Stochastic Geometry for Sensing Environmental Processes with a known Spatio-Temporal Profile

Abhishek K. Gupta, Kaushlendra Pandey, Harpreet S. Dhillon

***Abstract*—We consider the problem of sensing an environmental variable with a known spatial and temporal statistical profile. The main technical contribution is a stochastic geometry approach to characterizing the fraction of area that is successfully sensed by a given fraction of stationary or mobile agents for a specific error tolerance. Our results demonstrate that the knowledge of underlying spatio-temporal profile significantly improves the coverage of the agents. For the mobile case, we also compute optimal movement region of each agent to maximize the coverage performance for a given error tolerance.**

I. Introduction

Timely and accurate sensing of environmental processes is necessary for developing effective solutions for several global challenges, such as forest fires and the deteriorating quality of air, water, and soil. In its most general form, this sensing problem can be treated as a cyber physical system (CPS) problem where the *physical* aspects refer to the sensing part and the *cyber* aspects refer to conveying and processing the sensed information using a cyber communications network [1]. The relevant physical aspects of CPS for this problem include the design, placement, and trajectories (if mobile) of the sensors as well as the signal processing aspects of reconstructing the underlying stochastic process being sensed. On the other hand, the relevant cyber aspects include the design and performance analysis of the wireless sensor networks.

The problem studied in this paper is motivated by the fact that many key macro-environmental variables (MEVs) exhibit little variation over space and time. For example, environmental humidity and temperature of the Earth's surface would not change too dramatically over space and time. Because of this, there would be a tremendous amount of redundancy in the sensed data when such spatio-temporal processes are sampled too densely over space and time. Therefore, it is natural to wonder about the *optimal* density of sensing agents needed to cover a given area if the spatio-temporal profile of the variables to be sensed is available (which is the case for many variables of practical interest, as discussed shortly). This problem becomes even more interesting if one accounts for the fact that sensing agents many not be placed based on a deterministic regular pattern (such as a grid) because of multitude of factors, such as the the need for rapid deployment over a large area that could include sensing agents dropped from an aerial platform.

There is a rich literature on characterizing the spatial variation of MEVs. For example, in [2], authors present an approximate relation between the temperature of the soil and its depth. Similarly, a numerical study of variation of forest temperature over space and time was performed in [3]. As noted above already, this spatio-temporal profile information can be used to achieve better coverage of a region without increasing the sensor density by avoiding redundancy in the sensed data. However, there are only a handful of prior works that have incorporated this correlation in their system design. For instance, in [4], clusters of proximate sensors are formed as they have similarity in the sensed data. In order to form these clusters, the sink node relies on the correlation in the data streams coming from different source nodes. In [5], the authors used spatio-temporal correlation among the sensed data for dual prediction and data compression where sensors predict data based on the past observations. In [6], authors studied the coverage performance of a dynamic event that evolves with time (such as a forest fire). However, a systematic stochastic analysis of this general problem has not received much attention, which is the main focus of this paper.

Inspired by this, we develop a stochastic geometry based approach to study this deployment problem while accounting for the stochastic deployment of the agents, possibility of having mobile agents, as well as the knowledge of the underlying spatio-temporal profiles of the variables to be sensed. Using this approach, we derive closed form results for the fraction of area that can be sensed with a given density of sensing agents (in both stationary and mobile cases) for a given error tolerance level.

Notation: $x = \|\mathbf{x}\|$ denotes the norm of $\mathbf{x}$. $\mathbf{x} = x\angle\theta$ indicates that x is the radial distance and θ is the angle of the point $\mathbf{x}$. $\mathcal{B}(\mathbf{x}, r)$ is the ball with radius r and center $\mathbf{x}$.

