

Blinded challenge for precision cosmology with large-scale structure: results from effective field theory for the redshift-space galaxy power spectrum

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An accurate theoretical template for the galaxy power spectrum is a key for the success of ongoing and future spectroscopic surveys. We examine to what extent the Effective Field Theory of Large Scale Structure is able to provide such a template and correctly estimate cosmological parameters. To that end, we initiate a blinded challenge to infer cosmological parameters from the redshift-space power spectrum of high-resolution mock catalogs mimicking the BOSS galaxy sample but covering a hundred times larger cumulative volume. This gigantic simulation volume allows us to separate systematic bias due to theoretical modeling from the statistical error due to sample variance. The challenge task was to measure three unknown input parameters used in the simulation: the Hubble constant, the matter density fraction, and the clustering amplitude. We present analyses done by two independent teams, who have fitted the mock simulation data generated by yet another independent group. This allows us to avoid any confirmation bias by analyzers and pin down possible tuning of the specific EFT implementations. Both independent teams have recovered the true values of the input parameters within sub-percent statistical errors corresponding to the total simulation volume.

I. INTRODUCTION

Modern cosmology is getting more and more mature as accumulating observational data are available for us. We have however a fundamental lack of understanding of the physical nature of the dark components introduced to explain the dominant source of gravity that gathers material to form rich structures in the late universe (dark matter) as well as the accelerating cosmic expansion (dark energy), together filling the majority of the cosmological energy budget. Aiming at having further insights on those substances, growing number of large-scale observational programs are ongoing and planned [e.g. 1–3].

Of crucial importance from the theoretical point of view is our ability to prepare an accurate model template with which one can confront such observational data for their interpretation. Since a larger survey means

a smaller statistical error, the relative contribution from the systematic error arising from the inaccuracy of the template should be more important. Given the gigantic area coverage and depth of ambitious future programs, there is the necessity to come up with a really accurate theoretical framework to predict the observed large scale structure to attain their full potential to infer the underlying theory governing the universe.

One of the most difficult aspect of the large-scale structure prediction is the complicated relation between the matter density fluctuations dominated by invisible dark matter and visible structures such as galaxies [4]. The so-called galaxy bias cannot be predicted from first principles, unless one can model all the baryonic effects relevant for the formation and evolution of galaxies. While hydrodynamical simulations might be one way to proceed, the vastly large dynamical range, kpc to Gpc in

length scale, is a big obstacle. Typically, one comes up with empirical subgrid models and calibrate them against observed statistics of galaxies [see e.g., 5–12, for recent attempts].

Alternatively, one can formulate the statistical properties of galaxies on large scales via a perturbative expansion in which poorly known galaxy physics is parameterized by a set of effective bias operators. The strength of these operators is controlled by free coefficients, which should be treated as nuisance parameters. The recently developed Effective Field Theory of Large Scale Structure (EFTofLSS) provides a systematic way to derive all possible operators and corresponding bias coefficients that are allowed by symmetry [13–21] [also see 22, for a review]. Since this approach, in principle, does not assume any specific model of galaxy formation, it provides us with a conservative theoretical model for the galaxy density and velocity fields on large scales. The generality of the effective field theory approach comes at the price of having to marginalize over many free coefficients, which can compromise cosmological constraints. These constraints can become weaker compared to other theoretical templates in which a specific bias prescription is employed, such as halo model approaches. The detailed balance between the robustness and the tightness of the cosmological constraints has been addressed in recent studies [e.g., 23–25].

There are several non-trivial choices behind the application of the EFT to the data. First, one should determine the wavenumber up to which the EFT calculation up to a chosen perturbative order is reliable. This data cut should be carefully tested to avoid biased parameter estimates. Then, one has to decide how many nuisance parameters to keep in the fit (there are about 10 at the one-loop order) and what priors to use. Indeed, at the power spectrum level many EFT operators are degenerate among each other. Thus, one has to accurately determine their principal components to make the cosmological analysis efficient. All these subtleties should be examined and validated in a transparent manner to convince the community of the robustness of the EFT approach.

To that end, in this paper, we conduct a first *blind* test of EFTofLSS for clustering of galaxies in redshift space. Two independent groups (which will be referred to as “West Coast” and “East Coast”) have analyzed the mock data generated by yet another group (Kyoto and Tokyo, simply “Japan Team” hereafter). In this process, the true cosmological parameters used to generate the simulation mock data were known only to the Japan Team. The two analyzing teams have participated in the challenge on the condition that the results would be published regardless of the outcome, and the pipelines could not be modified after unblinding. We present these results in our paper in the original form. To complement the result of the blinded analysis and to get more insight on the origin of the cosmological information, we briefly discuss post-unblinding analyses.

The layout of this paper is as follows. We first describe the design of our mock challenge program in Sec. II. We then specify the mock simulations in Sec. III. The theoretical template and the method to conduct parameter inference are explained in Sec. IV. Then the results of the blinded analysis are summarized in Sec. V. We conclude this study in Sec. VI.

II. DESIGN OF BLINDED COSMOLOGY CHALLENGE

Throughout this paper, we consider a flat Λ CDM cosmology. This is motivated by the recently claimed tension in the values of the Hubble parameter, one from local measurements such as the distance ladder, and the other from the Cosmic Microwave Background (CMB) assuming a flat Λ CDM model [see 26, and references therein]. In such a situation, a robust measurement from other independent observable channels would be important, and indeed, the galaxy clustering, when the full shape information of its spectra is analysed, has been shown to serve as such a probe [27–30]. Also important might be a similar, but a weaker tension in the amplitude of the density fluctuations in the current universe [31–33]. This is known to be degenerate with the matter density parameter from the late-time observables. We wish to demonstrate through the challenge the current status of the use of galaxy clustering in particular with an EFT approach to describe the nonlinear nature of the cosmological large scale structure.

A. Cosmological parameters

To assess the reliability of the galaxy-clustering analyses within the flat Λ CDM model, three cosmological parameters, $\ln(10^{10}A_s)$, Ω_m and H_0 , are randomly drawn from independent normal distributions. These parameters are the logarithm of the amplitude of the primordial power spectrum at $k_0 = 0.05 \text{ Mpc}^{-1}$, the matter density parameter at present and the current Hubble expansion rate in km/s/Mpc , respectively. While the mean values of the normal distributions are set to be the best-fit values determined by Planck satellite [34], we consider the standard deviation four times larger than the same experiment to test the validity of the model in a broader parameter space. While all of the information above is shared among all the collaborators, the three random numbers drawn were kept only within the Japan Team until we finally unblinded them.

On the other hand, we fix the baryon fraction, $f_b = 0.1571$ and the spectral index $n_s = 0.9649$. These values are shared with the two US teams. For simplicity and to avoid the complication to deal with massive neutrinos both in theory and in simulations, we set the neutrino masses to be exactly zero. Under the above settings, the linear matter-density transfer function is computed using

the public Boltzmann solver CAMB [35]. The parameter file passed to this code by the Japan Team is provided to the US teams after the three blinded cosmological parameter values are erased.

The main goal of the challenge is to infer the three cosmological parameters A_s , Ω_m and H_0 . It was agreed among all the teams that, once these cosmological parameters are unblinded, the analysis pipeline used in the challenge may not be changed.

B. Target observables

We focus on the galaxy clustering in redshift space in the initial challenge presented in this paper. More specifically, we work in Fourier space and analyse the multipole moments of the galaxy power spectrum. This includes physical and observational effects such as the Baryon Acoustic Oscillation (BAOs; [36–40]), redshift-space distortions (RSD; [41, 42]) and the Alcock-Paczynski (AP; [43]) effect, where the AP is induced artificially by distorting the simulation boxes (see the next section for further detail). On top of these distinctive features, the mock data should contain the cosmological information through the overall shape of the power spectra, which might be hindered by the presence of various nonlinear effects. The aim of this challenge is to assess how robustly one may extract the fundamental cosmological parameters within the flat Λ CDM framework.

The Japan Team constructs mock galaxy catalogs and measures the multipole moments of the power spectra. To discriminate the *systematic* error from the statistical error, this experiment is done in huge simulation volumes much larger than the current surveys. The galaxy catalogs are constructed to roughly mimic the CMASS and the LOWZ catalog from the 12th Data Release of Sloan Digital Sky Survey [Ref. 44, hereafter SDSS DR12]. The details of these simulations will follow in the next section. Since the galaxy bias is formulated to be as general as possible in the EFT, based only on symmetry considerations without assuming any specific model with which galaxies are defined, the detail of the mock galaxies would not give a significant impact to the blinded analysis as long as one sticks to an EFT approach. However, other approaches such as the halo model would be directly impacted by the piece of information on the exact procedure with which the mock galaxies are distributed within the simulation volume. Therefore, any further information on the mock galaxies detailed in the next section was not provided to the US teams before unblinding.

For completeness, the set of mock data as well as the information on the simulations provided to the US teams are summarised at a dedicated website (<http://www2.yukawa.kyoto-u.ac.jp/~takahiro.nishimichi/data/PTchallenge/>). All the data and the information were shared through this website. Interested readers may download the same set of data and participate in the blinded challenge by

analysing the data using their own theoretical template, as the exact cosmological parameter values are not explicitly shown in this paper nor on the website.

III. GENERATING MOCK REDSHIFT-SPACE POWER SPECTRA OF BOSS-LIKE GALAXIES

The Japan Team works on the construction of mock galaxy catalogs and measurement of the power spectra. The settings of the numerical simulations, the prescription for the mock galaxies and the analysis methods to determine their statistics are described in this section.

A. Specification of simulations

We follow the gravitational dynamics of ten random realizations of the matter density field expressed by $3,072^3$ mass elements sampled in comoving periodic cubes with the side length $L = 3,840 h^{-1}\text{Mpc}$. The total volume, $566 (h^{-1}\text{Gpc})^3$, is about a hundred times that of the CMASS and LOWZ sample from SDSS BOSS DR12, which together have a volume coverage of $5.7 (h^{-1}\text{Gpc})^3$ [45]. The large volume of our simulations allows us to determine the statistics of the mock galaxies very precisely with little sample-variance error. Therefore, we can conduct a fairly stringent test of the systematic error due to an imperfect modeling of the target statistics.

The initial conditions are generated with a code developed in [46] and then parallelized in [47] based on the second-order Lagrangian Perturbation Theory (2LPT; [48, 49]). Following the result presented in [50], the starting redshift of the simulations are set at $z = 29$ to roughly optimize the total systematic error arising from the artificial growing mode due to the grid pre-initial condition [51–53] and the truncation of the LPT at the second order given the mean inter-particle distance of the simulations. We prepare ten independent random realizations, each of which is then evolved by a public Tree-Particle Mesh code GADGET2 [54] with $6,144^3$ grid points for fast Fourier transform (FFT) and the tree softening length of $62.5 h^{-1}\text{kpc}$. The other simulation parameters to control the force accuracy as well as the time-stepping criteria are the same as in [50]. We store the particle snapshots at $z = 3, 2, 1, 0.61, 0.51$ and 0.38 . We populate galaxies to the lowest three redshifts, and conventionally call the catalogs as CMASS2 ($z = 0.61$), CMASS1 ($z = 0.51$) and LOWZ ($z = 0.38$) in what follows.

B. Mock galaxy identification

After obtaining the particle snapshots, we run the ROCKSTAR halo finder [55], which is based on the six-dimensional phase space friends-of-friends algorithm. This code identifies not only isolated “central” halos but also

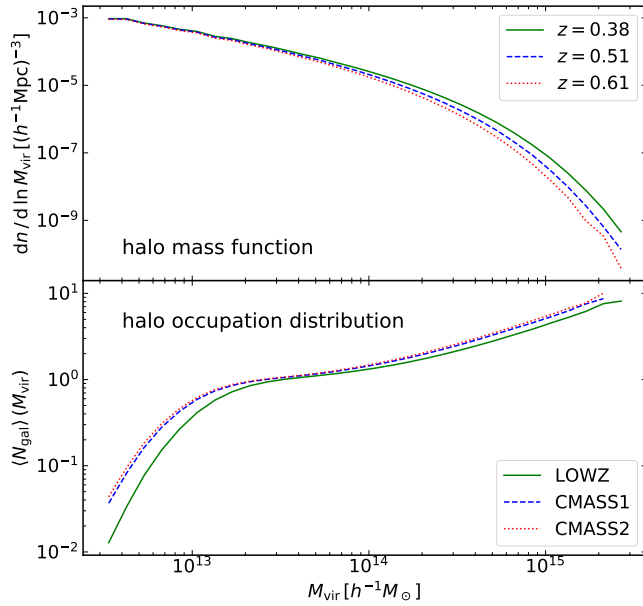


FIG. 1. The abundance of halos per unit logarithmic mass interval (upper) and the mean number of mock galaxies per halo (lower) as a function of the virial mass of halos. The mean of the ten random realizations are shown at three output redshifts of the simulations as indicated by the figure legend.

“satellite” halos existing as substructures of more massive halos without any distinction at first in the primary output files. For simplicity, we treat each of them irrespectively of whether it is a central or a satellite halo and populate a galaxy only according to the virial mass assigned by ROCKSTAR. We impose a soft cutoff to the virial mass to select massive halos to populate galaxies randomly with the probability

$$P(M_{\text{vir}}) = \frac{1}{2} \left[1 + \text{erf} \left(\frac{\log_{10} M_{\text{vir}} - \log_{10} M_{\text{min}}}{\sigma_{\log_{10} M}} \right) \right], \quad (1)$$

where $\text{erf}(x)$ is the error function. We have two parameters, $\log_{10} M_{\text{min}}$ and $\sigma_{\log_{10} M}$, which determine the typical minimum mass and the profile of the soft mass cutoff, respectively. We set $\log_{10} M_{\text{min}} = 13.08, 12.97$ and 12.95 for LOWZ, CMASS1 and CMASS2 (M_{min} is given in unit of $h^{-1} M_{\odot}$), respectively, while the value of $\sigma_{\log_{10} M}$ is fixed to 0.35 for all of the samples. These choices are made such that the resultant clustering signal of the mock galaxies, especially the amplitude of the power spectra at small k becomes roughly consistent with the observation (see the next subsection for more detail). We assume that the populated mock galaxies are located at the center-of-mass position of the core particles determined by ROCKSTAR. Similarly, we assign the the center-of-mass velocities of the same core particles to the mock galaxies, which are used when we displace the positions of mock galaxies to redshift space [24].

