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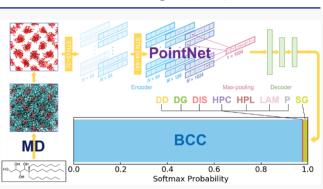
¹ Development of a PointNet for Detecting Morphologies of Self-² Assembled Block Oligomers in Atomistic Simulations

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7 predicting the self-assembly phase behavior of complex molecules. 8 Amphiphiles, block oligomers, and block polymers can form 9 mesophases with different ordered morphologies describing the 10 spatial distribution of the blocks, but entirely amorphous nature for 11 local packing and chain conformation. Screening block oligomer 12 chemistry and architecture through molecular simulations to find 13 promising candidates for functional materials is aided by effective 14 and straightforward morphology identification techniques. Captur-15 ing 3-dimensional periodic structures, such as ordered network 16 morphologies, is hampered by the requirement that the number of



17 molecules in the simulated system and the shape of the periodic simulation box need to be commensurate with those of the resulting 18 network phase. Common strategies for structure identification include structure factors and order parameters, but these fail to 19 identify imperfect structures in simulations with incorrect system sizes. Building upon pioneering work by DeFever et al. [*Chem. Sci.* 20 **2019**, *10*, 7503–7515] who implemented a PointNet (i.e., a neural network designed for computer vision applications using point 21 clouds) to detect local structure in simulations of single-bead particles and water molecules, we present a PointNet for detection of 22 nonlocal ordered morphologies of complex block oligomers. Our PointNet was trained using atomic coordinates from molecular 23 dynamics simulation trajectories and synthetic point clouds for ordered network morphologies that were absent from previous 24 simulations. In contrast to prior work on simple molecules, we observe that large point clouds with 1000 or more points are needed 25 for the more complex block oligomers. The trained PointNet model achieves an accuracy as high as 0.99 for globally ordered 26 morphologies formed by linear diblock, linear triblock, and 3-arm and 4-arm star-block oligomers, and it also allows for the discovery 27 of emerging ordered patterns from nonequilibrium systems.

28 INTRODUCTION

29 Self-assembling amphiphiles, block oligomers, and block 30 polymers that contain chemically distinct segments can form 31 a wide variety of structures across length scales from a few to 32 hundreds of nanometers. Depending on the self-assembled 33 morphologies and domain sizes, these classes of materials can 34 be targeted to numerous application including templates for 35 nanopatterning,¹⁻⁴ transport membranes,⁵⁻⁸ drug delivery,^{9,10} 36 and photonics.^{11,12} To accelerate material design and 37 discovery, molecular simulations can be used to efficiently 38 screen over molecular structures and provide detailed micro-39 scopic-level insights. In our recent studies, 13-15 molecular 40 dynamics (MD) simulations using transferable force fields^{16–18} 41 were performed to study the phase behavior of a class of block 42 oligomers with thermotropic liquid crystallinity. Multiple 43 mesophases were observed with domain sizes smaller than 4 44 nm, including lamellar (LAM), hexagonally packed cylinder 45 (HPC), hexagonally perforated lamellar (HPL), body-centered 46 cubic (BCC), and disordered states (DIS) (see Figure 1).

Although missing from our previous simulations, 3-dimen-⁴⁷ sional network structures (NET), which are often observed ⁴⁸ over narrow composition windows in self-assembling soft ⁴⁹ materials, are of increasing interest due to their inter-⁵⁰ penetrating domains that enable independent tuning of ⁵¹ orthogonal properties in a single material.¹⁹ Further exploring ⁵² the design space of these block oligomers could facilitate the ⁵³ computational discovery of NET-forming materials, as well as ⁵⁴ systems with larger NET composition windows. ⁵⁵

In molecular simulations, equilibrium mesophases can be 56 inferred from spatial information including atomic positions 57

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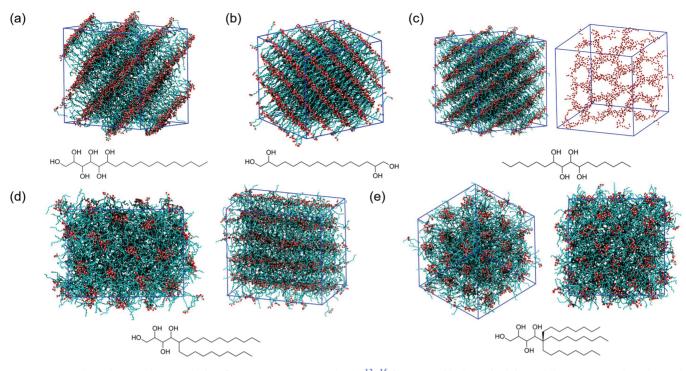


Figure 1. Snapshots showing the periodic box from previous MD simulations¹³⁻¹⁵ for various block amphiphiles and their corresponding chemical structures: (a) A_6B_{12} and (b) $A_2B_{12}A_2$ in the LAM phase, (c) $B_6A_4B_6$ in the HPL phase (the second snapshot shows one perforated layer containing O and H atoms); (d) $A_4B(B_{10})_2$ in the HPC phase, and (e) $A_4B(B_8)_3$ in the BCC phase. Hydroxyl hydrogen and oxygen atoms are shown as white and red spheres, respectively, and the alkyl tails as cyan lines.

58 and identities, which allows for quantitative analysis that can 59 bridge with macroscopic observations. One common techni-60 que is to compute the structure factor,²⁰⁻²² and the 61 morphology can be determined from the relative peak 62 positions and intensities. Importantly, the structure can be 63 readily compared with experimental scattering patterns. 64 However, in molecular simulations, the system dimensions 65 have to be delicately selected to match integer multiples of the 66 unit cell of the equilibrium structure.²³ When the system size is 67 chosen incorrectly, the system may adopt a distorted, 68 thermodynamically unstable structure in comparison to the 69 infinite system. In principle, the incommensurability effects can 70 be ameliorated by using a very large system size but, beyond 71 simple coarse-grained models, such simulations are unafford-72 able even with current computing hardware. For morphologies 73 that are anisotropic in one or two dimensions (e.g., LAM and 74 HPC), the commensurability issues can be accommodated by 75 using anisotropic orthorhombic simulation boxes that allow 76 independent fluctuations of the three dimensions.^{24,25} For 3-77 dimensional periodic structures, selecting the system param-78 eters is nontrivial, since the exact stable morphology and the 79 corresponding unit cell dimensions are not known a priori. In 80 our previous work, there have been successes in achieving 81 stable 3-dimensional periodic structures including BCC and 82 HPL¹⁵ by tuning the number of molecules after an initial guess 83 of the equilibrium morphology from the imperfect structures 84 resulting from arbitrary system sizes. Similarly, the system size 85 for the $A_4B(B_8)_3$ miktoarm tetrablock oligomer is tuned here 86 to yield a stable BCC morphology instead of disordered 87 micelles.¹³ However, such a human-based initial guess from 88 emergent NET structures can be far less accurate, due to the 89 existence of many possible NET geometries. Therefore, it is 90 beneficial to predict the likely stable structure of a NET 91 candidate system before fine-tuning the system size. The

structure factor, in this case, can fail to detect any nonglobal 92 features of the plausible but distorted structure. Another 93 structure detection tool that can be used for molecular 94 simulations is referred to as an "order parameter", which can be 95 some mathematical quantity such as a "signature vector" as a 96 function of atomic positions. The order parameter can tackle 97 local structure recognition but can only distinguish among very 98 few structures, and developing selective functional forms of 99 order parameters can be challenging.^{26–29} 100