II. System Model

We consider a CPS with movable sensing agents deployed in $\mathbb{R}^2$ to sense an environmental variable Θ that varies across space and time. Examples include temperature in a forest, soil moisture in an agricultural field, and humidity in a city. The initial locations of agents, termed *centers*, are modeled as a homogeneous Poisson point process (PPP) $\Psi = \{\mathbf{X}_i\}$ with intensity λ [7]. We assume that each agent $\mathbf{X}_i$ has a circular area $\mathsf{S}_i = \mathcal{B}(\mathbf{X}_i, R_v)$ around it in which it can move. The

A. K. Gupta and K. Pandey are with Indian Institute of Technology Kanpur, Kanpur, India. Email:{kpandey,gkrabhi}@iitk.ac.in.

H. S. Dhillon is with Wireless@VT, Department of ECE, Virginia Tech, VA. Email: hdhillon@vt.edu.

The support of DST-SERB (Grant SRG/2019/00459) and U.S. NSF (Grant CPS-1739642) is gratefully acknowledged.

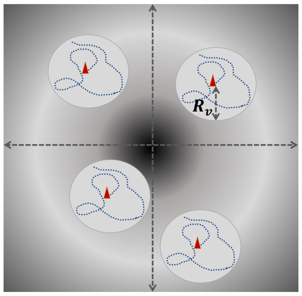

Fig. 1. Illustration of the system model of the considered CPS. Agents are shown as red triangles. Each agent has a trajectory in a circle of radius R_v around itself. A realization of Θ is also overlaid with the agent topology.

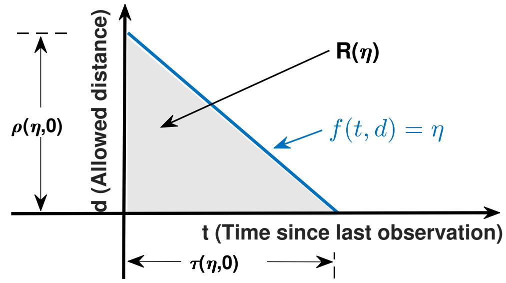

Fig. 2. Inverse region $\mathsf{R}(\eta)$ for tolerance function f_p. x-axis represents the time elapsed since the last observation and y-axis shows the allowable distance of the observation point for a certain η.

region physically covered by this CPS over time is Boolean-Poisson model ξ given as

$$\xi = \bigcup_{\mathbf{X}_i \in \Psi} \mathbf{X}_i + \mathsf{S}, \tag{1}$$

where $\mathsf{S} \equiv \mathcal{B}(\mathrm{o}, R_v)$ denotes the movement region of each agent around itself. Let the trajectory followed by the i-th agent around itself is given by $\mathsf{B}_i = \{B(t) : 0 \leq t \leq T_v\}$, which is assumed to be independent (and identically distributed) of other agents. Here T_v denotes the time period after which the trajectory is repeated. We assume continuous sensing in this paper although the analysis can be easily extended to the case of discrete-time sensing case as well.

III. Spatio-Temporal Profile of Environmental Variable And Sensor Breathing

We model the environmental variable as a spatio-temporal process $\Theta(t, \mathbf{x})$ which denotes its value at a location $\mathbf{x}$ at time t. We further assume that the variation of Θ is bounded which means that it can only vary by a finite value in a finite distance and time interval. In particular, we assume that for any two points $\mathbf{x}$ and $\mathbf{y}$ and time instants t_1 and t_2, the variation in the value of Θ satisfies:

$$|\Theta(t_1, \mathbf{x}) - \Theta(t_2, \mathbf{y})| \leq f(|t_1 - t_2|, \|\mathbf{x} - \mathbf{y}\|), \tag{2}$$

where $f(\cdot, \cdot)$ is the tolerance function [8]. Here, f is an increasing function with respect to both the arguments. Hence, the uncertainty in the value of $\Theta(t, \mathbf{x})$ conditioned on the knowledge of $\Theta(t_0, \mathbf{x}_0)$ is

$$\mathsf{Uncert}(\Theta(t, \mathbf{x}) | \Theta(t_0, \mathbf{x}_0)) = f(|t - t_0|, \|\mathbf{x} - \mathbf{x}_0\|).$$