We show the abundance of (central) halos as well as the mean number of galaxies per central halo as a func-

tion of the virial mass in Fig. 1. Here, we define “central” halos from the ROCKSTAR catalog as those satisfying the condition that any other halo is not more massive than the halo of interest to within a sphere of radius $R_{\text{vir}}^{\text{cen}}$, where $R_{\text{vir}}^{\text{cen}}$ is the virial radius of the central halo. Note that an isolated halo is also identified as a central halo according to this definition. On the other hand, the halos which reside around a more massive neighbor to within the neighbor’s virial radius are identified as “satellite” (sub)halos. The particular definition does not really affect our mock galaxy catalog due to our recipe (Eq. 1) to populate galaxies. The lower panel of Fig. 1 shows the average number of mock galaxies in central halos, i.e. the halo occupation distribution (HOD), as a function of central halo mass. Note that unlike the standard HOD prescription, the HOD of our mock catalog is not given a priori, and rather is *measured* from the mocks with the central/satellite split. Nevertheless, the shape of HOD in our mock catalogs looks similar to what can be found in the literature, e.g., [56, 57]. There appear two regimes; halos around the soft cutoff near $M_{\text{vir}} = 10^{13} h^{-1} M_{\odot}$ host only one galaxy (i.e., a central galaxy), while massive halos above $10^{14} h^{-1} M_{\odot}$ receive a significant contribution from satellite galaxies, displaying a power-law like form in the HOD.

C. Measurement of the mock signal and error

We here describe the method to measure the power spectra and estimate the data covariance from the mock galaxy catalogs.

The measurement is done based on FFT of the density field. We first assign the mock galaxies in redshift space to $n_g^3 = 2048^3$ grid points using the Cloud in Cell (CIC) interpolation scheme. We employ the distant observer approximation in the mapping to the redshift space. We follow Ref. [58] to correct for the aliasing effect [59], by the so-called interlacing method. To do this, we prepare another density grid but with mass assignment done after shifting the galaxy positions by half the grid size along all of the three Cartesian axes and then corrected for the phase shift by multiplying an appropriate factor to the field in Fourier space. By taking the average of the two density grids, the original and the interlaced, we can get rid of the aliasing effect due to the odd images, which would give the dominant aliasing source to standard cosmological power spectra with decaying amplitude toward higher wavenumbers. The effect of the CIC window function will eventually be removed later in Eq. (3).

We then reinterpret the wavenumbers by taking account of the AP effect. Namely, we rescale the fundamental modes along the each of the three axes as

$$\begin{aligned} \tilde{k}_{f,x} &= \tilde{k}_{f,y} = \frac{D_A^{(\text{true})}(z)}{D_A^{(\text{fid})}(z)} k_f, \\ \tilde{k}_{f,z} &= \frac{H^{(\text{fid})}(z)}{H^{(\text{true})}(z)} k_f, \end{aligned} \quad (2)$$

where $k_f = 2\pi/L$ is the original fundamental mode in the absence of AP effect. In the above, we take the z direction in the simulation box as the line-of-sight direction, and the upper scripts, (true) and (fid), indicate that the comoving angular diameter distance, $D_A(z)$, or the Hubble expansion rate, $H(z)$, are calculated assuming the correct, blinded cosmological parameters, and a fiducial cosmological parameters, respectively. Here, we adopt a flat Λ CDM cosmology with $\Omega_m^{(\text{fid})} = 0.3$ as the fiducial cosmology, and this information is shared with the two US analysis teams. $\Omega_m^{(\text{fid})}$ that was used to create the mock catalogs should not be confused with the true cosmological parameter Ω_m , which was used in the simulations and which was kept in secret to the analyzing teams.

The Japan Team then estimates the first three non-zero multipole moments, monopole ($\ell = 0$), quadrupole ($\ell = 2$) and hexadecapole ($\ell = 4$) by taking weighted averages of the squared Fourier modes:

$$\hat{P}_\ell(k_i) = \frac{2\ell+1}{N_i} \sum_{\tilde{\mathbf{k}} \in \text{bin } i} \mathcal{P}_\ell(\mu_{\tilde{\mathbf{k}}}) \hat{P}(\tilde{\mathbf{k}}), \quad (3)$$

with

$$\hat{P}(\tilde{\mathbf{k}}) = \frac{\tilde{V} |\delta_{\tilde{\mathbf{k}}}|^2 - \tilde{P}_{\text{shot}}(\tilde{\mathbf{k}})}{W_{\text{CIC}}^2(\tilde{\mathbf{k}})}, \quad (4)$$

where the distorted volume $\tilde{V}$ is given by

$$\tilde{V} = \left(\frac{D_A^{(\text{fid})}(z)}{D_A^{(\text{true})}(z)} \right)^2 \frac{H^{(\text{true})}(z)}{H^{(\text{fid})}(z)} L^3, \quad (5)$$

analogously to Eq. (2) to account for the AP effect, the summation runs over wavevectors $\tilde{\mathbf{k}} = (\tilde{k}_{f,x} i_x, \tilde{k}_{f,y} i_y, \tilde{k}_{f,z} i_z)$ specified by an integer vector (i_x, i_y, i_z) , $\mathcal{P}_\ell$ denotes the ℓ -th order Legendre polynomial, $\mu_{\tilde{\mathbf{k}}}$ is the cosine between the wavevector $\tilde{\mathbf{k}}$ and the z -direction, and N_i stands for the number of Fourier modes contained in the i -th wavenumber bin. In the above, we have subtracted the shot noise, $\tilde{P}_{\text{shot}}$, from the measured power spectrum. We evaluate the shot noise taking into account the interlacing technique for the aliasing correction and the CIC window function. Denoting

$$\tilde{\kappa}_a = \frac{\pi \tilde{k}_a}{2 \tilde{k}_{\text{Ny},a}}, \quad (6)$$

with $\tilde{k}_{\text{Ny},a} = \tilde{k}_{f,a} n_g/2$ being the direction-dependent Nyquist frequency ($a = x, y$ or z), the resultant expression for the wavevector-dependent shot noise contribution is given as

$$\begin{aligned} \tilde{P}_{\text{shot}}(\tilde{\mathbf{k}}) &= \sum_{n_x, n_y, n_z: \text{even}} W_{\text{CIC}}^2(\tilde{\mathbf{k}} + 2\tilde{\mathbf{k}}_{\text{Ny}} \cdot \mathbf{n}) \frac{\tilde{V}}{N_{\text{gal}}}, \\ &= \left[\prod_{a=x,y,z} C_a(\tilde{k}_a) \right] \frac{\tilde{V}}{N_{\text{gal}}}, \end{aligned} \quad (7)$$

with W_{CIC} being the CIC window function

$$W_{\text{CIC}}(\tilde{\mathbf{k}}) = \prod_{a=x,y,z} \text{sinc}^{-2} \tilde{\kappa}_a, \quad (8)$$

and the final shot-noise correction factor, C_a , given as the infinite summation over even integers can be computed analytically as

$$C_a(\tilde{k}_a) = \frac{1}{12} (1 + \cos \tilde{\kappa}_a)^2 (2 + \cos \tilde{\kappa}_a). \quad (9)$$

See Ref. [59] for a similar expression but without the interlacing correction that erases the odd images.

The estimator, Eq. (3), is computed at 100 wavenumber bins between the first bin edge taken at zero to the final bin edge at $1 h \text{ Mpc}^{-1}$ evenly spaced by $0.01 h \text{ Mpc}^{-1}$. The representative wavenumber of each bin, k_i in Eq. (3), is computed as the average of the norm of the wavevectors that actually enter the bin:

$$k_i = \frac{1}{N_i} \sum_{\tilde{\mathbf{k}} \in \text{bin } i} |\tilde{\mathbf{k}}|. \quad (10)$$

The pairs of numbers, $(k_i, \hat{P}_\ell(k_i))$, are provided to the analysis teams as the mock measurements, and the above way to compute the representative number of each k bin is informed to the analysis team. The data files also contain estimates of the covariance matrix. It is obtained assuming Gaussianity [24]:

$$\begin{aligned} \text{Cov}_{ij}^{\ell\ell'} &= \left\langle \left(\hat{P}_\ell(k_i) - \langle \hat{P}_\ell(k_i) \rangle \right) \left(\hat{P}_{\ell'}(k_j) - \langle \hat{P}_{\ell'}(k_j) \rangle \right) \right\rangle, \\ &= \delta_{ij}^K \frac{(2\ell+1)(2\ell'+1)}{N_i^2} \\ &\quad \times \sum_{\tilde{\mathbf{k}} \in \text{bin } i} \mathcal{P}_\ell(\mu_{\tilde{\mathbf{k}}}) \mathcal{P}_{\ell'}(\mu_{\tilde{\mathbf{k}}}) \left[P(\tilde{\mathbf{k}}) + P_{\text{shot}} \right]^2, \end{aligned} \quad (11)$$

where $P(\tilde{\mathbf{k}})$ is the expectation value of $\hat{P}(\tilde{\mathbf{k}})$. The expression reduces to the real-space formula by Ref. [60] when $\ell = \ell' = 0$. In reality, however, we have to make use of a noisy estimate of the power spectrum $\hat{P}(\tilde{\mathbf{k}})$ for each wavevector $\tilde{\mathbf{k}}$ instead of $P(\tilde{\mathbf{k}})$, and this can impact the estimation of the covariance matrix significantly. Therefore, instead of computing Eq. (11), we first bin the Fourier modes in 10 evenly-spaced $|\mu_{\tilde{\mathbf{k}}}|$ bins and take the average of $\hat{P}(\tilde{\mathbf{k}})$ within each bin to suppress the noise. The binned estimates are then used in Eq. (11), but the summation now runs over bins instead of individual wavevectors, to obtain our estimate of the covariance matrix.

The Japan Team considers two settings for the covariance matrix. The first is to use the volume and the shot noise consistent with the mock simulations. In addition, they provide another estimate scaled to the BOSS DR12 catalogs, by substituting the number density from the observation and then scaling the number of Fourier modes according to the ratio of the surveyed and the simulated volume. The set of estimates, $\hat{P}(k_i)$ and $\text{Cov}_{ij}^{\ell\ell'}$, with the

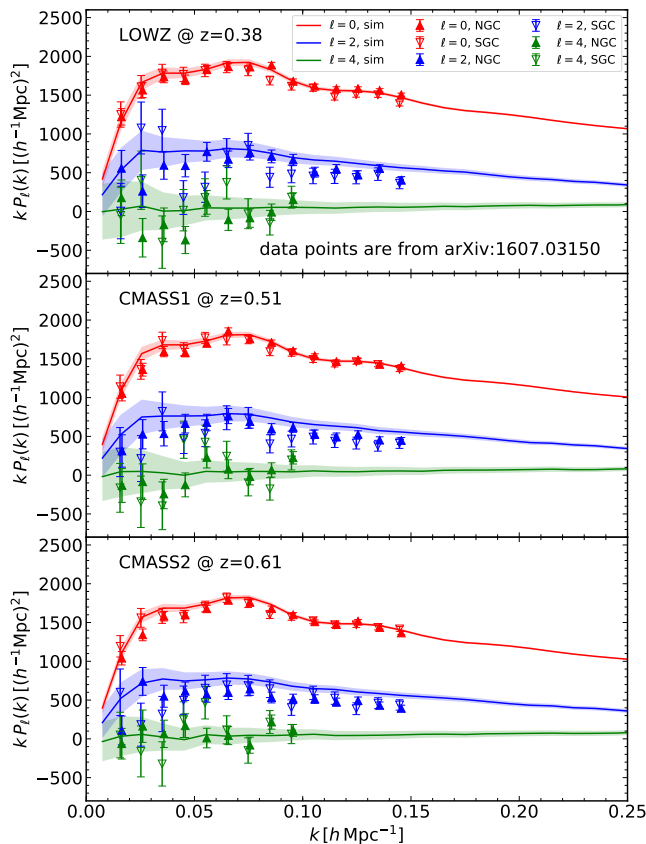


FIG. 2. First three multipole moments (monopole, quadrupole and hexadecapole) of the power spectrum in redshift space measured from our mock galaxy catalogs at three redshifts (the solid lines). The $1\text{-}\sigma$ uncertainty intervals assuming the survey parameter of the SDSS Data Release 12 are shown by the shaded regions. Also shown by the error bars are taken from Ref. [61] based on SDSS DR12. For these data points, the measurements from the sample in the North Galactic Cap (NGC) and the South Galactic Cap (SGC) are shown separately by different symbols as indicated by the figure legend. Note that the Alcock-Paczynski effect is artificially induced assuming $\Omega_m = 0.3$ in the redshift-distance conversion. The analysis teams can only access exactly the data vector shown in this figure. The analyses presented in this paper is based on the monopole and the quadrupole moments from the catalog at $z = 0.61$.

latter now has only diagonal entries with respect to the subscripts, i and j , due to the Gaussian approximation, are tabulated for each of the ten random realizations and provided through the website. The Japan Team leaves the decision to the US Teams on how to exactly use these estimates: which survey specification for the estimation of the covariance matrix to adopt, to combine the ten realization and analyse the averaged spectra just once or to analyse each realization one by one, or to further estimate the non-Gaussian error from the realization-to-realization scatter.

