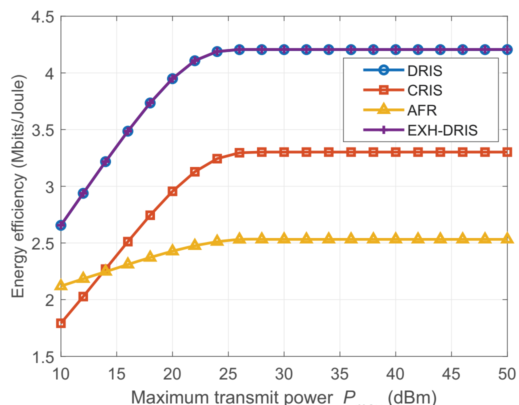

Fig. 2. Energy efficiency versus the maximum transmit power $P_{\max}$ of the BS.

the EXH-DRIS scheme is an exhaustive search method that can find a near optimal solution of problem (5). Hereinafter, the EXH-DRIS scheme refers to the proposed DRIS algorithm with 1000 initial starting points. In this simulation, EXH-DRIS can obtain 1000 solutions, and the solution with the highest energy efficiency is treated as the near optimal solution. It is shown that the energy efficiency of all schemes first increases and then remains stable as the maximum transmit power of the BS increases. This is because energy efficiency is not a monotonically increasing function of the maximum transmit power, as shown in (13). When $P_{\max} \sim 25$ dBm, the exceed transmit power is not used since it will decrease the energy efficiency. Fig. 2 also shows that the proposed DRIS scheme outperforms the CRIS and AFR schemes. For high maximum transmit power of the BS, DRIS can increase up to 27% and 68% energy efficiency compared to CRIS and AFR, respectively. Moreover, the proposed DRIS scheme achieves almost the same performance as the EXH-DRIS scheme, which indicates that the proposed DRIS can achieve the near optimum solution.

V. CONCLUSION

In this paper, we have investigated the resource allocation problem for a wireless communication network with distributed RISs. The RIS phase shifts, BS transmit power, and RIS on-off status were jointly optimized to maximize the system energy efficiency while satisfying minimum rate demand, maximum transmit power, and unit-modulus constraints. To solve this problem, we have proposed an iterative algorithm with low complexity. In particular, the phase optimization problem was solved by using the SCA method, where the closed-form solution was obtained at each step. Numerical results have shown that the proposed scheme outperforms conventional schemes in terms of energy efficiency, especially for small maximum transmit power.

REFERENCES

[1] W. Saad, M. Bennis, and M. Chen, "A vision of 6G wireless systems: Applications, trends, technologies, and open research problems," *IEEE Network*, 2020 (To appear).

[2] M. Chen, Z. Yang, W. Saad, C. Yin, H. V. Poor, and S. Cui, "A joint learning and communications framework for federated learning over wireless networks," *IEEE Trans. Wireless Commun.*, 2020 (To appear).

[3] M. Chen, U. Challita, W. Saad, C. Yin, and M. Debbah, "Artificial neural networks-based machine learning for wireless networks: A tutorial," *IEEE Commun. Surveys Tutorials*, vol. 21, no. 4, pp. 3039–3071, Fourthquarter 2019.

[4] E. Basar, M. Di Renzo, J. de Rosny, M. Debbah, M.-S. Alouini, and R. Zhang, "Wireless communications through reconfigurable intelligent surfaces," *arXiv preprint arXiv:1906.09490*, 2019.

[5] C. Huang, R. Mo, C. Yuen *et al.*, "Reconfigurable intelligent surface assisted multiuser miso systems exploiting deep reinforcement learning," *arXiv preprint arXiv:2002.10072*, 2020.

[6] Z. Yang, W. Xu, C. Huang, J. Shi, and M. Shikh-Bahaei, "Beamforming design for multiuser transmission through reconfigurable intelligent surface," *IEEE Trans. Commun.*, 2020 (To appear).

[7] H. Long, M. Chen, Z. Yang, B. Wang, Z. Li, X. Yun, and M. Shikh-Bahaei, "Reflections in the sky: Joint trajectory and passive beamforming design for secure uav networks with reconfigurable intelligent surface," *arXiv preprint arXiv:2005.10559*, 2020.

[8] C. Huang, A. Zappone, M. Debbah, and C. Yuen, "Achievable rate maximization by passive intelligent mirrors," in *Proc. IEEE Int. Conf. Acoust., Speech and Signal Process.*, Calgary, Canada, April 2018, pp. 3714–3718.

[9] M. Jung, W. Saad, Y. Jang, G. Kong, and S. Choi, "Performance analysis of large intelligence surfaces (LISs): Asymptotic data rate and channel hardening effects," *IEEE Trans. Wireless Commun.*, 2020 (To appear).

[10] M. Jung, W. Saad, M. Debbah, and C. S. Hong, "On the optimality of reconfigurable intelligent surfaces (RISs): Passive beamforming, modulation, and resource allocation," *arXiv preprint arXiv:1910.00968*, 2019.

[11] C. Pan, H. Ren, K. Wang, M. Elkashlan, A. Nallanathan, J. Wang, and L. Hanzo, "Intelligent reflecting surface enhanced MIMO broadcasting for simultaneous wireless information and power transfer," *arXiv preprint arXiv:1908.04863*, 2019.

[12] H. Shen, W. Xu, S. Gong, Z. He, and C. Zhao, "Secrecy rate maximization for intelligent reflecting surface assisted multi-antenna communications," *IEEE Commun. Lett.*, vol. 23, no. 9, pp. 1488–1492, Sep. 2019.

[13] X. Yu, D. Xu, Y. Sun, D. W. K. Ng, and R. Schober, "Robust and secure wireless communications via intelligent reflecting surfaces," *arXiv preprint arXiv:1912.01497*, 2019.

[14] C. Huang, A. Zappone, G. C. Alexandropoulos, M. Debbah, and C. Yuen, "Reconfigurable intelligent surfaces for energy efficiency in wireless communication," *IEEE Trans. Wireless Commun.*, vol. 18, no. 8, pp. 4157–4170, Aug. 2019.

[15] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.

[16] W. Dinkelbach, "On nonlinear fractional programming," *Management Science*, vol. 13, no. 7, pp. 492–498, 1967.

[17] Z. Yang, M. Chen, W. Saad, C. S. Hong, and M. Shikh-Bahaei, "Energy efficient federated learning over wireless communication networks," *arXiv preprint arXiv:1911.02417*, 2019.

[18] Y. Mao, B. Clerckx, and V. O. K. Li, "Rate-splitting for multi-antenna non-orthogonal unicast and multicast transmission: Spectral and energy efficiency analysis," *IEEE Trans. Commun.*, vol. 67, no. 12, pp. 8754–8770, Sept. 2019.

[19] A. Zappone, P. Cao, and E. A. Jorswieck, "Energy efficiency optimization in relay-assisted MIMO systems with perfect and statistical CSI," *IEEE Trans. Signal Process.*, vol. 62, no. 2, pp. 443–457, Jan. 2014.