One example of tolerance function $f(\cdot, \cdot)$ is

$$f_p(|t - t_0|, \|\mathbf{x} - \mathbf{x}_0\|) = A(e^{w\|\mathbf{x} - \mathbf{x}_0\| + v|t - t_0|} - 1), \tag{3}$$

where v and w are the temporal and spatial variation rates of Θ, respectively, and A is the scale coefficient. A high w means that Θ changes rapidly in space (likewise, v for the time axis). Assuming $\Theta(t_0, \mathbf{x}_0)$ is known, this tolerance function implies that, if $t = t_0$ and $\mathbf{x}$ is close to $\mathbf{x}_0$, $\Theta(t_0, \mathbf{x})$ is equal to $\Theta(t_0, \mathbf{x}_0)$ (*i.e.* $f(0, w) = 0$). In other words, $\Theta(t_0, \mathbf{x})$ can be predicted exactly. As $\mathbf{x}$ moves away from $\mathbf{x}_0$, the correlation between $\Theta(t_0, \mathbf{x})$ and $\Theta(t_0, \mathbf{x}_0)$ will reduce and hence we may not be able to accurately predict the value of $\Theta(t_0, \mathbf{x})$ from $\Theta(t_0, \mathbf{x}_0)$. Similarly, with increasing t, the uncertainty of the value at location $\mathbf{x}_0$ at time t increases.

We consider the case where we allow η tolerance in the estimation of Θ. To understand the spatio-temporal regions where uncertainty in Θ is within a certain tolerance η, it will be useful to understand the inverse of tolerance function f. Since f is a two-variable function, its inverse is a region defined as

$$\mathsf{F}(\eta) = f^{-1}(\eta) = \{(t, d) : f(t, d) = \eta\},$$

where t is the time elapsed since the last observation and d is the distance to the observation point. Fixing d or t gives the following two inverses which are scalar functions of η:

$$\tau(\eta, d) = \max_{t'} \{t' : f(t', d) = \eta\},$$
$$\rho(\eta, t) = \max_{d'} \{d' : f(t, d') = \eta\}.$$

These essentially give the largest values of the respective variables that keep the tolerance within η. Note that

$$\mathsf{R}(\eta) = \bigcup_{h \leq \eta} f^{-1}(h)$$

denotes the spatio-temporal region with tolerance within η.

Remark 1. *For tolerance function f_p, the inverses are given as (See Fig. 2)*

$$f^{-1}(\eta) = \{(t, d) : wd + vt = \ln(\eta/A + 1)\},$$
$$\tau(\eta, 0) = \frac{\ln(\eta/A + 1)}{v}, \ \rho(\eta, 0) = \frac{\ln(\eta/A + 1)}{w}.$$

In the subsequent lemmas, we now present some useful results on spatio-temporal characterization of uncertainty in Θ given an observation at $(t_0, \mathbf{x}_0)$.

Lemma 1 (Spatial breathing)**.** *If $\Theta(t_0, \mathbf{x}_0)$) is known, then the set of points where uncertainty in Θ is within η tolerance at the same time instant, is given by*

$$\begin{aligned}\mathsf{D}(t_0, \mathbf{x}_0, \eta) &\triangleq \{\mathbf{x} : \mathsf{Uncert}(\Theta(t_0, \mathbf{x}) | \Theta(t_0, \mathbf{x}_0)) \leq \eta\} \\ &= \{\mathbf{x} : f(0, \|\mathbf{x} - \mathbf{x}_0\|) \leq \eta\} \\ &= \mathcal{B}(\mathbf{x}_0, \rho(\eta, 0)),\end{aligned}$$

where $\rho(\eta, 0)$ is the inverse of $f(0, \cdot)$ with respect to the second argument.