Machine learning methods based on deep neural networks 101 have been widely employed in the prediction and design of 102 atomistic and molecular systems.^{30,31} Neural networks are a 103 class of mathematical models composed of multiple layers of 104 neurons, where each neuron outputs a linear combination of 105 the input from its previous layer followed by a nonlinear 106 transformation. Although a shallow neural network with two 107 layers is already sufficient to approximate any continuous 108 function, increasing the number of layers introduces a 109 hierarchy of representations of input data, which results in 110 strong performance in various complex tasks and alleviates the 111 need for feature engineering.³² Deep learning methods for 112 molecular systems are commonly based on atomic coordinates 113 as they directly represent the structure and geometry of the 114 system. For example, neural network potentials for MD and 115 Monte Carlo simulations typically take a series of symmetry 116 functions over atomic coordinates as input.^{33,34} Pairwise 117 distances have also been constructed as the features of neural 118 networks for molecular structure generation to utilize rota- 119 tional symmetry.³⁵ Recently, DeFever et al. reported a deep 120 learning method for identifying local crystal structures, 121 mesophases, and hydrophilic surfaces from MD simulations 122 for binary mixtures of single-bead particles and for multisite 123 water models directly from particle coordinates.³⁶ Their 124 method was based on PointNet, a highly efficient and effective 125

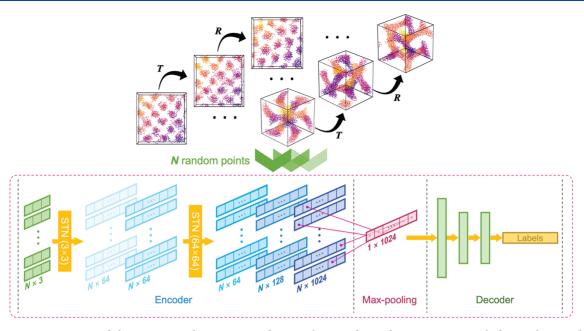


Figure 2. Data augmentation and the PointNet architecture. A random translation and a random rotation are applied on each point cloud, from which N points are randomly selected and fed into the PointNet. The N points are passed through the encoder which contains point-wise feedforward layers and spatial transformer networks (STN). The max-pooling layer performs permutation-invariant aggregations of the 1024dimensional point features, and the following feedforward network (512, 256 neurons) with dropout outputs predicts the scores for point cloud classification.

126 neural network architecture in computer vision applications for 127 point clouds,³⁷ and utilized the coordinates of neighboring 128 particles within a local spherical region centered on one 129 particle.

Here, we also adopt the PointNet architecture for 130 131 morphology classification of self-assembled block oligomers
 132 investigated by MD simulations.¹³⁻¹⁵ Apart from point clouds 133 consisting of sets of Cartesian coordinates, meshes and voxel 134 grids are also common representations of 3-dimensional spatial 135 data in computer vision. Meshes are the natural choice for 136 representing surfaces but, for the block oligomers investigated 137 here, the surfaces are locally very rough making identification 138 difficult. While voxel grids have been used for deep-learning-139 based generation of crystal structures,³⁸ they are computation-140 ally inefficient for larger systems and introduce discretization 141 errors making the detected morphology potentially ambiguous. 142 The PointNet model developed here is trained on atomic 143 structures of different morphologies from a combination of 144 MD simulation frames and synthetic point clouds of NET 145 structures to address the scarcity of NET geometries 146 encountered in our previous simulations.

METHODS 147

 f_2

PointNet Architecture. In this work, the standard settings 148 149 of the classification network in PointNet³⁷ are utilized (see 150 Figure 2). The input point clouds are represented by N151 Cartesian coordinates without additional features, but may 152 contain the positions of multiple beads taken from a given 153 block oligomer. The point clouds used here represent the 154 global structure of the system, but require a large number of 155 points. In contrast, the PointNet introduced by DeFever et 156 al.³⁶ utilizes only a local region with a small number of points 157 for structure identification of single-site particles or water 158 molecules. Although not guaranteeing better performance, the 159 global point clouds used here are closer to the original 160 computer vision application.³

In our approach, the input point clouds are then passed 161 through the encoder consisting of a series of pointwise 162 feedforward layers with 64, 64, 64, 128, and 1024 neurons with 163 weights shared among all points. Since atomic coordinates are 164 unordered, the network output should be invariant to the 165 permutation of atoms, such as exchanging a pair of coordinates 166 in the input. This is achieved in PointNet by applying a 167 symmetric function operated on the high-dimensional point 168 features produced from the pointwise network: 169

$$f(\{\mathbf{x}_{1}, ..., \mathbf{x}_{n}\}) \approx g(h(\mathbf{x}_{1}), ..., h(\mathbf{x}_{n}))$$
(1) 170

. .

where $\mathbf{x}_1, \dots, \mathbf{x}_n$ are input points from the point cloud, h 171 represents the pointwise network for feature extraction, and g 172 represents the symmetric function. The PointNet transforms 173 the 3-dimensional Cartesian coordinates into a 1024-dimen- 174 sional features space before performing max pooling. The max 175 pooling layer then takes the largest value for all points along 176 each dimension to give the global feature.³⁷ The points for 177 which the feature coordinates contribute to the global feature 178 are picked as the "critical points" regardless of the input order. 179 This also agrees with the physical intuition that the 180 morphology of the system can be identified with the most 181 important subset of atoms in the structure. Directly after the 182 max pooling layer, two dense feedforward layers with 512 and 183 256 neurons followed by a dropout layer with a 0.7 keep ratio 184 are used to calculate class scores and to infer class labels from 185 softmax probabilities. 186

In the PointNet structure used for the present work, two 187 spatial transformer networks (STNs) are applied before the 188 first and the third feature extraction layer. The STNs take a 189 similar structure as the main networks and comprise the same 190 types of modules including feature extraction, max pooling, and 191 fully connected layers. They are aimed at learning data- 192 dependent rigid or affine transformation matrices to align the 193 input (3×3) point sets and higher dimensional features (64×194) 64) into a consistent orientation to further improve the 195