We show in Fig. 2 the average multipole moments of

the power spectra at the three redshifts corresponding to LOWZ, CMASS1 and CMASS2. The solid lines show the mock measurements, where the shaded region around each line denotes the $1\text{-}\sigma$ error scaled to the SDSS BOSS DR12 survey parameters. The three lines in each panel depict the monopole, quadrupole and hexadecapole from top to bottom. Also shown by the symbols with error bars are the actual measurements from the BOSS data by [61]. The measurements from the North and South Galactic Cap are respectively plotted by the upward and the downward triangles.

Overall, the mock data follows the observed spectra. The monopole moment especially exhibits an excellent agreement, because the model parameters used to distribute the mock galaxies are chosen to match this moment. There is, however, small mismatch in the quadrupole moment: the observed data shows a stronger damping behaviour to the higher wavenumbers. It is out of the scope of the current investigation to see if or not this can be alleviated by further tuning the model parameters without spoiling the success in the monopole. This is nontrivial since the cosmological parameters adopted in the mock simulations could be off from the true unknown parameters governing our Universe, or the recipe to populate mock galaxies might not be flexible enough to meet the reality.

IV. THEORETICAL TEMPLATE

In this section we describe the implementation of the theoretical model by the two teams participating in the cosmological analysis challenge. The employed methodologies are almost identical to the ones used in the analysis of the actual BOSS data by the same teams [27–29].

Both teams participating in the PT challenge use, essentially, the same theoretical template. However, there are differences in the implementation of IR resummation, the choice of nuisance parameters and their priors. Besides, the two teams use absolutely independent pipelines based on different software. This section describes in detail the pipelines used by the two teams and focuses on methodological differences.

A. Common basis for the EFT formulation

On general grounds, it is believed that any physical system has a *unique* and *correct* description at long wavelengths where the microscopical details of the physical system under consideration can be encoded in just a few coefficients of the terms in the equations of motion. In the context of the long-distance universe, this description is believed to be the Effective Field Theory of Large-Scale Structure (EFTofLSS) [62, 63]. The originality of the EFTofLSS with respect to other pre-existing perturbative methods that were applied in the context of LSS is two-fold. First is the presence of suitable terms in

the equations of motion that encode the effect of short-distance non-linearities and galaxies at long distances, and that cannot be predicted without detailed knowledge of galaxy physics, and therefore are generically fit to observations. Second, the equations of motion in the EFTofLSS have non-linear terms that are proportional to some parameters. Due to the many phenomena that control the evolution of our universe, there are several of these parameters, such as the size of the density perturbation or the ratio of a given wavelength with respect to the size of the displacements induced by short distance modes [17]. For all of these parameters but one, an iterative solution is performed. Instead for one parameter, the one encoding the effect of long wavelength displacements, a non-linear solution is performed, which goes under the name of IR-Resummation [17, 64–67]. Different incarnations of the EFTofLSS make this expansion more or less manifest. For example, the Lagrangian-space EFTofLSS [68] automatically solves non-linearly in the effect of long-displacements, and so, it is identical to the Eulerian EFTofLSS that we use here after this has been IR-Resummed [17].

In the EFTofLSS, the description of the clustering of galaxies in redshift space is performed in the following way. First, the dark matter and baryonic fields are described in terms of fluids with a non-trivial stress tensor. Galaxies are biased tracers, in the sense that, if δ_g is the galaxy overdensity, we have that [18]

$$\begin{aligned}\delta_g(x, t) &= \sum_n \int dt' K_n(t, t') \tilde{\mathcal{O}}_n(x_{\text{fl}}, t') \\ &= \sum_{n, m} b_{n, m}(t) \mathcal{O}_{n, m}(x, t)\end{aligned}\quad (12)$$

where $\tilde{\mathcal{O}}_n$ are all possible fields, such as, for example, the dark matter density, that, by general relativity, can affect the formation of galaxies. $K_n(t, t')$ are some kernels that relate how a field at a certain time affects the galaxies at later times, and x_{fl} is the location at time t' of the fluid element that is at x at time t . The last step of the above equation can be performed using the perturbative expression for the matter and baryonic fields. In fact, in perturbation theory the time- and space-dependence parts factorize in a form, schematically, given by $\delta(\vec{k}, t) \sim \sum_n f_n(t) \delta^{(n)}(\vec{k})$, where $\delta^{(n)}$ is order n in the expansion parameters. This allows us to define the biases b as $b_{n, m}(t) \sim \int dt' K_n(t, t') f_m(t')$. This provides the first complete parametrization of the bias expansion, though many earlier attempts were made and substantial but partial successes were obtained.

Next, we need to describe the observed density field in redshift space. This is a combination of the density field in configuration space and density times powers of the velocity field of galaxies, such as $\rho(\vec{x}, t) v(\vec{x}, t)^i, \rho(\vec{x}, t) v^i(\vec{x}, t) v_i(\vec{x}, t), \dots$. Again, these short-distance-dependent terms are described as above as biased tracers of the density and baryonic fields [19].

Because of what we just discussed, the range over

which different implementations of the EFTofLSS can differ is extremely limited: they may choose a different basis for the EFT-parameters, they may add an incomplete, and therefore different, set of higher-order counterterms to partially include the effect of some higher order calculation that was not performed, or they may have different implementations or approximations for the IR-Resummation. We are going to list them in detail next.

B. Group dependent implementation

Although both teams use the same theoretical model, there are several important methodological differences. Moreover, the two groups have made very different choices in the model implementation and numerical algorithms. This section describes in detail the pipelines used by the two teams.

1. East Coast Team

The East Coast Team used only the monopole and the quadrupole in the analysis. The East Coast Team analyzed the challenge data with and without the hexadecapole moment and found identical constraints.¹ Given these reasons, the East Coast Team refrained from using the hexadecapole moment in the baseline analysis.

The theoretical model used by the East Coast Team for these two multipoles can be written schematically as

$$P_\ell(k) = P_\ell^{\text{tree}}(k) + P_\ell^{\text{loop}}(k) + P_\ell^{\text{ctr}}(k) + P_\ell^{\nabla_z^4 \delta}(k). \quad (13)$$

The tree-level contribution is given by the Kaiser formula [42]. The loop corrections are calculated using the standard one-loop power spectra for dark matter and biased tracers (see e.g., [22, 69, 70] and references therein). The bias model consists of the following bias operators [16, 18, 71]

$$\delta_g(\mathbf{k}) = b_1 \delta(\mathbf{k}) + \frac{b_2}{2} \delta^2(\mathbf{k}) + b_{\mathcal{G}_2} \mathcal{G}_2(\mathbf{k}), \quad (14)$$

where the momentum-space representation of $\mathcal{G}_2$ operator is given by

$$\mathcal{G}_2(\mathbf{k}) = \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left[\frac{(\mathbf{p} \cdot (\mathbf{k} - \mathbf{p}))^2}{p^2 |\mathbf{k} - \mathbf{p}|^2} - 1 \right] \delta(\mathbf{p}) \delta(\mathbf{k} - \mathbf{p}). \quad (15)$$

¹ On the scales of interest the hexadecapole signal is dominated by leakage contributions from the monopole and quadrupole. These contributions appear due to discreteness effects, i.e. because the monopole and quadrupole are not exactly orthogonal to the hexadecapole on a finite grid. Even with the gigantic volume of the challenge simulation and the wide binning the hexadecapole moment happened to be dominated by the systematic leakage from lower multipole moments.

The one-loop power spectrum has one extra bias operator multiplied by an additional parameter b_{Γ_3} . However, this contribution is almost fully degenerate with the counterterms and $\mathcal{G}_2$ operator on the scales of interest. Given this strong degeneracy, the East Coast Team has set $b_{\Gamma_3} = 0$ in the baseline analysis. Running the MCMC chains with and without b_{Γ_3} , it was checked that this choice does not affect constraints on cosmological parameters.

The standard one-loop counterterms for the monopole and the quadrupole are [19]

$$P_0^{\text{ctr}}(k) = -2c_0^2 k^2 P_{11}(k), \quad P_2^{\text{ctr}}(k) = -\frac{4f}{3} c_2^2 k^2 P_{11}(k), \quad (16)$$

where $f = d \ln D_+ / d \ln a$ is the logarithmic growth rate, D_+ denotes the linear growth factor and $P_{11}(k)$ is the linear power spectrum. The purpose of these counterterms is to fix the UV-dependence of the loops and to partly take into account the effects of the fingers-of-God [41]. The East Coast Team also added an extra k^4 term shared between the multipoles,

$$P^{\nabla^4 \delta}(k, \mu) = -c(\mu k f)^4 (b_1 + f\mu)^2 P_{11}(k). \quad (17)$$

This new counterterm takes into account next-to-leading order of the fingers-of-God. Note that on general grounds one also expects the presence of the stochastic contribution of the form [19, 72],

$$P_{\text{RSD, stoch}} = -c_\epsilon k^2 \mu^2. \quad (18)$$

This contribution happens to be very degenerate with the counterterm (17) on the scales of interest for the analysis and it was not included in the model by the East Coast Team.

The East Coast Team has implemented IR-Resummation and the Alcock-Paczynski effect as explained in detail in Refs. [73, 74]. Importantly, the East Coast team has used the IR resummation algorithm based on the wiggly-smooth decomposition directly in Fourier space [64, 67, 75], which allowed for a significant boost of computational speed. This scheme is efficient and numerically stable. Moreover, it is based on solid systematic parametric expansion that guarantees that the error is under control at every order of IR resummation. It was explicitly checked that the residuals introduced by our procedure are much smaller than the 2-loop contributions which are not included in the model, in full agreement with theoretical expectations [67, 75]. The labels that indicate IR-Resummation and the AP effect were omitted in all equations in this section to avoid clutter. However, the reader should keep in mind that they are always included in the model.

The total number of nuisance parameters used in the blinded analysis of the East Coast Team is 6: three counterterms (c_0^2 , c_2^2 , c) and three bias parameters (b_1 , b_2 , $b_{\mathcal{G}_2}$). Since the shot noise contribution has been subtracted from the measured spectra, the corresponding parameter was not fitted, in contrast to Ref. [74]. As far

as the cosmological parameters are concerned, the basis that was used consists of the dimensionless Hubble constant h ($H_0 = h \cdot 100$ km/s/Mpc), the physical matter density ω_m , and the normalization $A^{1/2}$ defined with respect to the best-fit Planck value for the base Λ CDM cosmology,

$$A^{1/2} \equiv \left(\frac{A_s}{A_{s, \text{Planck}}} \right)^{1/2}, \quad (19)$$

$$\text{where } A_{s, \text{Planck}} = 2.0989 \cdot 10^{-9}.$$

All varied cosmological and nuisance parameters were assigned flat priors without boundaries, i.e. $(-\infty, \infty)$.

The evaluation of perturbation theory integrals was performed using the FFTLog method of [76] implemented as a module in the CLASS Boltzmann solver [77]. Using the IR-Resummation based on wiggly-smooth decomposition, a single evaluation of a theory model is of the order $\mathcal{O}(1)$ sec for high precision settings. This allows a new evaluation of the non-linear power spectra at every step of the MCMC chain, which is what is done in the East Coast Team analysis. The MCMC analysis was performed using the MONTEPYTHON v3.0 [78, 79] sampler interfaced with the modified version of the CLASS code. The nuisance parameters were sampled in the “fast mode” [80] at a negligible computational cost.

Since the k -binning of the challenge spectra is very wide ($\Delta k = 0.01 h \text{ Mpc}^{-1}$) compared to the fundamental mode of the box, the theoretical predictions had to be properly averaged over each bin. The boundaries of the bins were estimated using the simulation volume, known to both teams. The East Coast Team checked that the estimated boundaries allow one to accurately reproduce the provided weighted means of the k -bins and found that averaging the theory over the bin versus evaluating it in the mean can induce roughly $\mathcal{O}(0.5)\sigma$ shifts in cosmological parameters.

2. West Coast Team

The implementation of the West Coast Team is the result of a long journey where each of ingredients of the EFTofLSS that is necessary to apply it to data was one-by-one subsequently developed, tested on simulations, shown to be successful. Though not all those results are directly used in the analysis, the West Coast Team, and probably nobody, would simply have never applied the model to the data without those intermediate successes. We therefore find it nice to add, in each instance where the EFTofLSS is applied to data, the following footnote where we acknowledge at least a fraction of those important developments².

² The initial formulation of the EFTofLSS was performed in Eulerian space in [62, 63], and then extended to Lagrangian space

The model for the West Coast Team and the analysis techniques are the same as the one used in [29, 111], to which we refer for details. The one-loop redshift-space galaxy power spectrum reads:

$$\begin{aligned}
P_g(k, \mu) = & Z_1(\mu)^2 P_{11}(k) \\
& + 2 \int \frac{d^3 q}{(2\pi)^3} Z_2(\mathbf{q}, \mathbf{k} - \mathbf{q}, \mu)^2 P_{11}(|\mathbf{k} - \mathbf{q}|) P_{11}(q) \\
& + 6 Z_1(\mu) P_{11}(k) \int \frac{d^3 q}{(2\pi)^3} Z_3(\mathbf{q}, -\mathbf{q}, \mathbf{k}, \mu) P_{11}(q) \\
& + 2 Z_1(\mu) P_{11}(k) \left(c_{\text{ct}} \frac{k^2}{k_{\text{M}}^2} + c_{r,1} \mu^2 \frac{k^2}{k_{\text{M}}^2} + c_{r,2} \mu^4 \frac{k^2}{k_{\text{M}}^2} \right) \\
& + \frac{1}{\bar{n}_g} \left(c_{\epsilon,1} + c_{\epsilon,2} \frac{k^2}{k_{\text{M}}^2} + c_{\epsilon,3} f \mu^2 \frac{k^2}{k_{\text{M}}^2} \right). \quad (20)
\end{aligned}$$

k_{M}^{-1} controls the bias derivative expansion and we set it to be $\simeq k_{\text{NL}}^{-1}$, which is the scale controlling the expansion of the dark matter derivative expansion. We set $k_{\text{NL}} = 0.7 h \text{Mpc}^{-1}$. $\bar{n}_g$ is the mean galaxy density.