Lemma 1 shows that if a certain tolerance is allowed in sensing, a circular region around each agent can also be covered by the current observation of that agent. Naturally, the radius of this region increases with the increase in the tolerance level. The following Lemma shows a similar behavior for the temporal domain. When a certain tolerance is allowed, the agents need to sense a location only after a certain time.

Lemma 2 (Temporal breathing)**.** *If $\Theta(t_0, \mathbf{x}_0)$) is known, then the time after which the uncertainty in $\Theta(t, \mathbf{x}_0)$ grows more than η tolerance is given as*

$$\begin{aligned}\mathsf{T}(t_0, \mathbf{x}_0, \eta) &\triangleq \max_{t>t_0} \{t : \mathsf{Uncert}(\Theta(t, \mathbf{x}_0)|\Theta(t_0, \mathbf{x}_0)) \leq \eta\} \\ &= \max_{t>t_0} \{t : f(t - t_0, 0) \leq \eta\} \\ &= t_0 + \tau(\eta, 0),\end{aligned}$$

where $\tau(\eta, 0)$ is the inverse of $f(\cdot, 0)$ with respect to the first argument.

Theorem 1. *Consider a point of interest* P *at the origin at time t. Consider a sensor with center location $\mathbf{x}$ and arbitrary trajectory $B(t)$, which could be deterministic or a given realization of a random trajectory. The event that uncertainty in Θ at this location at the current time is within η tolerance is equivalent to each of the following events:*

1) *The sensor agent is present in the region $\mathcal{B}(\mathrm{o}, \rho(\eta, t - t'))$ at $t - t'$ time for some $t' \geq 0$.*
2) *At least, one observation by the sensor agent should occur at a point $\mathbf{z}$ at t' time ago such that $(t', \|\mathbf{z}\|) \in \mathsf{R}(\eta)$.*
3) *Let the distance of the sensor agent from the point of interest at t' time ago is denoted as*
$$y_t(t') = \|\mathbf{x} + B(t - t')\|.$$
Then, the curve $y_t(t')$ should intersect region $\mathsf{R}(\eta)$ i.e.
$$y_t(t') \leq \rho(\eta, t') \text{ for some } t'.$$
4) *Let the uncertainty due to an observation at time $t - t'$ by the sensor agent (with its initial location $\mathbf{x}$) is*
$$u_t(t') = f(t', \|\mathbf{x} + B(t - t')\|).$$
Then, the minimum uncertainty offered by any observation should be less than η i.e.
$$M_{\mathbf{x}}(t) = \min_{t' \geq 0} u_t(t') \leq \eta.$$

Example 1. *Let us consider a scenario where the agent is following a circular trajectory around its center location $\mathbf{x} = x\angle\theta$. Hence, $B(t) = (R_v \cos(2\pi t/T_v), R_v \sin(2\pi t/T_v))$. The distance trajectory can be expressed as*

$$y_t(t') = \left(x^2 + R_v^2 + 2xR_v \cos\left(2\pi \frac{t'}{T_v} + \theta - 2\pi \frac{t}{T_v}\right)\right)^{1/2}.$$

Fig. 3(a) shows the distance trajectory for two values of t, where the one with $t = 2$ intersects the $\mathsf{R}(\eta)$ showing the uncertainty in Θ at $t = 2$ is within η. Fig. 3(b) shows the uncertainty $u_t(t')$ for two values of t showing the uncertainty in Θ at $t = 2$ is within η. Fig. 3(c) shows the variation of $M_{\mathbf{x}}(t)$ with time t.