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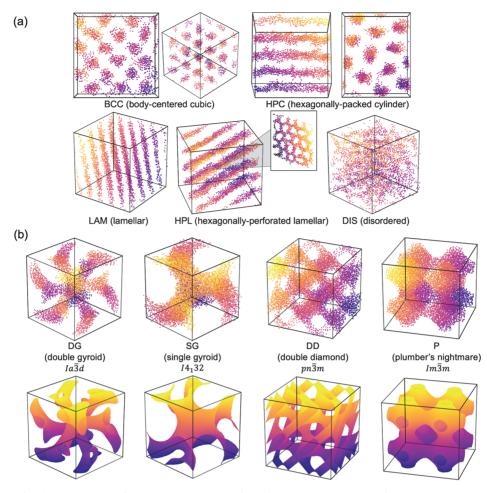


Figure 3. (a) Examples for the point clouds of the minor component of the block oligomers retrieved from MD simulation trajectories (see Table S1). (b) Generated point clouds and their corresponding surfaces for four ordered NET structures. To guide the eye, the point clouds are colored from yellow to purple according to a point's average values of its x, y, and z coordinates.

196 results.³⁷ The PointNet implementation by DeFever et al.³⁶
197 does not include such STNs. Because molecular systems are
198 less orientation-dependent than real-life objects, we also
199 evaluate the effectiveness of STNs in morphology detection
200 in this work.

Batch normalization and rectified linear unit (ReLU) 202 activation functions are applied throughout all fully connected 203 layers. We use the Adam optimizer³⁹ with an initial learning 204 rate of 0.001, which is halved every 20 training epochs. The 205 exponential decay rates for the first and the second moment 206 are set to values of 0.9 and 0.999, respectively. The model is 207 implemented in PyTorch and trained on an NVIDIA Titan 208 RTX GPU.

Data Augmentation and Learning Formalism. We use 209 210 equilibrium trajectories obtained from NpT MD simulations of 211 diblock, symmetric triblock, miktoarm triblock, and miktoarm 212 tetrablock oligomers (for examples, see Figure 1). These amphiphiles consist of oligo-ol blocks (A_x with $2 \le x \le 6$ 213 CH,OH repeat units, where r is 1 or 2) and linear alkyl blocks 214 (B_v with $6 \le y \le 12$ CH_s repeat units, where s is 2 or 3), and 215 216 they assemble into various morphologies including body-217 centered cubic (BCC), hexagonally packed cylinders (HPC), 218 lamellar (LAM), hexagonally perforated lamellar (HPL) and 219 disordered (DIS).¹³⁻¹⁵ The simulation frames used for the 220 generation of the point clouds are selected from multiple 221 systems (see Table S2) spanning 15 different block oligomers

and, for all but one compound, two temperatures are included 222 that represent one of the ordered morphologies and the DIS 223 morphology. LAM, HPL, HPC, and BCC morphologies are 224 represented by 10, 1, 2, and 1 systems, respectively, in the 225 training set. For each morphology, 3000 point clouds for the 226 minor component (the positions of O atoms) are extracted 227 from frames across the different MD trajectories, with 228 examples illustrated in Figure 3. 229 f3

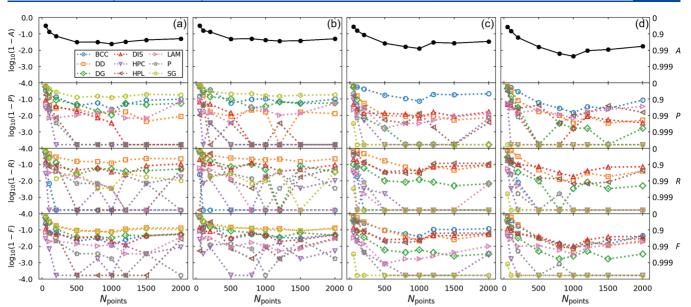
Due to the strict commensurability requirement for NET 230 morphologies with crystallographic periodicity, our previous 231 simulations on the specific class of self-assembling block 232 oligomers did not yield any stable ordered NET structures. To 233 train a model that can recognize common NET structures seen 234 in other self-assembling systems, we generate synthetic data for 235 point clouds representing minor components of double gyroid 236 (DG), single gyroid (SG), double diamond (DD), and 237 plumber's nightmare (P) morphologies using the following 238 criteria, respectively: 239

$$|\sin(x)\cos(y) + \sin(y)\cos(z) + \sin(z)\cos(x)| > t$$
 (2a) 240

 $\sin(x)\cos(y) + \sin(y)\cos(z) + \sin(z)\cos(x) > t$ (2b) ₂₄₁

lsin(x)sin(y)sin(z) + sin(x)cos(y)cos(z)

$$+\cos(x)\sin(y)\cos(z) + \cos(x)\cos(y)\sin(z)| > t \qquad (2c)_{242}$$



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Figure 4. Overall test accuracy, *A*, and test precision, *P*, recall, *R*, and F1 score for individual morphologies as functions of input point cloud size for the models trained with (a) only random translation, (b) random translation and including STN in the PointNet, (c) random translation and rotation, (d) random translation, rotation, and including STN in the PointNet. The original point clouds of the minor components contain 2000–6000 points. Because of the finite size of the test set (600 point clouds per morphology), it is possible for recall, precision, and F1 score to reach a value of exactly 1.0000. For the logarithmic plot, this value is replaced with log_{10} (1/6000) = -3.778.

$$cos(x) + cos(y) + cos(z) > t$$
 (2d)

where x, y, and z are point coordinates for the minor 244 component, and t is an adjustable parameter to control the 245 volume fraction of the minor component. Values of t are 246 randomly selected within practical ranges to account for 247 ariations in volume fraction for the self-assembled systems. 248 The *t* ranges are [0.9, 1.2], [0.6, 1.0], [0.6, 0.9], and [0.0, 0.4] 249 for DG, SG, DD, and P morphologies, respectively. Points are 2.50 uniformly sampled within these confined regions, and small 251 random displacements are added to account for local 2.52 composition fluctuations. Examples of the point clouds for 253 the NET morphologies are shown in Figure 3. 254