In the next to the last line of Eq. (20), the term in c_{ct} represents a linear combination of a higher derivative bias [18] that appears in Eq. (12) and the speed of sound of dark matter [62, 63]: $\delta(\vec{k}, t) \supset k^2 \delta_{\text{lin}}(\vec{k}, t)$. The terms in $c_{r,1}$ and $c_{r,2}$ represent the redshift-space counterterms [19]: $\delta_{\text{redshift}}(\vec{k}, t) \supset k^2 \mu^2 \delta(k, t)$, $k^2 \mu^4 \delta(k, t)$. In the last line of Eq. (20), we have the stochastic counterterms: $c_{\epsilon,1}$ and $c_{\epsilon,2}$ originate from Taylor expansion of Eq. (12) [18], while $c_{\epsilon,3}$ originates from the redshift-space expressions [19].

The redshift-space galaxy density kernels Z_1, Z_2 and Z_3 are given in Appendix A. These kernels depend on

the bias coefficients that we define as explained below Eq. (12). By choosing only the linearly-independent ones, this gives rise to the so-called base of descendants. While up to cubic order this base is equivalent to more standard bases, already at quartic perturbative order new terms appear.

The IR-resummation is performed in a numerically efficient way using the original method for configuration and redshift space developed in [17, 65, 66], where all the errors are parametrically controlled by the perturbative order of the calculation (*i.e.* no uncontrolled approximations are present)³.

We define the following combination of parameters: $c_2 = (b_2 + b_4)/\sqrt{2}$, $c_4 = (b_2 - b_4)/\sqrt{2}$, $c_{\epsilon, \text{mono}} = c_{\epsilon,2} + f c_{\epsilon,3}/3$ and $c_{\epsilon, \text{quad}} = 2 f c_{\epsilon,3}/3$. As we analyze only the monopole and the quadrupole, we set $c_{r,2} = 0$ since the two redshift-space counterterms are degenerate in this case, but we allow a larger prior on $c_{r,1}$ to absorb the contribution of $c_{r,2}$ in the quadrupole. Additionally, since the shot noise is known and has been subtracted from the data, we set $c_{\epsilon,1} = 0$. This leaves us with the set $(b_1, c_2, b_3, c_4, c_{\text{ct}}, c_{r,1}, c_{\epsilon, \text{mono}}, c_{\epsilon, \text{quad}})$ of 8 parameters. The PT challenge data are precise enough to determine all EFT parameters with no priors. However, we impose the following priors motivated by the fact that all EFT parameters are expected to be $\mathcal{O}(1)$ ⁴:

$$\begin{aligned}
b_1 \in [0, 4]_{\text{flat}}, \quad c_2 \in [-4, 4]_{\text{flat}}, \quad b_3 \in 10_{\text{gauss}}, \\
c_4 \in 2_{\text{gauss}}, \quad c_{\text{ct}} \in 4_{\text{gauss}}, \quad c_{r,1} \in 8_{\text{gauss}}, \\
c_{\epsilon, \text{mono}} \in 2_{\text{gauss}}, \quad c_{\epsilon, \text{quad}} \in 4_{\text{gauss}}. \quad (21)
\end{aligned}$$

As it is evident from Eqs. (20) and (A1), some EFT-parameters appear linearly in the model power spectrum, and therefore appear quadratically in the Likelihood. If we are not interested in the actual value of these parameters, as it is our case, we can marginalize over these parameters analytically, obtaining a marginalized likelihood that is a function of only 3 parameters: b_1, c_2 and c_4 .

Given that the k -bins ($\Delta k = 0.01 h/\text{Mpc}$) contain many fundamental modes, the West Coast Team averages the predictions of the model over each bin. As a

in [68]. The dark matter power spectrum has been computed at one-, two- and three-loop orders in [17, 63, 81–89]. Some additional theoretical developments of the EFTofLSS that accompanied these calculations were a careful understanding of renormalization [63, 90, 91] (including rather-subtle aspects such as lattice-running [63] and a better understanding of the velocity field [81, 92]), of the several ways for extracting the value of the counterterms from simulations [63, 93], and of the non-locality in time of the EFTofLSS [18, 81, 83]. These theoretical explorations also include an instructive study in 1+1 dimensions [93]. In order to correctly describe the Baryon Acoustic Oscillation (BAO) peak, an IR-resummation of the long displacement fields had to be performed. This has led to the so-called IR-Resummed EFTofLSS [17, 64–67]. A method to account for baryonic effects was presented in [20]. The dark-matter bispectrum has been computed at one-loop in [94, 95], the one-loop trispectrum in [96], and the displacement field in [97]. The lensing power spectrum has been computed at two loops in [98]. Biased tracers, such as halos and galaxies, have been studied in the context of the EFTofLSS in [18, 71, 72, 99–101] (see also [13]), the halo and matter power spectra and bispectra (including all cross correlations) in [18, 99]. Redshift space distortions have been developed in [19, 21, 72]. Clustering dark energy has been included in the formalism in [88, 102–104], primordial non-Gaussianities in [21, 99, 105–108], and neutrinos in [109, 110]. Faster evaluation schemes for evaluation for some of the loop integrals have been developed in [76].

³ Especially within the observational community, a non-linear treatment of the BAO based on the decomposition of the wiggle and smooth part of the power spectrum has been popular for a long time (see for example [112]). However, this Team does not find this decomposition to be under parametric control (*i.e.* there is no small parameter controlling its correctness). It is possible to go from the original IR-Resummation to the simplified ones based on the decomposition by performing a series of approximations (see Appendix of [66]). Of course, this does not mean that the errors which are introduced are large or significant, as can be a-posteriori checked on numerical simulations.

⁴ Notice that the consistency of the EFTofLSS is based on a power counting argument that assumes that the subsequent terms of the perturbative expansion are much smaller than the ones that are kept. In order for this to be the case, it is essential that the physical nuisance parameter are kept $\mathcal{O}(1)$, once the relevant physical scales have been factorized.

check, the Team verified that the provided effective k of the bin was correctly reproduced.

In terms of the cosmological parameters, the West Coast Team has parameterized their analysis in terms of the dimensionless Hubble constant h ($H_0 = h \cdot 100$ km/s/Mpc), the present-day matter density fraction Ω_m , and the normalization of the power spectrum A_s . The evaluation of the perturbation theory integrals were performed either by direct numerical integration, or by the FFTLog method of [76], obtaining the same result.

V. RESULTS OF BLINDED ANALYSIS

In this section we display the results obtained by the two teams. The input values of the cosmological parameters were unblinded after each team has submitted its results for consensus data cuts. We present these results in the original form prepared by either team independently. Both teams have chosen to analyze the mean power spectrum (at $z = 0.61$) over 10 realizations with the covariance estimated from the inverse sum of covariances for 10 single boxes,

$$\bar{C} = \left(\sum_i C_i^{-1} \right)^{-1}, \quad \bar{P} = \bar{C} * \sum_i C_i^{-1} P_i, \quad (22)$$

where P_i , C_i are the power spectrum and covariance of the i 'th box and $\bar{P}$, $\bar{C}$ are the final mean and covariance that have been analyzed.

This procedure ensures that the analysis is approximately equivalent to fitting the spectrum from a single simulation box of $566 (h^{-1}\text{Gpc})^3$ volume. We stress that the obtained statistical errors on cosmological parameters correspond to the total volume of 10 simulation boxes, i.e. $566 (h^{-1}\text{Gpc})^3$.

A. East Coast Team

Although the East Coast Team submitted its baseline results for the average over 10 challenge boxes at $z = 0.61$, they have also analyzed the data for other redshifts and found consistent results across all challenge spectra. Prior to unblinding, the East Coast Team has submitted results for 8 different evenly-spaced values of k_{max} in the range $(0.08 - 0.2) h \text{ Mpc}^{-1}$.

The marginalized posteriors for the three cosmological parameters are shown in Fig. 3 for several choices of k_{max} . Between $k_{\text{max}} = 0.08 h \text{ Mpc}^{-1}$ and $k_{\text{max}} = 0.14 h \text{ Mpc}^{-1}$ the different contours are compatible within 1σ . When pushing to higher values of k_{max} , the shifts in the central values of the posterior distributions become significant. Note that for $k_{\text{max}} > 0.14 h \text{ Mpc}^{-1}$ the contours of h and ω_m remain consistent even though the other parameter exhibit clear shifts. The East Coast Team quoted its final results for a conservative choice of $k_{\text{max}} = 0.12 h \text{ Mpc}^{-1}$ because this is the scale up to which the Team believed

the theoretical modeling is sufficiently accurate given sub-percent statistical error bars and the size of neglected nonlinear corrections (see Fig. 4, in which we display an estimate of the two-loop correction from [113]). The 1d marginalized limits for the cosmological and parameters and the linear bias b_1 are given in Table I. After the true parameters were unblinded, the values obtained by the East Coast Team were replaced by relative differences. For convenience, the values of σ_8 , Ω_m and $\ln(10^{10} A_s)$ derived from the East Coast Team MCMC chains are also quoted. As we have seen after unblinding, the true values of ω_m and h reside within 2σ posterior regions even at $k_{\text{max}} = 0.2 h \text{ Mpc}^{-1}$, while the clustering amplitude measurement is consistent up to $k_{\text{max}} = 0.14 h \text{ Mpc}^{-1}$. Importantly, the Team has also inferred a correct value of the linear bias⁵ coefficient b_1 .

Fig. 4 shows the comparison of the best-fit model at $k_{\text{max}} = 0.12 h \text{ Mpc}^{-1}$ to the data and the residuals. The quality of the fit is quite good, $\chi^2/\text{dof} = 12/(24 - 9)$. It is consistent with the hypothesis that the data follow the χ^2 -distribution with 15 degrees of freedom. The lower panel of Fig. 4 displays a breakdown of different contributions to the best-fit model. The linear theory contribution dominates on all scales, which is consistent with the applicability of perturbation theory. Towards $k_{\text{max}} = 0.12 h \text{ Mpc}^{-1}$ the loop corrections (including the k^2 -counterterms) become progressively important. Note that the one loop corrections are detectable already on very large scales, $\sim 0.02 h \text{ Mpc}^{-1}$. The k^4 -counterterm is important only for the quadrupole around $k_{\text{max}} = 0.12 h \text{ Mpc}^{-1}$, where it dominates over the other loop corrections.

B. West Coast Team

As specified before, the West Coast Team has analyzed the mean over the 10 boxes in the high redshift bin at $z = 0.61$, using the covariance on the mean. Originally, for the purpose of parameter estimation, the Team presented the results up to $k_{\text{max}} = 0.12 h \text{ Mpc}^{-1}$ since this is the k_{max} at which the Team predicted the estimates to be still unbiased. The marginalized posteriors for the cosmological parameters are shown in Fig. 5, and best fit and means are listed in Table II. When the true results were revealed, it is found that A_s and H_0 lie within the 1σ region of the estimates of the West Coast Team, and Ω_m within the 1.5σ region. b_1 is also correctly reproduced within the 1σ interval. Additionally, one can see that the pre-unblinding results at $k_{\text{max}} = 0.14 h \text{ Mpc}^{-1}$,

⁵ The true value of the linear bias was estimated as follows. The Japan team has measured the real space galaxy-galaxy and matter-matter auto-spectra along with the galaxy-matter cross spectrum. Then, we took the average over three estimators for linear bias, $b_1^{(1)} = P_{gg}/P_{gm}$, $b_1^{(2)} = (P_{gg}/P_{mm})^{1/2}$, $b_1^{(3)} = P_{gm}/P_{mm}$ evaluated in the very first k -bin.

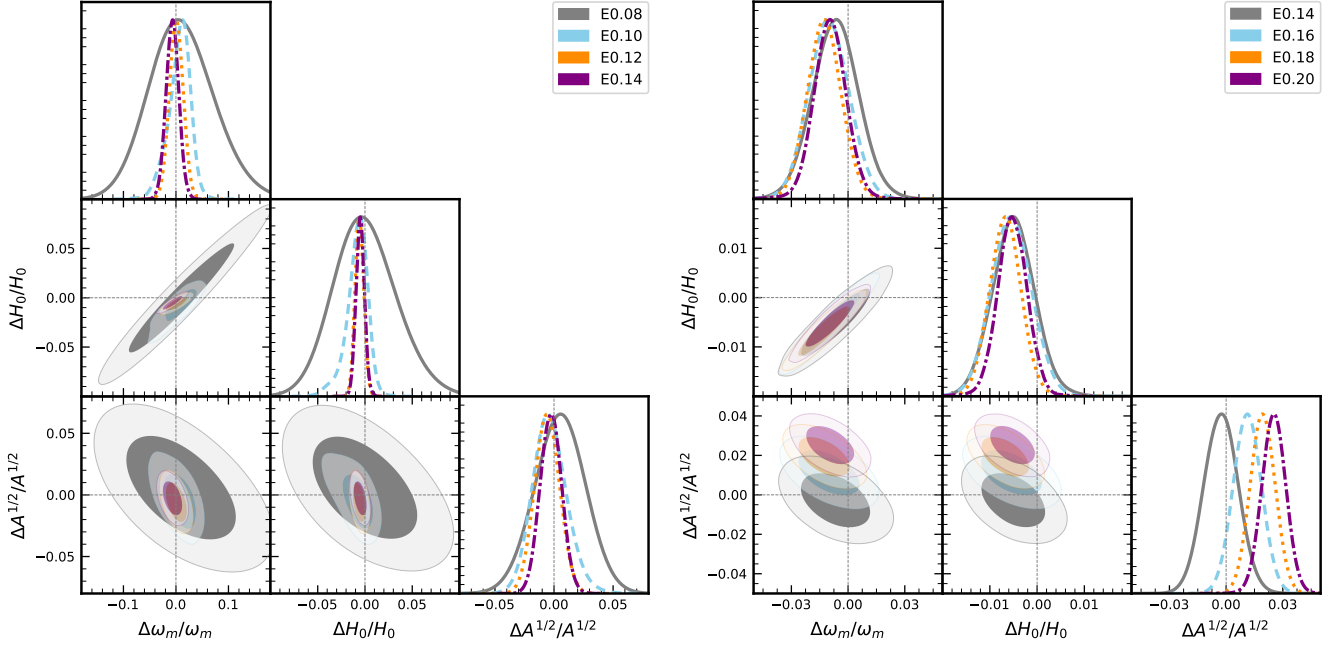


FIG. 3. Marginalized posteriors for the three varied cosmological parameters as a function of $k_{\max}$ (quoted in $h \text{ Mpc}^{-1}$ in the figure legend) obtained by the East Coast Team. Dashed lines mark the input parameters which were revealed once the Team submitted its final result.