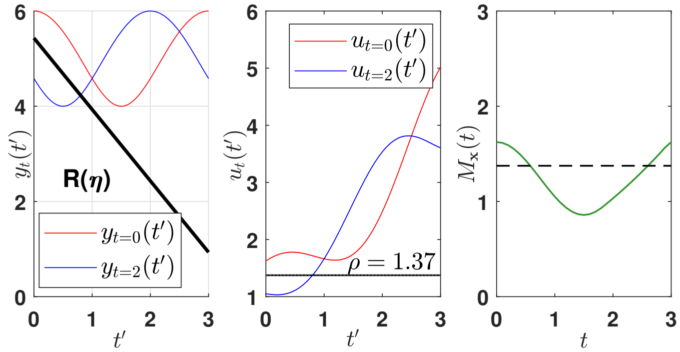

Fig. 3. Uncertainty for circularly moving sensor agents for Θ with tolerance function f_p. (a) Distance trajectory $y_t(t')$ for two values of t, (b) Uncertainty $u_t(t')$, (c) Variation of $M_{\mathbf{x}}(t)$ with time t. Here, $w = 0.2\text{m}^{-1}$, $v = 0.3\text{s}^{-1}$, $A = 0.7$, $\eta = 1.37$, $T = 3$. The sensor's center is at $\mathbf{x} = (5, 0)$ and $R_v = 1$.

IV. Coverage Analysis

In this section, we will derive the $\eta-$tolerance coverage probability which is defined as the probability that the environmental variable Θ is known at an arbitrary point of interest P with an error tolerance of η.

A. Case I: Stationary Agents

We first consider the case when all sensing agents are stationary *i.e.* $\mathsf{B}_i = \{0\}$.

Theorem 2. *If the agents are stationary, the probability that at an arbitrary location* P*, the value of Θ is known with η tolerance, is*

$$\mu_0(\eta) = 1 - \exp\left(-\lambda\pi\rho^2(\eta, 0)\right)$$

which is also equal to the average fraction of the area that is covered with η tolerance.

Proof. See Appendix A. □

B. Case II: Mobile Agents

We now present the main results assuming that all sensing agents are mobile with i.i.d. trajectories.

Theorem 3. *If the agents are moving in a 2D region according to their trajectories B_i, the probability that the value of Θ is known with η tolerance at an arbitrary location* P *at time t is*

$$\mu(t, \eta) = 1 - \exp\left(-\lambda \int_{\mathbb{R}^2} (1 - \mathbb{E}[M_{\mathbf{x}}(t) > \eta]) \mathrm{d}\mathbf{x}\right).$$

where inner expectation is with respect to the random trajectory B.

Proof. See Appendix B. □

As evident from Theorem 3, the $\eta-$tolerance coverage probability depends on t and the trajectory. The following result shows how this general result can be applied to various trajectories.

Lemma 3. *Consider the scenario where the agents are sweeping the 2D ball of radius R_v completely and with infinite speed* (i.e. *they immediately return to their centers). The probability that the value of* Θ *is known with* η *tolerance at an arbitrary location* P *is*

$$\mu(\eta, t) = 1 - \exp\left(-\lambda\pi(R_v + \rho(\eta, 0))^2\right).$$

Proof. See Appendix C. □

It may be difficult to solve $\mu(\eta, t)$ derived in Theorem 3 for a general trajectory. There are also scenarios when the trajectory is not known. Many times, a guarantee to achieve at least a certain coverage can suffice which is given by the lower bound of $\mu(\eta, t)$. The following theorem present a simple lower bound for general trajectories.

Theorem 4 (Current time coverage)**.** *Consider the scenario where the agents are moving in a 2D region. The probability that at an arbitrary location* P*, the value of* Θ *is known with* η *tolerance at any time instant, is*

$$\mu(\eta, t) \geq 1 - \exp\left(-\lambda\pi\rho^2(\eta, 0)\right).$$

Proof. The sufficient condition to ensure that the uncertainty in Θ is within η is that there is a sensor in $\mathcal{B}(\mathrm{o}, \rho(\eta, 0)$ at the current time instant. Since B_i's are independent to each other, at any time instant, the current locations of sensors form a PPP with density λ, owing to the displacement theorem [7]. Using the void probability of PPP, we get the desired result. □