Before feeding the point clouds into the PointNet, 255 256 normalization and augmentation are applied on all raw point 257 clouds. First, each point cloud is min-max scaled such that |x|, | yl, $|z| \leq 1$. In addition to the permutation invariance achieved 258 259 by the PointNet structure, the model prediction should not 260 change with translation or rotation of a point cloud. Therefore, random translation vector $[\Delta x, \Delta y, \Delta z]^{T}$ is applied to all а 261 262 points in a given cloud, which satisfies $|\Delta x| \leq L_x/2$, $|\Delta y| \leq L_y/2$ $_{263}$ 2, $|\Delta z| \leq L_z/2$, where L_x , L_y , and L_z are the lengths of the x, y, $_{264}$ and z dimensions for the orthorhombic simulation box. The 265 transformed coordinates are then wrapped into the original 266 bounding box using the periodic boundary conditions. A $_{267}$ spatially uniformly random rotation matrix, M_{rot} , is sub-268 sequently applied on the wrapped coordinates. To ensure a 269 uniformly distributed rotation (see Figure S1), $M_{\rm rot}$ is obtained 270 from performing a random rotation about the vertical axis ²⁷¹ followed by rotating the north pole to a random position,⁴⁰ 272 and can be described by

$$\boldsymbol{R} = \begin{bmatrix} \cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1 \end{bmatrix}$$
(3a)

273

$$H = I - 2 \begin{bmatrix} \cos \phi \sqrt{z} \\ \sin \phi \sqrt{z} \\ \sqrt{1 - z} \end{bmatrix} [\cos \phi \sqrt{z} \sin \phi \sqrt{z} \sqrt{1 - z}]$$
(3b) 274

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$$\mathbf{M}_{\rm rot} = -\mathbf{H}\mathbf{R} \tag{3c} \ _{275}$$

where **R** is a simple rotation matrix around the vertical axis, **H** 276 is the Householder matrix, **I** is the identity matrix, and θ , ϕ , 277 and z are randomly chosen azimuthal angle ($[0, \pi]$), polar 278 angle ($[0, 2\pi]$), and radial distance ([0, 1]), respectively. The 279 transformed points are again wrapped using the periodic 280 boundary conditions. 281

To train the PointNet, 3000 point clouds are selected for 282 each of the nine morphologies obtained from the MD 283 simulation trajectories and the generated NET structures. 284 The data are split into training and test sets containing 80% 285 and 20% of the data, respectively. A 5-fold cross-validation is 286 performed to detect overfitting. After the data augmentation, 287 the network is trained for 100 epochs with a batch size of 64 288 point clouds. Depending on the chemical compound and the 289 number of molecules used in the simulations (see Table S2) 290 and the volume fraction for the generated NET morphologies, 291 each point cloud representing the minor component contains 292 on average 4500 points with a standard deviation of 1920 293 points. For consistency of the input point cloud dimensions, a 294 constant number of points ranging from 50 to 2000 is 295 randomly drawn from each of the augmented point clouds. In 296 addition to the standard workflow including random trans- 297 lation and rotation in data augmentation, and STN in the 298 PointNet, we also compare the performance when excluding 299 random rotation and/or STN. 300

RESULTS AND DISCUSSION

301

We examine the performance and robustness of the PointNet 302 by training the network with the point clouds retrieved from 303 MD simulations and the generated NET structures. These 304

305 point clouds represent the spatial distribution of oxygen atoms, 306 part of the minor CH_OH block in the self-assembled systems 307 with a volume fraction less than 0.5. All point clouds supplied 308 to the PointNet contain equal number of points (N_{points}) 309 sampled from the augmented point sets. Each point cloud is 310 labeled as one of nine different morphologies: BCC, DD, DG, 311 DIS, HPC, HPL, LAM, P, and SG. Figure 4 shows the 312 evaluation metrics for morphology classification including test 313 accuracy, precision, recall, and F1 scores of the PointNet 314 models trained by four strategies: (A) only applying random 315 translation, (B) random translation and STN, (C) random 316 translation and rotation, (D) random translation and rotation 317 and STN. In each case, the model accuracy grows initially as 318 N_{points} is increased, reaches a plateau with only small changes in 319 the accuracy, and then drops slightly when the number of 320 points is further increased. Confusion matrices for the four 321 strategies are given in Figures S2 to S5. For all four models, the 322 best performance is reached when 1000 or 1200 random 323 points are provided as input point clouds, that is, when on 324 average about 75% of the oxygen positions are not included in 325 the point clouds. This peak in accuracy likely reflects the 1024 326 dimensions before the max pooling layer in the PointNet, but 327 we also note that the average number of molecules in the 328 simulated systems is close to 1000. To obtain a more reliable 329 estimate of the accuracy of the models, we trained each model 330 10 times using different random seeds and randomly selected 331 configurations with 1024 points to form the training set data. 332 The numerical values of the average achieved accuracy and the 333 corresponding 95% confidence interval are provided in Table 334 1.

Table 1. Accuracy Obtained for Different Training Strategies. The 95% Confidence Interval (CI) Is Estimated from Training Each Model 10 Times

model	strategy	accuracy	CI
А	translation	0.973	0.010
В	translation + STN	0.957	0.011
С	translation + rotation	0.983	0.009
D	translation + rotation + STN	0.990	0.005

As can be seen in Figure 4, the accuracy generally starts to as exceed 90% when only 500 points are extracted, regardless of are training approach, which corresponded to a nearly 90% sector of the PointNet against input corruption presented in the at set, and the PointNet against input corruption presented in the original paper³⁷ for benchmarks with the ModelNet40 data set, and a accuracy of 0.75 was reported when only 256 are points were used (75% missing data ratio). Interestingly, the are obustness in the current study is significantly higher, which and may be attributed to two factors: (i) the larger number of points in the training clouds (an average of 4500 vs 1024), and (ii) the point clouds in this study containing atomic positions that span the entire volume of the minority region instead of an only representing the surface as in the ModelNet data.

The PointNet is designed to discover perceptually so interesting points with the highest contribution to the max pooled features.³⁷ Given the max pooling dimension (see S2 Figure 2), up to 1024 points that contribute to the max pooling layer can be selected as critical points from among the N_{points} points in a point cloud. Since a given point can take the largest value in more than one max pooling dimension, the lower bound for the number of unique critical points, N_{crity} is the size of the input point cloud, N_{points}.³⁷ Figure S6 illustrates input 357 point clouds with $N_{\text{points}} = 2000$ and the corresponding critical 358 points. Although we find $N_{crit} < 1024 < N_{points}$, the global shape 359 features are not substantially changed even when most of the 360 noncritical points are missing. Among the critical points 361 contributing to the max-pooled features, only a small portion 362 of points are close to domain interfaces, which also indicates 363 that PointNet is potentially more robust to volumetric data 364 than surface data. Indeed, initial tests generating point clouds 365 only from the position of the center of the bond linking the 366 oligo-ol and alkyl regions showed less promise than utilizing 367 the locations of the oxygen atoms. Similarly, focusing on only 368 the local environment of this center point yielded poor 369 performance in particular for HPL and NET structures. That 370 is, local fluctuations and feature size for the multibead 371 oligomers investigated here necessitate a nonlocal PointNet. 372