$k_{\max} = 0.12 \text{ h Mpc}^{-1}$	best-fit	mean $\pm 1\sigma$
$\Delta A^{1/2}/A^{1/2} \cdot 10^2$	-0.15	-0.16 ± 1.0
$\Delta h/h \cdot 10^2$	-0.55	-0.59 ± 0.46
$\Delta \omega_m/\omega_m \cdot 10^2$	0.2	0.15 ± 1.4
$\Delta b_1/b_1 \cdot 10^2$	1.2	1.2 ± 1.2
$\Delta \Omega_m/\Omega_m \cdot 10^2$	1.3	1.2 ± 0.9
$\Delta \ln(10^{10} A_s)/\ln(10^{10} A_s) \cdot 10^2$	-0.098	-0.11 ± 0.69
$\Delta \sigma_8/\sigma_8 \cdot 10^2$	-0.094	-0.022 ± 0.92

TABLE I. The baseline results obtained by the East Coast Team for $k_{\max} = 0.12 h \text{ Mpc}^{-1}$ at $z = 0.61$. Only the cosmological parameters and b_1 are shown. Note that Ω_m , $\ln(10^{10} A_s)$ and σ_8 in the lower disjoint table shows the results for derived parameters.

which however was not the $k_{\max}$ at which the Team anticipated to be most accurate, are even closer to the true values.

In Fig. 6 the Team shows that the data are well fitted by the theoretical model with the best-fit parameters, with $-2 \log \mathcal{L}/\text{dof} = 16/(24 - 6)$, corresponding to a very good p -value⁶. In the lower panel, different contributions to the best fit power spectra are shown, to check the self-consistency of the perturbative expansion. It is apparent that the one-loop term is safely less than 10% of the linear one at all k 's. In addition to the one-loop term, an estimate of the two-loop contribution, i.e. $P_{1\text{-loop}}^2/P_{\text{lin}}$, is shown: clearly, at least for the quadrupole, this esti-

mate is of the order of the error on the data at the highest k . This is an additional indication that for roughly $k_{\max} \gtrsim 0.12\text{--}0.14 h \text{ Mpc}^{-1}$ the one-loop model will not be an accurate description of the data, and parameter estimation will suffer from theory systematics.

After *unblinding*, the West Coast Team submitted additional results at $k_{\max} = 0.14, 0.16, 0.18, 0.20 h \text{ Mpc}^{-1}$. This is because it was subsequently decided that it was interesting to explore the $k_{\max}$ -dependence of the theory-systematic error. In fact, though this has already been analyzed by the Team in both their original papers [27, 29], the challenge simulation is different and its volume larger. At the higher $k_{\max}$'s, the Team performs the (analytical) marginalization over the additional $c_{\epsilon, \text{mono}}$ parameter, with a Gaussian prior with $\sigma_{c_{\epsilon, \text{mono}}} = 2$. The effect of adding this parameter is completely negligible at low $k_{\max}$: in fact, the Team chose

⁶ Notice that the Likelihood of this team is not Gaussian.

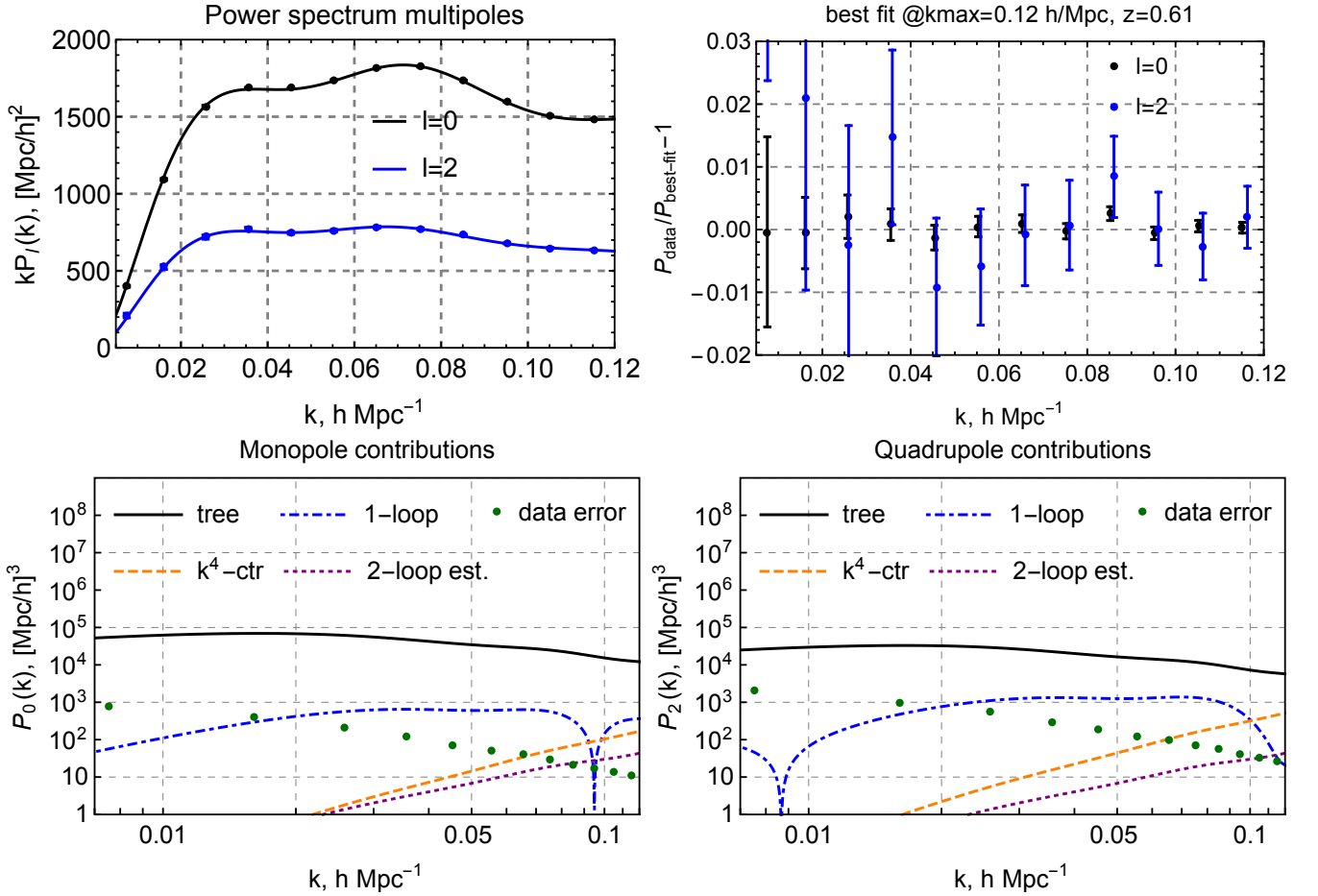


FIG. 4. *Upper panel:* Comparison of the data for the monopole and the quadrupole (the error bars are there, albeit barely visible) with the best-fit model (left panel) obtained by the East Coast Team. The residuals for the monopole and the quadrupole for the best-fit model with $\chi^2/\text{dof} = 12/(24 - 9)$ (right panel). Note that the quadrupole data points are slightly shifted for better visibility. *Lower panel:* Different contributions to the monopole (left panel) and quadrupole (right panel) power spectra. The data errors and the two-loop estimate are also displayed. We plot the absolute values, some terms are negative.

to safely set it to zero for the original chains. Indeed one can check that the results are unchanged at low k_{max} when adding this parameter. However, because of the small error bars of the simulation data, at higher k_{max} this parameter has to be added to the model.

The trend as a function of k_{max} is apparent from Fig. 5. Ω_m and H_0 are well recovered up to $k_{\text{max}} = 0.18 h \text{ Mpc}^{-1}$, approximately within the $1\text{-}\sigma$ region, the estimate of clustering amplitude A_s starts to deviate significantly from the true value after $k_{\text{max}} \gtrsim 0.14 h \text{ Mpc}^{-1}$.

C. Comparison of the two analyses

So far we have presented the analyses done by two teams. We now compare the two and discuss how different model assumptions lead to the different cosmological-parameter constraints.

First, since the two teams employ different sets of cos-

mological parameters as the varied parameters, a direct comparison between Figs. 3 and 5 is not very clear. We stick here instead to the parameter space (Ω_m, H_0, A_s) to see the constraints. We first show in Fig. 7 the one dimensional marginalized error on these parameters as a function of the maximum wavenumber, k_{max} , used in the analysis. The $1\text{-}\sigma$ credible intervals by the East (West) Coast Team are shown by the upward (downward) triangles with error bars. Also shown by the shades are the same intervals but scaled for the SDSS BOSS DR12 according to the ratio of the simulated and the observed volume⁷.

Overall, the ground truth values of the three cosmological parameters stay within or slightly off from the $1\text{-}\sigma$ interval up to $k_{\text{max}} = 0.14 h \text{ Mpc}^{-1}$. The inferred

⁷ We adopt the total volume of SDSS BOSS DR12, $5.7(h^{-1}\text{Gpc})^3$, instead of that of CMASS2.

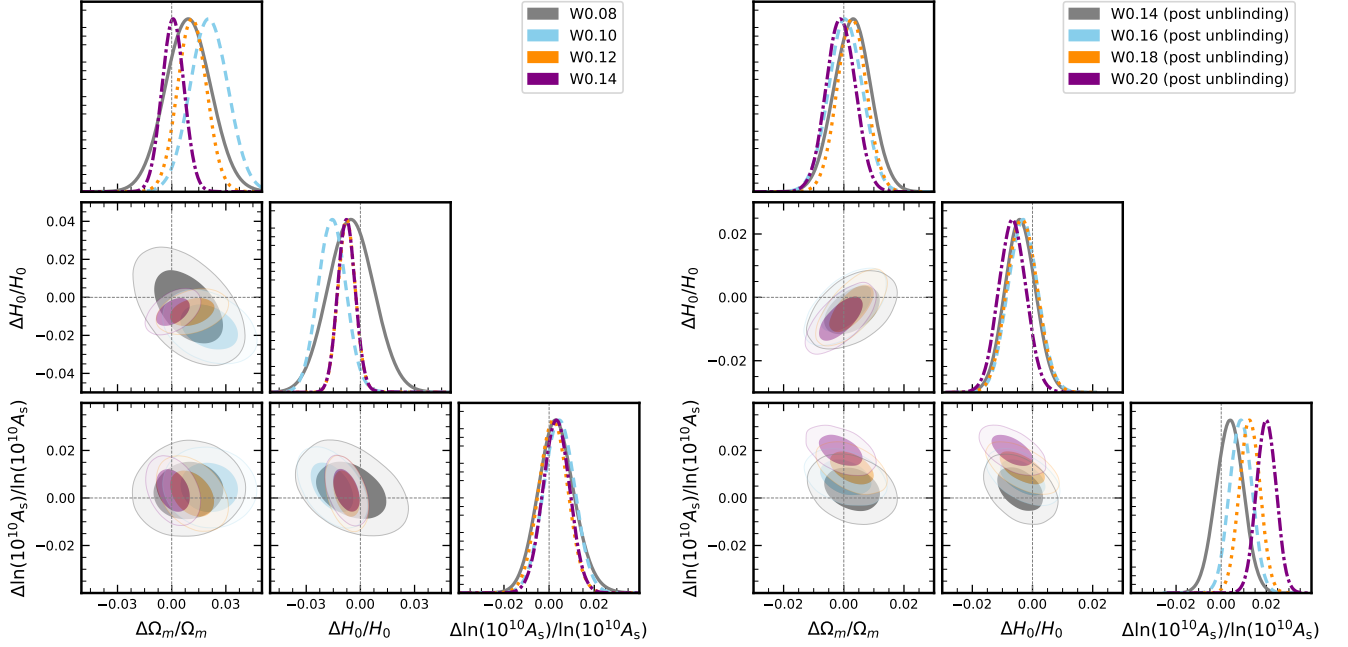


FIG. 5. Marginalized posteriors for the three varied cosmological parameters as a function of $k_{\max}$ (quoted in $h \text{ Mpc}^{-1}$ in the figure legend) obtained by the West Coast Team. Dashed lines mark the input parameters which were revealed once the Team submitted its final result, similarly to Fig. 3.