Theorem 5 (All times coverage)**.** *Consider the scenario where the agents are moving in a 2D ball of radius* R_v *with time period* T_v *and at least sweeping the boundary completely. The probability that at an arbitrary location* P*, the value of* Θ *is known with* η *tolerance at all times, is*

$$\nu(\eta) \geq 1 - \exp\left(-\lambda\pi a^2(\eta)\right) \quad \textit{with}$$

$$a(\eta) = \begin{cases} \max\left(\rho(\eta, T_v) + R_v,\ \rho(\eta, 0) - R_v\right) & \textit{if } f(T_v, 0) \leq \eta \\ \max\left(0,\ \rho(\eta, 0) - R_v\right) & \textit{if } f(T_v, 0) > \eta \end{cases}.$$

Proof. The sufficient condition (A) to ensure uncertainty in Θ within η at all times, is that there is a sensor with its center located in $\mathcal{B}(\mathrm{o}, r_1)$ with $r_1 = \max(\rho(\eta, 0) - R_v, 0)$. This will ensure that the sensor is always within distance $\rho(\eta, 0)$ from P at the current time instant. The probability that the condition is met is

$$v_1(\eta) = 1 - \exp\left(-\lambda\pi r_1^2\right).$$

Another sufficient condition (B) to ensure uncertainty in Θ within η at all times, is that there is a sensor with its center located in $\mathcal{B}(\mathrm{o}, r_2)$ with $r_2 = \rho(\eta, T_v) + R_v$. This will ensure that the sensor visits $\mathcal{B}(\mathsf{P}, \rho(\eta, T_v))$ neighborhood of P once in the last T_v. The probability that the condition is met is

$$v_1(\eta) = 1 - \exp\left(-\lambda\pi r_2^2\right).$$

Note that above condition (B) is only true if $\rho(\eta, T_v)$ exists which is only possible if $f(T_v, 0) \leq \eta$, otherwise it is not possible to achieve sub$-\eta$ uncertainty from a T_v time old observation. Combining these arguments, we get the desired result. □

To demonstrate the remarkable tractability of the aforementioned setup, we consider a trajectory which is uniformly sweeping the $\mathcal{B}(\mathrm{o}, R_v)$ in time T_v. Assuming a constant area sweeping rate V_v, T_v is related to R_v as

$$T_v = \frac{\pi R_v^2}{V_v}.$$

We can observe that increasing the sweep radius R_v has following three effects on the coverage probability:

1) First, as agents are able to cover larger area, spatial distance of an arbitrary point from physically covered area ξ decreases which has a positive effect on the coverage.
2) Second, as R_v increases, the farthest possible location of sensor agents increases, thereby decreasing coverage.
3) Third, an increase in T_v results in the data sensed during the last visit being stale, which increases the uncertainty in Θ. It decreases $\rho(\eta, T_v)$ and hence has a negative impact on the coverage probability.

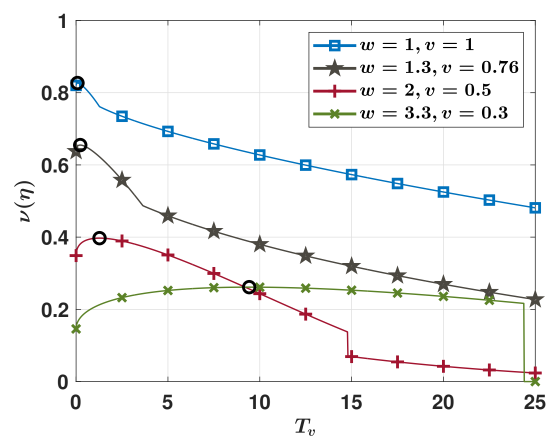

Fig. 4. The variation of $\eta-$tolerance coverage probability $\nu(\eta)$ with the sweep time T_v for different values of w and v. Here $A = 0.0006, V_v = 1,\ \lambda = 0.01/m^2$ and allowed tolerance $\eta = 1$. The tolerance function is taken as (3).