Besides the difference in the spatial distribution of points 373 (surface vs volumetric), another major distinction between the 374 point clouds from molecular simulations and those from real-375 world objects is periodicity. In most molecular simulations, 3dimensional periodic boundary conditions are utilized to allow 377 the finite system in the simulation box to better mimic bulk 378 systems. Therefore, the classification model should give exactly 379 the same label when a simulated system is shifted along, or 380 rotated by any angles around an arbitrary vector. Examples of 381 applying such operations to BCC and DG structures are shown 382 in Figure 2, which is analogous to achieving equivalent 383 morphologies from independent simulations under the same 384 thermodynamic conditions. 385

The STNs are designed to disentangle part deformations of 386 the objects by aligning input data with affine transformations.⁴² 387 While real-world objects subject to gravitational forces usually 388 have a strongly preferred orientation in the direction parallel to 389 the gravitational field, molecular systems in the absence of an 390 aligning field may take any orientation. Given this difference in 391 orientational preferences, it is interesting to compare the 392 impact of random rotations and STNs on the model 393 performance. As can be observed from the data presented in 394 Figure 4 and Table 1, introducing STNs slightly reduces the 395 highest accuracy from 0.973 to 0.957 when the point clouds 396 are augmented only with random translations. Models A and B 397 both incorrectly classify some of the DD structures as SG, as 398 can be inferred from the low precision for SG and the low 399 recall for DD. In contrast, the overall accuracy increases from 400 model C to D (from 0.983 to 0.990 at $N_{\rm points}$ = 1000) when 401 STNs are applied along with point clouds that have undergone 402 both translations and rotations. Precision and recall are now 403 near-perfect for the SG structures, but about 2% of DD 404 structures are classifieded as BCC in model C. 405

The difference in model performance when applying random 406 rotations and/or STNs can be understood using the orienta- 407 tional distributions of the simulated and generated morphol- 408 ogies. In the point cloud data set, all generated NET 409 morphologies possess the same center and orientation, and 410 multiple frames are taken from the same MD trajectory. When 411 a stable ordered structure is formed in an MD simulation, then 412 the overall structural orientation changes only very slowly; this 413 results in correlation between samples in the data set, making 414 the orientational distribution of the simulation samples highly 415 nonuniform and multimodal due to different orientations 416 encountered for different systems (see Figure S7). Due to the 417 limited alignment capability of the STN originally designed for 418 correcting slight rotations and perspective transformations,⁴² it 419

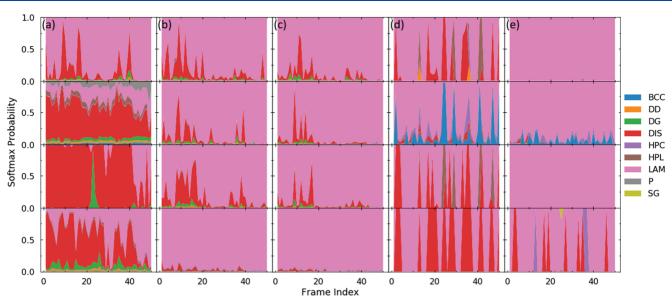


Figure 5. Stack plots of the predicted softmax classification probabilities obtained with models A, B, C, and D (top to bottom) for 50 frames taken at 10 ns intervals during a 500 ns MD trajectory. Data in columns a–c are for a 600-molecule $A_{10}B_{20}$ system with 1000 oxygen atoms for the point cloud selected at random from (a) all oxygen atoms, (b) only from oxygen atoms in positions 1, 4, 7, and 10 of the polar group, and (c) only from oxygen atoms in positions 2, 5, and 8 of the polar group. Data in columns d and e are for a 8000-molecule system of the $A_2B_8A_2$ block oligomer with 1000 oxygen atoms for the point cloud selected at random from (d) all oxygen atoms and (e) only from oxygen atoms located in a subvolume with linear dimensions of $L_x/2$, $L_y/2$, and $L_z/2$, where L_x , L_y are the box lengths for the entire simulation box.

420 is only able to assist in classification of the model when the orientational distribution of input data is well behaved. 421 Therefore, when comparing models A and B, application of 422 the STN converts a multipeaked orientational distribution 423 (resulting from an insufficiently diverse set of point clouds) 424 into a more uniform distribution (see Figure S7) that, in turn, 425 may introduce extra noise from structural distortion and lead 426 to lower accuracy. A comparison of the performance of models 427 B and C shows that randomly rotating the point clouds before 42.8 429 feeding into the PointNet tends to be more effective than solely applying STNs, since the network was encouraged to 430 capture rotational invariance from the rotated point clouds. In 431 contrast, applying STNs for model D indeed enhances the 432 accuracy beyond applying only translations and rotations for 433 434 model C, and also leads to higher precision for classification of 435 DD and BCC structures with model D. This can be explained 436 by the effectiveness of STNs on a near-uniform orientational distribution after random rotations are applied. In this case, the 437 STNs align the arbitrarily rotated point cloud onto one or a 438 439 few canonical orientations (see Figure S7), which stabilizes the network and further improves the performance. 440

The block oligomer systems used as training sets contain 44 442 between 2000 and 6016 oxygen atoms with mean and median of 4500 and 4000, respectively, and their polar blocks contain a 443 total of four or six oxygen atoms (see Table S2). Thus, it is 444 important to assess whether the trained PointNet models can 445 also be applied to block oligomers with larger numbers of 446 447 repeat units or to larger systems. To this extent, we performed 448 new simulations for four systems using the same force fields 449 and MD parameters as in prior work.¹³⁻¹⁵ Specifically, we 450 investigated 600-molecule systems for $A_{10}B_{20}$ and $A_8B(B_{18})_2$ 451 (i.e., approximately doubling the number of A segments in the 452 block oligomers), and eight times larger systems for A2B8A2 453 and A₄B(B₈)₂ (8000 and 4000 molecules, respectively). In 454 both cases, the linear block oligomers assemble in LAM 455 morphology, whereas the double-tailed oligomers assemble in

HPC morphology. Figure 5 and Figure S8 illustrate the 456 f5 performance of the four PointNet models. Despite that the 457 total numbers of oxygen atoms for the $A_{10}B_{20}$ and $A_8B(B_{18})_2$ 458 systems fall within the range used for the training structures, 459 the classification performance is quite poor when the 1000 460 points are drawn randomly from the positions of all oxygen 461 atoms; models A and C perform best for $A_{10}B_{20}$ and $A_8B(B_{18})_{27}$, 462 respectively. That is, the PointNet models are confused when 463 applied to larger block oligomers without pretreatment, and 464 the number of oxygen atoms per oligomer appears to play a 465 role. Thus, we tested pretreatments where only either four or 466 three oxygen atoms in specific positions of the polar group are 467 considered for selection of the point cloud. This pretreatment 468 leads to a marked increase in performance. Model D classifies 469 all 50 frames correctly for $A_{10}B_{20}$, and all but one frame 470 correctly for $A_8B(B_{18})_2$. For the latter, however, models A and 471 C classify all 50 frames correctly. 472