Param	best-fit	mean $\pm\sigma$
$\Delta\Omega_m/\Omega_m \cdot 10^2$	1.3	$1.2^{+0.8}_{-0.8}$
$\Delta h/h \cdot 10^2$	0.7	$0.6^{+0.6}_{-0.6}$
$\Delta \ln(10^{10} A_s)/\ln(10^{10} A_s) \cdot 10^2$	0.1	$0.1^{+0.7}_{-0.7}$
$\Delta b_1/b_1 \cdot 10^2$	0.8	$0.7^{+1.0}_{-1.1}$

TABLE II. Similar to Table I, but the results obtained by the West Coast Team for $k_{\max} = 0.12 h \text{ Mpc}^{-1}$ at $z = 0.61$. Only cosmological parameters and b_1 are shown.

primordial scalar amplitude, A_s , in particular, is always within the interval up to this $k_{\max}$ from both teams. On the one hand, A_s starts to deviate from the ground truth in a systematic way with statistical significance above this $k_{\max}$. This is consistent with the expectation that two-loop corrections become important at these scales. On the other hand, H_0 and Ω_m stay roughly within 1σ from the true value all the way up to $k_{\max} = 0.2 h \text{ Mpc}^{-1}$. However, if one focuses on the shaded regions corresponding to the statistical error from the actual BOSS survey, the ground truth values are always well within the 1σ interval, which justifies the $k_{\max}$ choice of the analyses from the same teams in Refs. [27–29].

While the size of the error bars shrinks towards higher $k_{\max}$, the gain is small after $k_{\max} \gtrsim 0.14 h \text{ Mpc}^{-1}$. This could be caused by the combination of two effects. Firstly, the relative contribution of the shot noise in the data covariance becomes important. Secondly, the EFT parameters controlling the nonlinear corrections become important in such a way that the additional information coming from small-scale modes mainly determines these

parameters rather than the cosmological parameters. If one looks into the trend in the error bars more in detail, the results from the two teams are clearly different, especially when $k_{\max} \lesssim 0.1 h \text{ Mpc}^{-1}$, up to factor ~ 2 smaller by the West Coast Team. This difference is driven by the prior treatment. The East Coast Team had no priors on the chosen set of nuisance parameters, whereas the West Coast Team has always kept the nuisance parameters within physically-motivated bounds. Thus, the observed difference of the results between the two teams implies that on scales larger than $0.1 h \text{ Mpc}^{-1}$ the data are not good enough to break degeneracies between the cosmological and nuisance parameters. These degeneracies get broken at larger wavenumbers, where the results of the two teams agree regardless of the nuisance parameters' priors.⁸ We discuss the issue of priors by the East Coast Team in the low $k_{\max}$ analyses in Appendix B.

⁸ Note that at $k_{\max} = 0.1 h \text{ Mpc}^{-1}$ both teams have measured Ω_m to be 1.6σ higher than the true value. However, as one can

Let us briefly discuss some cosmological implications of our blinded analysis. The cosmological information probed by redshift galaxy surveys can be crudely divided into four different categories:

- **Shape information.** The shape of the galaxy power spectrum is controlled by the physical matter density ω_m . This parameter is measured from the data regardless of the choice of rulers such as H_0 . ω_m is extracted from the features of the power spectrum, such as the form of the BAO peaks, the baryonic suppression, the turnover, and the overall slope.
- **Distance information,** mainly encoded through the volume-average distance⁹ $D_V(z)$. This parameter, essentially, controls the freedom to shift the power spectrum along the k axis. In the flat Λ CDM framework this distance depends only on two cosmological parameters, ω_m and H_0 . Since ω_m is measured from the shape, the constraint on D_V translates directly into a constraint on H_0 . Note that Ω_m in this picture can be seen as a parameter derived from a combination of the shape and distance parameters.
- **Redshift space distortions.** Observing galaxies in redshift space allows one to measure unbiased rms velocity fluctuation $f\sigma_8(z) = f(z)D_+(z)\sigma_8$. In Λ CDM D_+ and f depend only on Ω_m , which is constrained from the shape and the distance. This way the RSD measurements constrain directly A_s .
- **The Alcock-Paczynski geometric distance information.** The AP effect allows one to measure the combination $H(z)D_A(z)$. However, in Λ CDM this combination is a slow function of cosmological parameters at small redshifts. Thus, it does not contribute significantly to the overall constraints on Ω_m , see Ref. [28] for more detail.

We can see in Fig. 8 that indeed our results are fully in line with these theoretical expectations. First, let us focus on the two-dimensional posterior in the $(\Omega_m - H_0)$ plane. The change in the degeneracy direction is observed to rotate with increasing the maximum wavenumber. When $k_{\max} = 0.08 h \text{ Mpc}^{-1}$, Ω_m and H_0 are negatively correlated. At the other end, the correlation turns to be a positive one for $k_{\max} = 0.16$ and $0.2 h \text{ Mpc}^{-1}$. We can interpret this as the outcome of

see from the results of the East Coast Team, the measured value of ω_m is perfectly within 1σ from the truth, but h is shifted from the truth by almost 1σ . Since $\Omega_m = \omega_m/h^2$ the 1σ bias in h causes the 2σ bias in ω_m . Thus, this bias can be viewed as a marginalization effect.

⁹ It is defined as $D_V(z) = (z(1+z)^2 D_A^2(z)/H(z))^{1/3}$, where $D_A(z) = \frac{1}{1+z} \int_0^z \frac{dz'}{H(z')}$ and $H^2(z) = H_0^2(\Omega_m(1+z)^3 + 1 - \Omega_m)$ in flat Λ CDM.

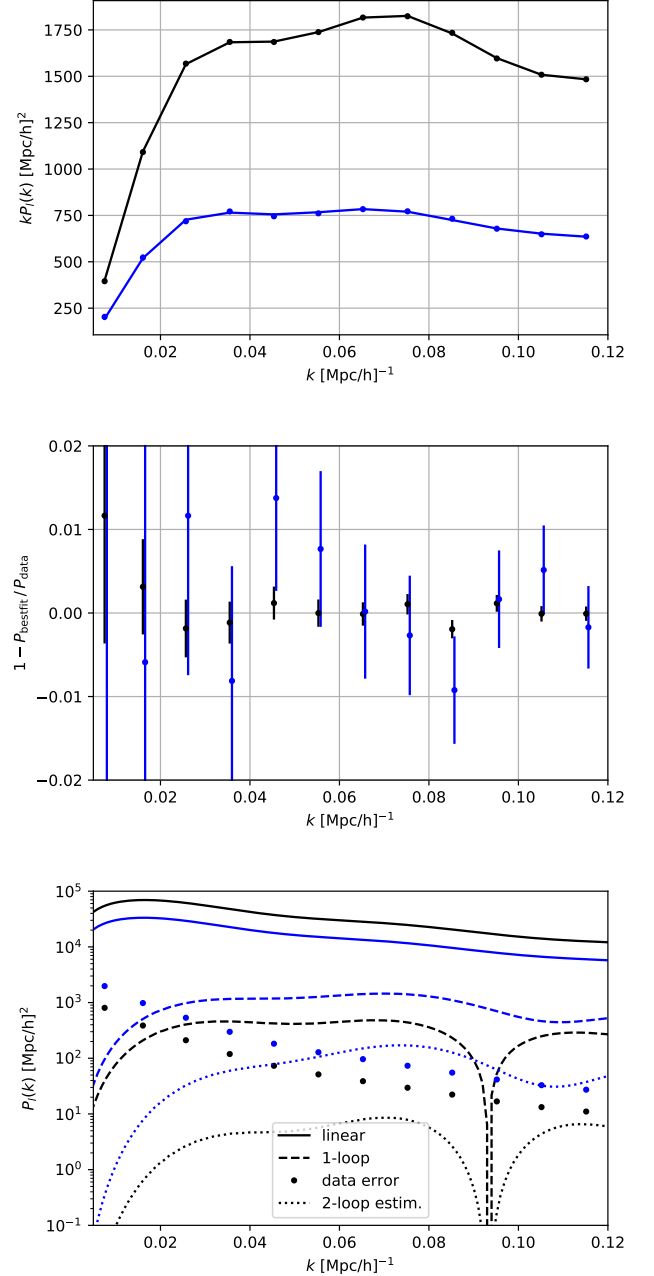


FIG. 6. *Upper panel:* Comparison of the data for the monopole (black) and the quadrupole (blue) with the best-fit model obtained by the West Coast Team. *Middle panel:* Residuals for the monopole and the quadrupole for the best-fit model with the partially-marginalized Likelihood giving $-2\log \mathcal{L}/\text{dof} = 16/(24 - 6)$ for $k_{\max} = 0.12 h \text{ Mpc}^{-1}$. *Lower panel:* Different contributions to the monopole and quadrupole power spectra. We plot just the absolute values, some terms are negative.

the change in the relative importance of the BAO feature. Although the first BAO peak is already included

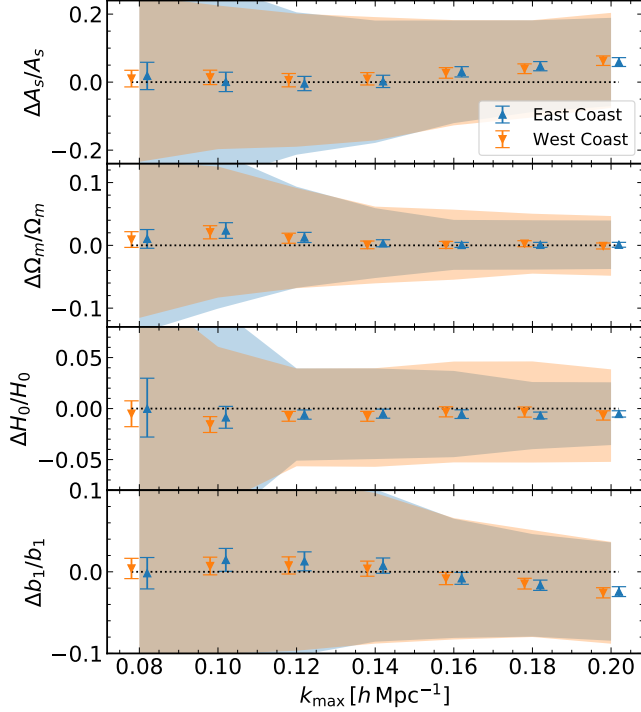


FIG. 7. One dimensional marginalized posterior distributions for the three main cosmological parameters as well as the linear bias parameter as a function of the maximum wavenumber $k_{\max}$ considered in the analysis. The 68% credible intervals derived by the East and West Coast Team are shown respectively by the blue and red error bars with the mean marked by the upward and downward triangles, respectively. Overplotted by the shaded regions are those scaled to the volume of SDSS DR12. The error bars are slightly shifted horizontally to avoid a heavy overlap.

at $k_{\max} = 0.08 h \text{ Mpc}^{-1}$, the dominant constraint is coming from the overall shape information, e.g., the matter-radiation equality scale ($\theta_{\text{eq}} = 1/(k_{\text{eq}} D_V) \propto \Omega_m^{-0.83} h^{-1}$, where k_{eq} denotes the equality wavenumber) at this maximum wavenumber. Indeed, the contours from the two teams are roughly oriented along this direction depicted by the red dashed line. At $k_{\max} = 0.2 h \text{ Mpc}^{-1}$, as we can clearly see the BAO feature up to the third peak (see Fig. 2), the BAO scale (the blue dashed line in Fig. 8: $\theta_{\text{BAO}} = r_s/D_V$ with the sound horizon scale r_s) plays a more significant role. The measurement of the relative location of these two characteristic scales allow us to determine the physical density $\omega_m = \Omega_m h^2$, and together with the distance measurement through cosmology dependence of the redshift-distance conversion (i.e., a measurement of D_V), we can break the degeneracy between Ω_m and H_0 .

Once $D_V(z)$ and ω_m are fixed, the other parameters such as the distance parameters, $H(z)$, $D_A(z)$ (with h kept in the unit as $h \text{ Mpc}^{-1}$ or $h^{-1} \text{ Mpc}$) or the growth parameter, $f(z)$, are merely dependent parameters fully

determined by Ω_m given that we stick to the flat Λ CDM cosmology. Had we fitted the data with a more general expansion model, e.g. dynamical dark energy or modified gravity models, the posterior distribution of these parameters would have been different. These parameters extracted from our MCMC chains, together with some other useful parameters, are displayed in Fig. 9.

Apart from the shape-related parameters, the determination of the amplitude parameter is of interest. We can see in Fig. 8 that the posterior of the amplitude parameter, A_s , is strongly correlated with the linear bias parameter b_1 . To understand this more clearly, we show the constraints on the parameters relevant for the measurement of RSD (the one-dimensional and the two-dimensional marginalized posterior in Figs. 9 and 10, respectively). In the two-dimensional contour plot, we can see that the amplitude parameter scaled to the redshift of the survey volume, $\sigma_8(z) = [D_+(z)/D_+(z=0)] \sigma_8$, is strongly degenerate with the linear bias parameter, b_1 , just as we have seen for A_s and b_1 . In fact, they are expected to be fully degenerate in the absence of RSD information in linear theory. We can also see in Fig. 7 that b_1 starts to depend weakly on $k_{\max}$ above $\sim 0.14 h \text{ Mpc}^{-1}$ with statistical significance, and a similar departure from the ground truth value happens at the same place but to the opposite direction in $\sigma_8(z)$ as shown in Fig. 9. The other famous degeneracy directions, $f\sigma_8$ or f/b , which are the direct observables from linear RSD, do not appear in our contours in Fig. 10. This is again due to the fact that the flat Λ CDM assumption makes f a dependent variable fully determined by Ω_m . What we see here is that the constraint on Ω_m through the shape and distance measurement discussed above, combined with the measurement of $f\sigma_8$ from RSD, allows us to constrain σ_8 (and thus A_s) directly.