Fig 4 shows the variation of $\nu(\eta)$ with the sweep time T_v for the tolerance function in (3) for different values of w and v. These different sets of w and v show different rates of spatial and temporal correlation. The optimal value of T_v is indicated by a black circular marker for each curve. As discussed earlier, the trade-off in ν with respect to T_v can be observed here. For the case when spatial variation rate w is higher, the optimal strategy would be to cover more area (thus large T_v). When temporal variation rate in Θ is higher than the spatial variation rate, the optimal strategy is to perform sensing more frequently, and hence to use smaller T_v.

V. Conclusion

In this paper, we considered a CPS with movable agents deployed to sense an environmental variable that exhibits correlation across both space and time. We demonstrated that the information about its spatio-temporal profile improves the coverage probability and reduces the required sensor density to cover a given area of observation. There are numerous possible extension of this work. For instance, one can consider a problem of determining the trajectory that provides the best coverage performance for a particular spatio-temporal profile. Further, understanding the impact of various spatio-temporal variation profiles on the performance is another possible extension of this work.

Appendix

A. Proof of Theorem 2

Due to stationarity of Ψ, we can consider the point of interest at the origin o. From Theorem 1, we know that to estimate the value of Θ with η uncertainty at o, there should be at least one agent in $\mathcal{B}(\mathrm{o}, \rho(\eta, t')$ at t' time ago for some t'. Since agents are not moving, it is equivalent to the condition that there should be at least one agent in $\mathcal{B}(\mathrm{o}, \rho(\eta, 0)$. Mathematically, the condition can be written as

$$\begin{aligned}\mathsf{E} &= \bigcup_{\mathbf{X}_i \in \Psi} \{\mathbf{X}_i \in \mathcal{B}(\mathrm{o}, \rho(\eta, 0))\} \\ &= \left(\bigcap_{\mathbf{X}_i \in \Psi} \{\mathbf{X}_i \notin \mathcal{B}(\mathrm{o}, \rho(\eta, 0))\}\right)^{\mathrm{c}}.\end{aligned}$$

Hence, the probability that the value of Θ is known with η uncertainty at o is

$$\begin{aligned}\mu_0(\eta) &= \mathbb{P}[\mathsf{E}] = 1 - \mathbb{E}\left[\prod_{\mathbf{X}_i \in \Psi} \mathbb{1}\left(\mathbf{X}_i \notin \mathcal{B}(\mathrm{o}, \rho(\eta, 0))\right)\right] \\ &\overset{(a)}{=} 1 - \exp\left(-\lambda \int_{\mathbb{R}^2} \left(1 - \mathbb{1}\left(\mathbf{x} \notin \mathcal{B}(\mathrm{o}, \rho(\eta, 0))\right)\right) \mathrm{d}\mathbf{x}\right) \\ &= 1 - \exp\left(-\lambda \int_{\mathbb{R}^2} \mathbb{1}\left(\mathbf{x} \in \mathcal{B}(\mathrm{o}, \rho(\eta, 0))\right) \mathrm{d}\mathbf{x}\right) \\ &= 1 - \exp\left(-\lambda \left|\mathcal{B}(\mathrm{o}, \rho(\eta, 0))\right|\right) = 1 - \exp\left(-\lambda\pi\rho^2(\eta, 0)\right),\end{aligned}$$

where (a) is due to the probability generating functional (PGFL) of the PPP [7].

B. Proof of Theorem 3

Let the point of interest be at the origin o. To ensure that the value of Θ is known with η uncertainty at o due to observations from the sensor (with its center at $\mathbf{x}$), the minimum uncertainty offered by its observations should be less than η, *i.e.*

$$M_{\mathbf{x}}(t) = \min_{t' \geq 0} u_t(t') \leq \eta.$$

In case of multiple sensor agents, at least one sensor needs to satisfy the above condition, *i.e.*

$$\mathsf{E} = \bigcup_{\mathbf{X}_i \in \Psi} \{M_{\mathbf{X}_i}(t) \leq \eta\} = \left(\bigcap_{\mathbf{X}_i \in \Psi} \{M_{\mathbf{X}_i}(t) > \eta\}\right)^{\mathrm{c}}.$$