Considering the systems with larger numbers of molecules, 473 all four models without pretreatment indicate that A2B8A2 is 474 likely a LAM morphology, but at least eight out of the 50 475 frames are incorrectly assigned. For $A_4B(B_8)_2$ with HPC 476 morphology, models A, B, and D incorrectly indicate a 477 preference for the DIS morphology, and model B points to 478 either BCC or LAM. Despite that this $A_4B(B_8)_2$ system 479 contains half the number of oxygen atoms compared to the 480 A₂B₈A₂ system, the correct classification of the HPC 481 morphology from a spatially sparse point cloud of 1000 482 oxygen atoms (selected from a total of 16000 oxygen atoms) 483 appears to be more challenging. Thus, a pretreatment is also 484 needed for the larger system sizes. In this case, our 485 pretreatment consists of selecting the 1000 oxygen atoms for 486 the point cloud only from a subvolume that is one-eighth of 487 the volume of the entire system but has the same orientation. 488 Again, pretreatment vastly improves the classification perform- 489 ance with models A and C giving the correct morphology for 490 all 50 frames for both systems. Model D yields some 491

We also apply the models trained using the four strategies to 501 502 detect morphology changes encountered in a simulation 503 trajectory for a LAM-to-DIS transition. The simulated compound, 1,2,11,12-dodecanetetrol $(A_2B_8A_2)$, forms a 504 thermotropic liquid crystal that self-assembles into the LAM 505 $_{506}$ morphology at T_{SIM} = 400 and 430 K.¹⁵ Here, an equilibrated 507 LAM structure is simulated in the isobaric-isothermal ensemble for 10 ns at T = 430 K; at this point, a step increase 508 is applied to raise the kinetic temperature and the thermostat 509 510 temperature to 490 K (i.e., above the order-disorder s11 temperature, T_{ODT}), followed by another 30 ns at T = 490512 K. One thousand frames (spaced at 40 ps intervals) taken from the entire trajectory are used to generate point clouds for the 513 minor component and analyzed by the four models trained 514 515 only on equilibrium structures.

⁵¹⁶ The time evolution of the predicted softmax scores is ⁵¹⁷ illustrated in Figure 6, for which each point cloud ($N_{\text{points}} =$ ⁵¹⁸ 1000) is classified as belonging to a particular morphology if

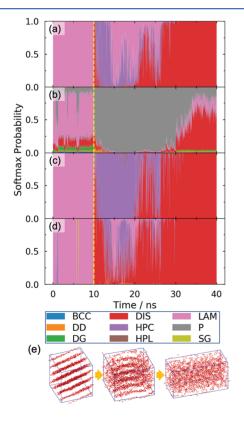


Figure 6. Stack plots of the predicted softmax classification probabilities obtained with models A, B, C, and D (subgraphs a to d) for 1000 frames taken at regular intervals during a 40 ns MD trajectory for which the kinetic temperature is increased above T_{ODT} at t = 10 ns (marked by yellow dashed line) to induce a LAM-to-DIS transition. (e) Point clouds of the minor component sampled at 5 ns (left), 20 ns (middle), and 35 ns (right).

the softmax probability for that morphology is greater than 0.5. 519 Models A and C consistently yield LAM softmax scores close 520 to unity for the initial 10 ns period below T_{ODT} . For the 521 models involving STNs, model D yields a few frames with 522 HPC and SG false positives, but otherwise LAM softmax 523 scores close to unity. In contrast, model B consistently shows 524 nonzero softmax scores for other morphologies (mainly DIS 525 and P) and also periods of false positives for the P morphology. 526 Models A, C, and D indicate softmax scores near unity for DIS 527 during the final 13 ns of the trajectory (t > 27 ns). In contrast, 528 model B recognizes DIS morphologies only later (t > 35 ns) 529 and with softmax scores significantly smaller than unity. 530

Since the training data do not contain any point clouds 531 reflecting a "transition" phase, it is of interest to compare the 532 model predictions for the transition period (10 to 27 ns). All 533 four models yield a sudden change to either DIS or P 534 morphology immediately after the step increase in temper- 535 ature, presumably because the higher temperature is almost 536 instantaneously reflected in a change in the local structure 537 and/or interfacial roughness (e.g., buckling of the lamellae). 538 During the transition period, models A, C, and D indicate a 539 mixture of LAM, HPC, and DIS structures, whereas model B 540 predicts high probability for the P morphology with softmax 541 scores mostly above 0.9. Models A and D show fleeting 542 reappearance of the LAM morphology at $t \approx 10, 20, \text{ and } 25 \text{ ns. } 543$ Overall, models C and D show strong preferences for HPC and 544 DIS, respectively, during most of the transition period, whereas 545 model A yields more similar fractions of HPC, LAM, and DIS. 546 These morphology classications suggest that the transition 547 from the LAM to the DIS phase is not instantaneous, but 548 rather involves a process of disruption and final disintegration 549 of the lamellar planes. For the A2B8A2 system, the DIS 550 morphology is bicontinuous,¹⁵ and the local packing in the DIS 551 phase exhibits similarities to disordered cylindrical micelles and 552 the HPC phase. Furthermore, order-order transitions from 553 LAM to gyroid to HPC can occur before reaching T_{ODT} , and 554 are predicted by self-consistent mean field theory for coil-coil 555 and rod-coil block polymers within certain volume-fraction 556 ranges.^{43,44} Although PointNet classifications for the point 557 clouds during the transition period are generalizations from 558 models only trained on equilibrium structures, they provide 559 additional support for the observation that STNs should not be 560 included when the point clouds are not augmented by rotation 561 (i.e., model B). 562

CONCLUSION

In this work, we train a deep neural network, PointNet, to 564 identify morphologies of self-assembling block oligomers using 565 points clouds taken from atomic coordinates of the minor 566 component obtained by molecular simulations. To expand the 567 scope of structure detection, we include synthetic point clouds 568 of NET structures commonly observed for self-assembly of soft 569 materials. The performances of the models trained using 570 different strategies in performing data augmentation, building 571 the PointNet architecture, and the number of points in the 572 cloud are compared. A classification accuracy of 0.990 is 573 achieved using 1000 coordinates (a missing data ratio of about 574 75%, but close to the number of dimensions in the max 575 pooling layer), applying random translations and rotations 576 under periodic boundary conditions to the training data, and 577 including spatial transformer networks in the PointNet. With 578 judicious pretreatment, the PointNet models can also be 579

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580 applied to oligomers with twice the number of repeat units and 581 an eight times larger system than included in the training set. 582 The generalization ability of the trained models is tested 583 using new point clouds from an MD trajectory sampling the 584 lamellar-to-disorder transition of a block oligomer. We 585 demonstrate that the PointNet models successfully predict 586 the initial and final equilibrium structures and reflect the phase 587 transition during intermediate time frames. The PointNet 588 models presented in this study are generalizable and potentially 589 transferable to discovering emerging structures of other shape-590 filling amphiphiles and block oligomers in molecular 591 simulations and may guide the discovery of block oligomers 592 forming ordered NET morphologies.