VI. CONCLUSION AND OUTLOOK

In this paper we have presented results of the blinded cosmology challenge initiated to test theoretical models for redshift-space galaxy clustering. The task was to assess whether the theoretical model, here EFTofLSS, can recover the blinded cosmological parameters in N-body simulation from the mock data of redshift-space power spectrum multipoles for BOSS-like galaxies. The sufficiently large volume, dynamical range and high resolution of the challenge simulation allow one to pin down any potential inaccuracy of theoretical modeling, compared to the statistical errors for the BOSS-like survey.

The simulations were run by a team (“Japan Team”) that kept the true parameters in secret. The mock data were analyzed by two other independent teams (“East Coast Team” and “West Coast Team”) who volunteered to participate in the challenge. The rule of the challenge is that the true parameters can be unblinded only when the analyzing teams submit their final results to the simulation team. All the three teams agreed that the

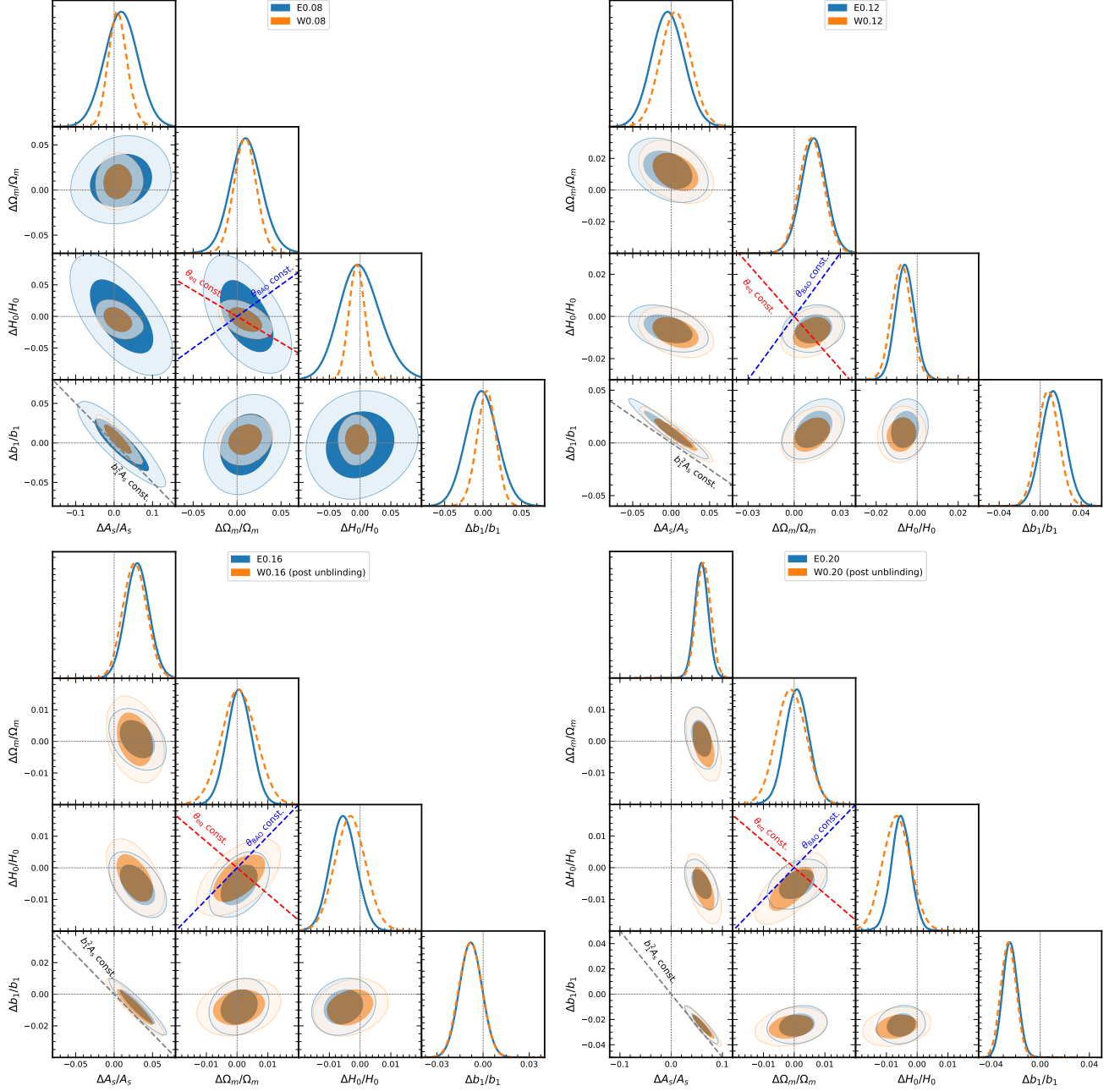


FIG. 8. Two dimensional marginal posterior distributions for the three main cosmological parameters and the linear bias parameter. The 68% and 95% credible intervals derived by the East and West Coast Team are shown respectively by the cyan and orange contours. The corresponding one dimensional marginal distributions are shown in the diagonal panels by the solid and dashed lines. The maximum wave number included in this analysis is $k_{\text{max}} = 0.08$ (upper left), 0.12 (upper right), 0.16 (lower left) and $0.2 h \text{ Mpc}^{-1}$ (lower right). Three degeneracy directions for some parameter combinations are also displayed in the contour panels by the thick dashed lines (see text for more detail).

submitted results be presented in this paper, without any change, after the unblinding.

Both analyzing teams used the same theoretical model based on the effective field theory of large-scale structure. However, there exist some nontrivial differences, whose impact on the final cosmological inference should

be tested quantitatively with care. The corresponding pipelines were the ones applied to the real BOSS data in Refs. [27–29]. We have discussed in detail methodological and technical differences between these two pipelines. Despite these differences, both teams have successfully recovered the true cosmological parameters within ex-

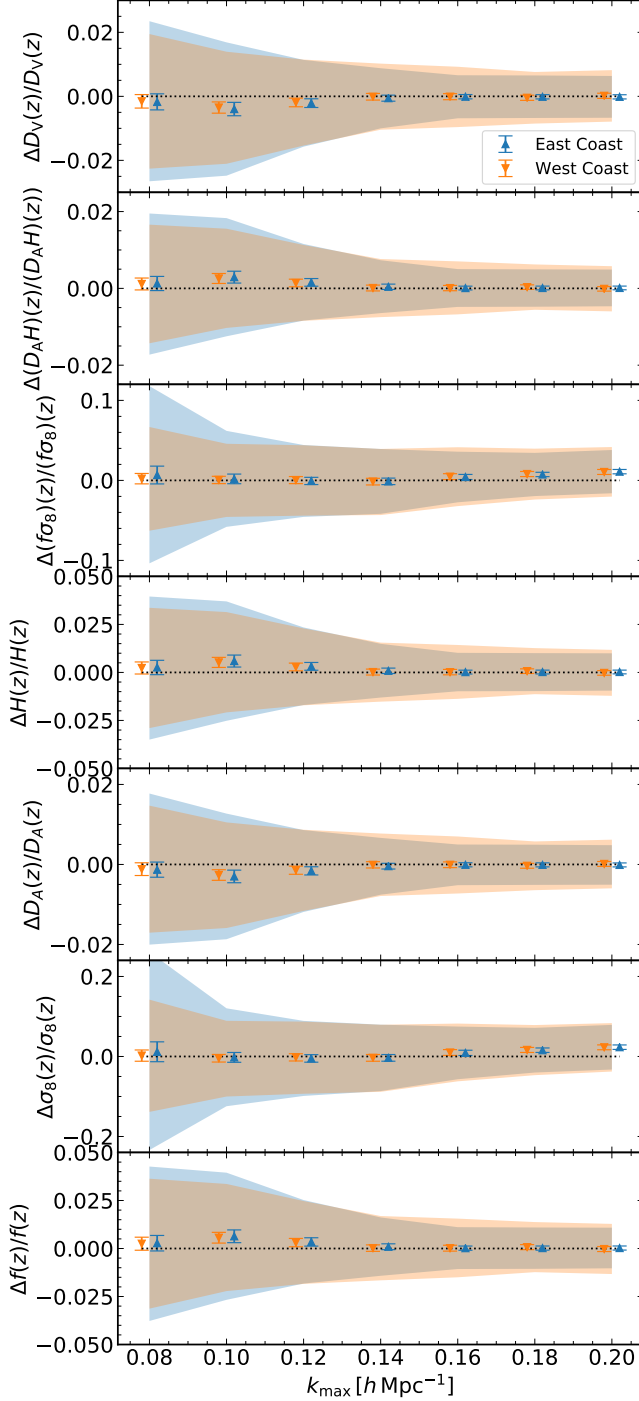


FIG. 9. One dimensional marginalized posterior distributions of *derived* parameters for flat Λ CDM model as a function of the maximum wavenumber included in the analysis, $k_{\max}$. The fractional error is shown with the uncertainty in H_0 that is kept in the unit for the distance parameters (i.e., D_A is expressed in $h^{-1}\text{Mpc}$ and H is in $h\text{Mpc}^{-1}$).

pected statistical error bars. This suggests that perturbation theory, once consistently implemented, can be used

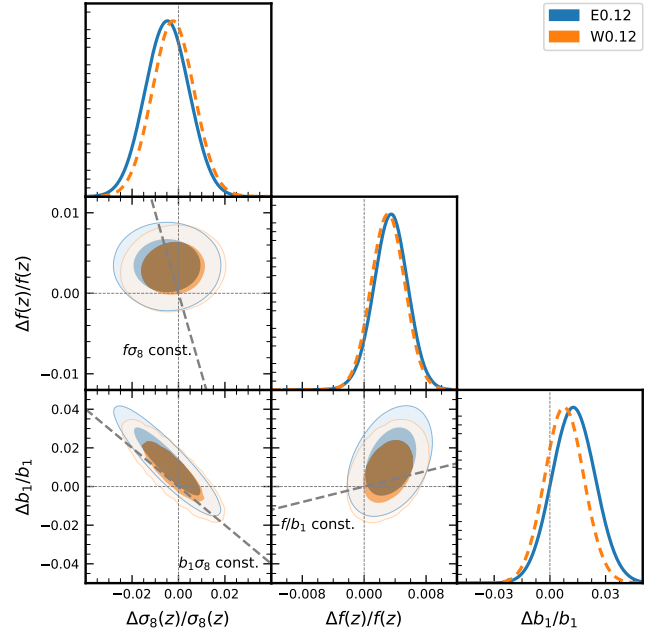


FIG. 10. Two dimensional marginalized posterior distributions for amplitude-related parameters relevant for the RSD measurement from the analyses at $k_{\max} = 0.12 h\text{Mpc}^{-1}$. The expected degeneracy directions, $f\sigma_8$, $b_1\sigma_8$ or f/b_1 is constant, expected from linear RSD measurements are shown by the dashed lines. Note that $f(z)$ and $\sigma_8(z)$ are derived parameters fully fixed once Ω_m and A_s are given within the flat Λ CDM model.

as a standard tool for unbiased estimation of cosmological parameters from galaxy surveys.

We are currently exploring a number of various post-blinded research directions. The first one includes a thorough investigation of the information content of redshift galaxy surveys. Second, it would be curious to see how much the $k_{\max}$ value where one-loop perturbation theory breaks down depends on the properties of the galaxy population, i.e. assembly bias or satellite fraction. Third, it will be interesting to see how well perturbation theory performs for other observables, e.g. the galaxy-galaxy weak lensing or the redshift-space bispectrum. These research avenues are left for future work.

We have presented the results obtained by analyzing teams in the way such that the true parameters are still blinded to the readers. This is done in case some other researchers would like to test their theory models on the challenge spectra. All challenge data are available online at <http://www2.yukawa.kyoto-u.ac.jp/~takahiro.nishimichi/data/PTchallenge/>. We encourage all groups working on galaxy clustering analysis to participate in the challenge.

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Appendix A: Galaxy kernels

The explicit expressions for the galaxy kernels appearing in the one-loop power spectrum are given here (see for a derivation [72]):

$$\begin{aligned} Z_1(\mathbf{q}_1) &= K_1(\mathbf{q}_1) + f\mu_1^2 G_1(\mathbf{q}_1) = b_1 + f\mu_1^2, \\ Z_2(\mathbf{q}_1, \mathbf{q}_2, \mu) &= K_2(\mathbf{q}_1, \mathbf{q}_2) + f\mu_{12}^2 G_2(\mathbf{q}_1, \mathbf{q}_2) \\ &\quad + \frac{1}{2}f\mu q \left(\frac{\mu_2}{q_2} G_1(\mathbf{q}_2) Z_1(\mathbf{q}_1) + \text{perm.} \right), \\ Z_3(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mu) &= K_3(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) + f\mu_{123}^2 G_3(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) \\ &\quad + \frac{1}{3}f\mu q \left(\frac{\mu_3}{q_3} G_1(\mathbf{q}_3) Z_2(\mathbf{q}_1, \mathbf{q}_2, \mu_{123}) \right. \\ &\quad \left. + \frac{\mu_{23}}{q_{23}} G_2(\mathbf{q}_2, \mathbf{q}_3) Z_1(\mathbf{q}_1) + \text{cyc.} \right), \quad (\text{A1}) \end{aligned}$$

where here $\mu = \mathbf{q} \cdot \hat{\mathbf{z}}/q$, $\mathbf{q} = \mathbf{q}_1 + \dots + \mathbf{q}_n$, and $\mu_{i_1 \dots i_n} = \mathbf{q}_{i_1 \dots i_n} \cdot \hat{\mathbf{z}}/q_{i_1 \dots i_n}$, $\mathbf{q}_{i_1 \dots i_m} = \mathbf{q}_{i_1} + \dots + \mathbf{q}_{i_m}$, with $\hat{\mathbf{z}}$ being the unit vector in the direction of the line of sight, and n is the order of the kernel Z_n . K_i and G_i are the galaxy density and velocity kernels, respectively. We choose to work in the basis of descendants (this is the first complete set of bias coefficient for LSS, established in [18, 99] and with some typos corrected in [100]; see [18, 99] for connection to former bases of bias coefficients, as for example [13]). Notice that while the new terms introduced in [18] happen to be degenerate with

the standard bias terms at one-loop order, this will not be the case anymore once one goes to higher orders. For the one-loop power spectrum, all kernels can be described with 4 bias parameters b_i .