Hence, the probability that the value of Θ is known within η uncertainty at o is

$$\begin{aligned}\mu(\eta, t) &= \mathbb{P}[\mathsf{E}] = 1 - \mathbb{E}\left[\prod_{\mathbf{X}_i \in \Psi} \mathbb{1}\left(M_{\mathbf{X}_i}(t) > \eta\right)\right] \\ &\overset{(a)}{=} 1 - \exp\left(-\lambda \int_{\mathbb{R}^2} \left(1 - \mathbb{P}\left[M_{\mathbf{x}}(t) > \eta\right]\right) \mathrm{d}\mathbf{x}\right),\end{aligned}$$

where (a) is due to PGFL of the marked PPP. Here, $m_{\mathbf{x}} = \mathbb{1}\left(M_{\mathbf{x}}(t) > \eta\right)$ is taken as the mark of the point at $\mathbf{x}$ of the point process.

C. Proof of Lemma 3

Let us assume that the sensor agents sweep their movement region completely in T_v time with $T_v \to 0$. It is sufficient to consider only one time period. Hence, the minimum uncertainty offered by any observation in this time period is

$$N_{\mathbf{x}}(t) = \min_{T_v \geq t' \geq 0} u_t(t') = \min_{T_v \geq t' \geq 0} f(t', \|\mathbf{x} + B(t - t')\|).$$

As $T_v \to 0$,

$$\begin{aligned}M_{\mathbf{x}}(t) &= \lim_{T_v \to 0} N_{\mathbf{x}}(t) = \lim_{T_v \to 0} \min_{T_v \geq t' \geq 0} f(t', \|\mathbf{x} + B(t - t')\|) \\ &= \lim_{T_v \to 0} \min_{T_v \geq t' \geq 0} f(0, \|\mathbf{x} + B(t - t')\|),\end{aligned}$$

which is due to the fact that all observations are at the current time owing to infinitesimal sweep time T_v. Due to lack of the time delay term in the above expression, the minimum is achieved when distance term $\|\mathbf{x} + B(t - t')\|$ is minimized. This occurs when the sensor is at distance $\min(\|\mathbf{x}\| - R_v, 0)$. Therefore,

$$M_{\mathbf{x}}(t) = \lim_{T_v \to 0} N_{\mathbf{x}}(t) = f(0, \min(\|\mathbf{x}\| - R_v, 0)).$$

Now, the event that $M_{\mathbf{x}}(t) > \eta$ is equivalent to

$$\begin{aligned}\{M_{\mathbf{x}}(t) > \eta\} &= \{f(0, \min(\|\mathbf{x}\| - R_v, 0)) > \eta\} \\ &= \{\min(\|\mathbf{x}\| - R_v, 0) > \rho(\eta, 0)\} \\ &= \{\|\mathbf{x}\| > R_v + \rho(\eta, 0)\} \\ &= \{\mathbf{x} \notin \mathcal{B}(0, R_v + \rho(\eta, 0)\}.\end{aligned}$$

Now, from Theorem 3,

$$\begin{aligned}\mu(t, \eta) &= 1 - \exp\left(-\lambda \int_{\mathbb{R}^2} (1 - \mathbb{E}\left[M_{\mathbf{x}}(t) > \eta\right]) \mathrm{d}\mathbf{x}\right) \\ &= 1 - \exp\left(-\lambda \int_{\mathbb{R}^2} \mathbb{1}\left(\mathbf{x} \in \mathcal{B}(0, R_v + \rho(\eta, 0)\right) \mathrm{d}\mathbf{x}\right) \\ &= 1 - \exp\left(-\lambda |\mathcal{B}(0, R_v + \rho(\eta, 0)|\right)\end{aligned}$$

which gives the desired result.

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