593 ASSOCIATED CONTENT

594 Supporting Information

595 The Supporting Information is available free of charge at 596 https://pubs.acs.org/doi/10.1021/acs.jpcb.1c02389.

597 Information on PointNet model and data availability; 598 tables providing details for the simulations of the block 599 oligomers; figures illustrating the performance of the 600 rotation matrix, the critical points with the highest 601 contributions to the max-pooled features, and confusion 602 matrices for the four model strategies (PDF)

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632 Notes

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REFERENCES

Feng, X.; Tousley, M. E.; Cowan, M. G.; Wiesenauer, B. R.; 642
 Nejati, S.; Choo, Y.; Noble, R. D.; Elimelech, M.; Gin, D. L.; Osuji, C. 643
 O. Scalable Fabrication of Polymer Membranes with Vertically 644
 Aligned 1 nm Pores by Magnetic Field Directed Self-Assembly. ACS 645
 Nano 2014, 8, 11977–11986. 646

(2) Feng, X.; Nejati, S.; Cowan, M. G.; Tousley, M. E.; Wiesenauer, 647 B. R.; Noble, R. D.; Elimelech, M.; Gin, D. L.; Osuji, C. O. Thin 648 Polymer Films with Continuous Vertically Aligned 1 nm Pores 649 Fabricated by Soft Confinement. *ACS Nano* **2016**, *10*, 150–158. 650

(3) Nickmans, K.; Schenning, A. P. H. J. Directed Self-Assembly of 651 Liquid-Crystalline Molecular Building Blocks for Sub-5 nm Nano- 652 patterning. *Adv. Mater.* **2015**, *30*, 1703713. 653

(4) Sinturel, C.; Bates, F. S.; Hillmyer, M. A. High χ -Low N Block 654 Polymers: How Far Can We Go. ACS Macro Lett. **2015**, 4, 1044–655 1050. 656

(5) Jo, G.; Ahn, H.; Park, M. J. Simple Route for Tuning the 657
Morphology and Conductivity of Polymer Electrolytes: One End 658
Functional Group is Enough. ACS Macro Lett. 2013, 2, 990–995. 659
(6) Ichikawa, T.; Kato, T.; Ohno, H. 3D Continuous Water 660
Nanosheet as a Gyroid Minimal Surface Formed by Bicontinuous 661
Cubic Liquid-Crystalline Zwitterions. J. Am. Chem. Soc. 2012, 134, 662
11354–11357. 663

(7) Jackson, G. L.; Perroni, D. V.; Mahanthappa, M. K. Roles of 664 Chemical Functionality and Pore Curvature in the Design of 665 Nanoporous Proton Conductors. *J. Phys. Chem. B* **2017**, *121*, 666 9429–9436. 667

(8) Orilall, M. C.; Wiesner, U. Block copolymer based composition 668 and morphology control in nanostructured hybrid materials for energy 669 conversion and storage: solar cells, batteries, and fuel cells. *Chem. Soc.* 670 *Rev.* **2011**, *40*, 520–535. 671

(9) Kluzek, M.; Tyler, A. I. I.; Wang, S.; Chen, R.; Marques, C. M.; 672 Thalmann, F.; Seddon, J. M.; Schmutz, M. Influence of a pH-sensitive 673 polymer on the structure of monoolein cubosomes. *Soft Matter* **2017**, 674 *13*, 7571–7577. 675

(10) Kumar, R.; Le, N.; Tan, Z.; Brown, M. E.; Jiang, S.; Reineke, T. 676
M. Efficient Polymer-Mediated Delivery of Gene-Editing Ribonu- 677
cleoprotein Payloads through Combinatorial Design, Parallelized 678
Experimentation, and Machine Learning. ACS. ACS Nano 2020, 14, 679
17626–17639. 680

(11) Hsueh, H.-Y.; Yao, C.-T.; Ho, R.-M. Well-ordered nanohybrids 681 and nanoporous materials from gyroid block copolymer templates. 682 *Chem. Soc. Rev.* **2015**, 44, 1974–2018. 683

(12) Stefik, M.; Guldin, S.; Vignolini, S.; Wiesner, U.; Steiner, U. 684 Block copolymer self-assembly for nanophotonics. *Chem. Soc. Rev.* 685 **2015**, 44, 5076–5091. 686

(13) Chen, Q. P.; Barreda, L.; Oquendo, L. E.; Hillmyer, M. A.; 687 Lodge, T. P.; Siepmann, J. I. Computational Design of High- χ Block 688 Oligomers for Accessing 1 nm Domains. *ACS Nano* **2018**, *12*, 4351–689 4361. 690

(14) Barreda, L.; Shen, Z.; Chen, Q. P.; Lodge, T. P.; Siepmann, J. I.; 691 Hillmyer, M. A. Synthesis, Simulation, and Self-Assembly of a Model 692 Amphiphile To Push the Limits of Block Polymer Nanopatterning. 693 *Nano Lett.* **2019**, *19*, 4458–4462. 694

(15) Shen, Z.; Chen, J. L.; Vernadskaia, V.; Ertem, S. P.; 695 Mahanthappa, M. K.; Hillmyer, M. A.; Reineke, T. M.; Lodge, T. 696 P.; Siepmann, J. I. From Order to Disorder: Computational Design of 697 Triblock Amphiphiles with 1 nm Domains. *J. Am. Chem. Soc.* **2020**, 698 *142*, 9352–9362. 699

(16) Martin, M. G.; Siepmann, J. I. Transferable Potentials for Phase 700 Equilibria. 1. United-Atom Description of n-Alkanes. *J. Phys. Chem. B* 701 **1998**, *102*, 2569–2577. 702 703 (17) Chen, B.; Potoff, J. J.; Siepmann, J. I. Monte Carlo Calculations 704 for Alcohols and Their Mixtures with Alkanes. Transferable Potentials 705 for Phase Equilibria. 5. United-Atom Description of Primary, 706 Secondary, and Tertiary Alcohols. *J. Phys. Chem. B* **2001**, *105*, 707 3093–3104.