The first and second order galaxy density kernel are:

$$\begin{aligned} K_1 &= b_1, \quad (\text{A2}) \\ K_2(\mathbf{q}_1, \mathbf{q}_2) &= b_1 \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_1^2} + b_2 \left(F_2(\mathbf{q}_1, \mathbf{q}_2) - \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_1^2} \right) \\ &\quad + b_4 + \text{perm.} \end{aligned}$$

The galaxy velocity kernels G_n are simply the standard perturbation theory ones since the galaxy velocity field follows the dark matter velocity field, up to higher-derivative terms which are degenerate with other counterterms that appear in the renormalization of the redshift space expression (see e.g. [114] for the expressions of F_n and G_n).

The third-order galaxy density kernel has a much more involved expression. However, for the one-loop calculation, degeneracies appear in the one-loop diagram obtained from $\langle \delta^{(3)} \delta^{(1)} \rangle$, when UV-divergences are removed and the integral over the angular coordinates is performed, leading to the following simple expression:

$$\begin{aligned} K_3(k, q) &= \frac{b_1}{504k^3q^3} (-38k^5q + 48k^3q^3 - 18kq^5) \quad (\text{A3}) \\ &\quad + 9(k^2 - q^2)^3 \log \left[\frac{k - q}{k + q} \right] \\ &\quad + \frac{b_3}{756k^3q^5} (2kq(k^2 + q^2)(3k^4 - 14k^2q^2 + 3q^4) \\ &\quad + 3(k^2 - q^2)^4 \log \left[\frac{k - q}{k + q} \right]). \end{aligned}$$

Appendix B: A comment on priors by the East Coast Team

As mentioned, the West Coast team imposed priors on the nuisance parameters that are physically motivated, apart for setting to zero (which is an infinitely strong prior) the parameters that could not be measured. This last case is what happened for the term proportional to $c_{\epsilon, \text{mono}}$, which is included only at higher wavenumbers. As argued in the former section, the West Coast team believes this is a way to consistently match the EFTofLSS to data. In this Appendix, the East Coast Team advocates an alternative method of efficiently implementing the priors for nuisance parameters. This approach is presented here as part of post-unblinding study.

In its baseline analysis, the East Coast Team has assumed no priors on the chosen nuisance parameters. This resulted in large errors on the cosmological parameters when the analysis was restricted to very low k_{max} cuts. Let us now discuss a practical data-driven way to impose priors on nuisance parameters without biasing the inferred cosmology. In order to keep the nuisance parameters blinded to the reader, we have shifted each of them

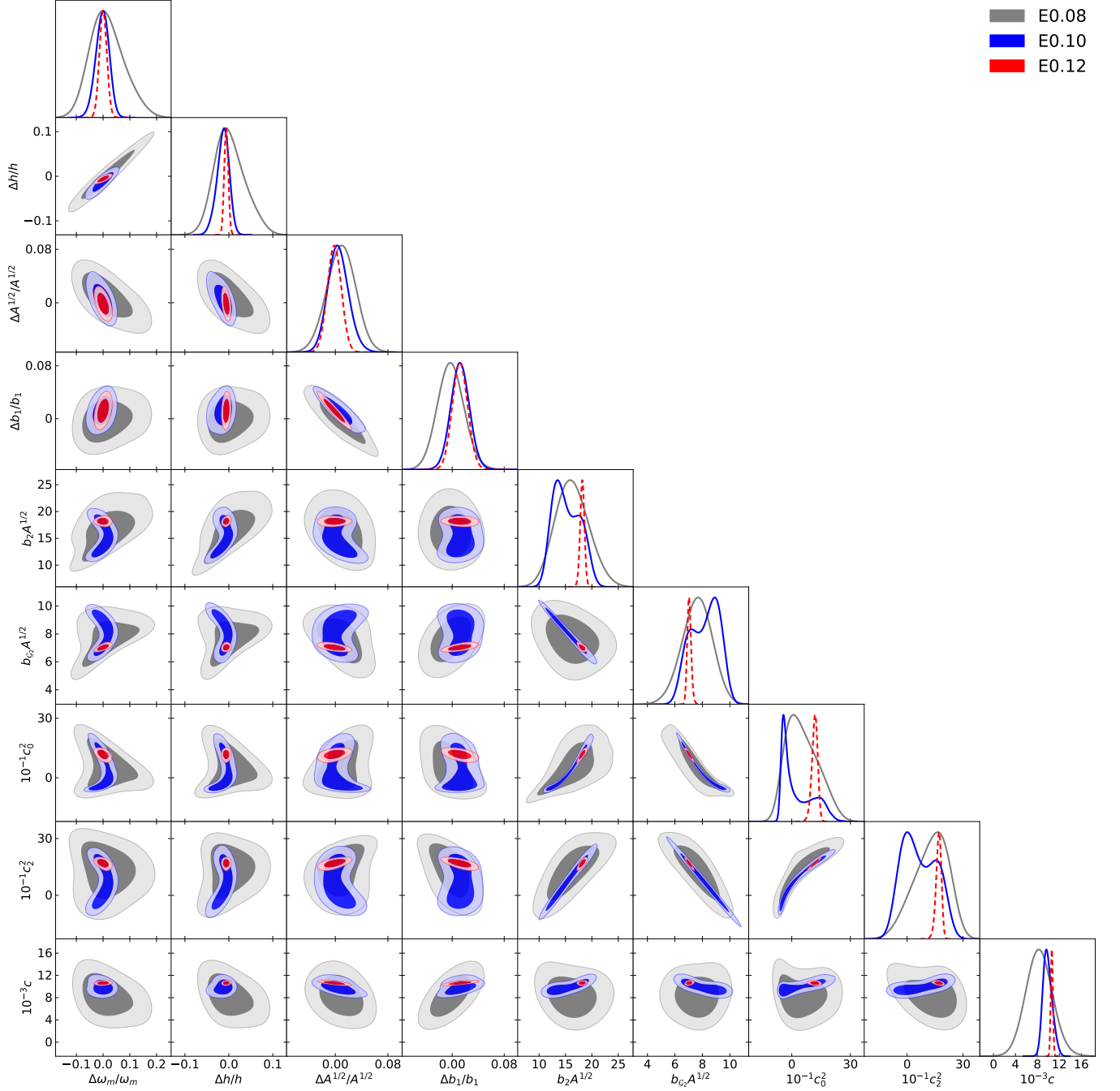


FIG. 11. Posterior distribution of cosmological and nuisance parameters obtained by the East Coast Team in the baseline analysis without priors on the nuisance parameters. Note that the displayed nuisance parameters were shifted by random numbers.

by a randomly drawn number. Let us first take a look at the corner plot obtained by the East Coast Team for the first three values of k_{max} , as shown in Fig. 11. One can clearly see degeneracies between the nuisance and cosmological parameters at $k_{\text{max}} = 0.08 \, h \text{ Mpc}^{-1}$ and $k_{\text{max}} = 0.10 \, h \text{ Mpc}^{-1}$. Moreover, at $k_{\text{max}} = 0.10 \, h \text{ Mpc}^{-1}$ the posterior develops two connected islands. Obviously,

one of them is unphysical, as it may be seen from the $k_{\text{max}} = 0.12 \, h \text{ Mpc}^{-1}$ case, where the degeneracies between the cosmological and nuisance parameters are broken. This suggests that not assuming priors on nuisance parameters at low k_{max} cuts makes the analysis quite inefficient. First, it compromises the cosmological constraints. Second, it makes the MCMC sampler explore

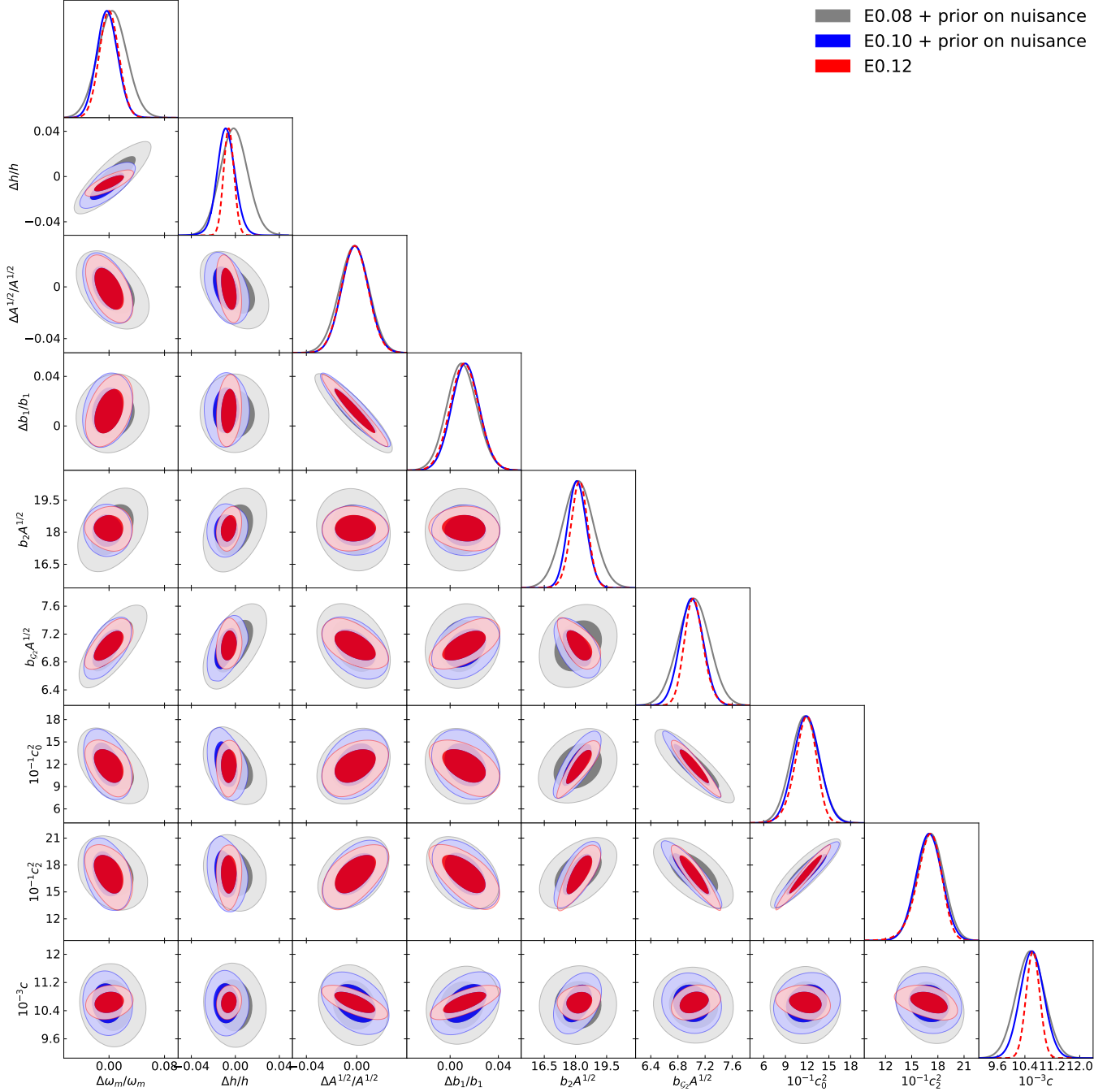


FIG. 12. Posterior distribution of cosmological and nuisance parameters obtained by the East Coast Team in the analysis with priors on the nuisance parameters inferred from the $k_{\text{max}} = 0.12 \, h \, \text{Mpc}^{-1}$ analysis. Note that the displayed nuisance parameters were shifted by random numbers.

unphysical regions of parameter space (which can also be highly non-Gaussian), and hence can lead to a significant waste of computational time.

A way to optimize the analysis is to impose an educated prior on the nuisance parameters from the high- k measurements. Let us take marginalized limits on the nuisance parameters from $k_{\text{max}} = 0.12 \, h \, \text{Mpc}^{-1}$, where

we believe that their measurements are unbiased. Then, one can increase the standard deviations by some factor and impose these limits as Gaussian priors in the analyses with low k_{max} cuts. In our particular example, we inflate all errors on nuisance parameters by a factor of 2. By importance sampling our posteriors with these priors we obtain the new posterior shown in Fig. 12. Curiously,

the contours for some of nuisance parameters shrink more than imposed by the priors as a result of additional degeneracy breaking between them. We see that the constraints on the cosmological parameters dramatically improved compared to the no-prior case. In particular, at $k_{\text{max}} = 0.08 \text{ h Mpc}^{-1}$ we find

$$\begin{aligned}\Delta\omega_m/\omega_m &= (0.46 \pm 1.9) \cdot 10^{-2}, \\ \Delta h/h &= (-0.13 \pm 1.1) \cdot 10^{-2}, \\ \Delta A^{1/2}/A^{1/2} &= (-0.21 \pm 1.1) \cdot 10^{-2},\end{aligned}\quad (\text{B1})$$

which can be compared with the no-prior measurements,

$$\begin{aligned}\Delta\omega_m/\omega_m &= (1.4 \pm 6.2) \cdot 10^{-2}, \\ \Delta h/h &= (0.19 \pm 3.4) \cdot 10^{-2}, \\ \Delta A^{1/2}/A^{1/2} &= (0.9 \pm 2.1) \cdot 10^{-2}.\end{aligned}\quad (\text{B2})$$

The East Coast Team believes that the presented strategy of imposing the priors on nuisance parameters can be useful for future analyses.

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