708 (18) Stubbs, J. M.; Potoff, J. J.; Siepmann, J. I. Transferable 709 Potentials for Phase Equilibria. 6. United-Atom Description for 710 Ethers, Glycols, Ketones, and Aldehydes. *J. Phys. Chem. B* **2004**, *108*, 711 17596–17605.

712 (19) Meuler, A. J.; Hillmyer, M. A.; Bates, F. S. Ordered Network 713 Mesostructures in Block Polymer Materials. *Macromolecules* **2009**, *42*, 714 7221–7250.

715 (20) Schultz, A. J.; Hall, C. K.; Genzer, J. Obtaining Concentration 716 Profiles from Computer Simulation Structure Factors. *Macromolecules* 717 **2007**, *40*, 2629–2632.

718 (21) Overduin, S. D.; Patey, G. N. Understanding the Structure 719 Factor and Isothermal Compressibility of Ambient Water in Terms of 720 Local Structural Environments. *J. Phys. Chem. B* **2012**, *116*, 12014– 721 12020.

722 (22) Liu, H.; Paddison, S. J. Direct calculation of the X-ray structure 723 factor of ionic liquids. *Phys. Chem. Chem. Phys.* **2016**, *18*, 11000– 724 11007.

725 (23) Schönhöfer, P. W. A.; Ellison, L. J.; Marechal, M.; Cleaver, D. 726 J.; Schröder-Turk, G. E. Purely entropic self-assembly of the 727 bicontinuous Ia3d gyroid phase in equilibrium hard-pear systems. 728 *Interface Focus* **2017**, *7*, 20160161.

729 (24) Arora, A.; Morse, D. C.; Bates, F. S.; Dorfman, K. D. 730 Commensurability and finite size effects in lattice simulations of 731 diblock copolymers. *Soft Matter* **2015**, *11*, 4862–4867.

732 (25) Medapuram, P.; Glaser, J.; Morse, D. C. Universal 733 Phenomenology of Symmetric Diblock Copolymers near the Order-734 Disorder Transition. *Macromolecules* **2015**, *48*, 819–839.

735 (26) Steinhardt, P. J.; Nelson, D. R.; Ronchetti, M. Bond-736 Orientational Order in Liquids and Glasses. *Phys. Rev. B: Condens.* 737 *Matter Mater. Phys.* **1983**, *28*, 784.

738 (27) Larsen, P. M.; Schmidt, S.; Schiøtz, J. Robust structural
739 identification via polyhedral template matching. *Modell. Simul. Mater.*740 Sci. Eng. 2016, 24, 055007.

741 (28) Mukhtyar, A. J.; Escobedo, F. A. Developing Local Order 742 Parameters for Order-Disorder Transitions From Particles to Block 743 Copolymers: Methodological Framework. *Macromolecules* **2018**, *51*, 744 9769–9780.

745 (29) Barnes, B. C.; Beckham, G. T.; Wu, D. T.; Sum, J. Two-746 component order parameter for quantifying clathrate hydrate 747 nucleation and growth. *J. Chem. Phys.* **2014**, *140*, 164506.

748 (30) Sanchez-Lengeling, B.; Aspuru-Guzik, A. Inverse Molecular 749 Design Using Machine Learning: Generative Models for Matter 750 Engineering. *Science* **2018**, *361*, 360–365.

751 (31) Noé, F.; Tkatchenko, A.; Müller, K.-R.; Clementi, C. Machine
752 Learning for Molecular Simulation. *Annu. Rev. Phys. Chem.* 2020, 71,
753 361–390.

754 (32) Goodfellow, I.; Bengio, Y.; Courville, A. *Deep Learning*; MIT 755 Press: Cambridge, MA, 2016.

756 (33) Behler, J.; Parrinello, M. Generalized Neural-Network 757 Representation of High-Dimensional Potential-Energy Surfaces. 758 *Phys. Rev. Lett.* **2007**, *98*, 146401.

759 (34) Chmiela, S.; Sauceda, H. E.; Müller, K.-R.; Tkatchenko, A. 760 Towards exact molecular dynamics simulations with machine-learned 761 force fields. *Nat. Commun.* **2018**, *9*, 3887.

762 (35) Gebauer, N.; Gastegger, M.; Schütt, K. Symmetry-adapted 763 generation of 3d point sets for the targeted discovery of molecules. 764 *Adv. Neural Inf. Process Syst.* **2019**, *32*, 7566–7578.

765 (36) DeFever, R. S.; Targonski, C.; Hall, S. W.; Smith, M. C.; 766 Sarupria, S. A generalized deep learning approach for local structure 767 identification in molecular simulations. *Chem. Sci.* **2019**, *10*, 7503– 768 7515.

769 (37) Charles, R. Q.; Su, H.; Kaichun, M.; Guibas, L. J. PointNet:

770 Deep Learning on Point Sets for 3D Classification and Segmentation.

771 Computer Vision and Pattern Recognition. Honululu, HI 2017, 77.

(38) Kim, B.; Lee, S.; Kim, J. Inverse Design of Porous Materials 772

Using Artificial Neural Networks. *Sci. Adv.* **2020**, *6*, eaax9324. 773 (39) Kingma, D. P.; Ba, J. A method for stochastic optimization. 774

arXiv:1412.6980; https://arxiv.org/abs/1412.6980v1, **2014**. 775 (40) Arvo, J. In *Fast Random Rotation Matrices*; Kirk, D., Ed.; 776 Academic Press, 1992; pp 117–120. 777

(41) Wu, Z.; Song, S.; Khosla, A.; Yu, F.; Zhang, L.; Tang, X.; Xiao, 778 J. 3D ShapeNets: A deep representation for volumetric shapes. 2015 779 IEEE Conf. on Computer Vision and Pattern Recognition (CVPR) 2015, 780 1912–1920. 781

(42) Jaderberg, M.; Simonyan, K.; Zisserman, A.; Kavukcuoglu, K. 782 Spatial Transformer Networks. *Advances in Neural Information* 783 *Processing Systems* 28. *Montreal, Canada* **2015**, 2017–2025. 784

(43) Matsen, M. W.; Bates, F. S. Unifying Weak- and Strong- 785 Segregation Block Copolymer Theories. *Macromolecules* **1996**, 29, 786 1091–1098. 787

(44) Shao, J.; Jiang, N.; Zhang, H.; Yang, Y.; Tang, P. Target- 788 Directed Design of Phase Transition Path for Complex Structures of 789 Rod-Coil Block Copolymers. *ACS Omega* **2019**, *4*, 20367–20380. 790