

# Pedagogical Affordance Analysis: Leveraging Teachers' Pedagogical Knowledge to Elicit Pedagogical Affordances and Constraints of Instructional Tools

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**Abstract:** When designing an instructional tool and using it in pedagogical activities, it is essential that designers and users understand what pedagogical affordances and constraints the tool provides to support its successful integration into targeted pedagogical activities. Toward this end, we developed *Pedagogical Affordance Analysis (PAA)*. PAA involves analyzing teachers' Pedagogical Content Knowledge and/or Technological Pedagogical Content Knowledge to elicit pedagogical affordances and constraints that are specific to a given instructional goal. Information obtained through PAA can help in designing, refining, and/or evaluating instructional tools. We present a case study in which we used PAA to successfully design a visual representation for middle-school algebra. To the best of our knowledge, PAA is the only available systematic method that leverages teachers' pedagogical knowledge in identifying pedagogical affordances and constraints. PAA can be used across a wide range of existing tools and prototypes of to-be-designed tools.

## Introduction

When instructional tools are used to support teaching and learning, it is important that they are designed, adapted, and refined in a way that is well aligned with real-world pedagogical practices (Bell & Gresalfi, 2017). In particular, it is essential to understand a tool's pedagogical affordances and constraints (Martin, Gnesdilow, & Puntambekar, 2018), by which we mean, respectively, properties of an instructional tool that could help achieve instructional goals, or that would put a limit on achieving the goals. Identifying pedagogical affordances and constraints of an instructional tool can increase the likelihood that the tool will benefit learning while helping to avoid situations in which the tool affects learning in an undesired way.

However, not all instructional tools we have today are designed and adapted based on a thorough understanding of their pedagogical affordances and constraints in the intended context of use (e.g., Dyckhoff, Lukarov, Muslim, Chatti, & Schroeder, 2013). Prior studies have often identified pedagogical affordances through literature review or examination of technological features (e.g., Wu & Puntambekar, 2012). This approach, however, tends to ignore instructional goals or how they might affect the way pedagogical affordances are activated and perceived. Thus, it may insufficiently inform real-world pedagogical practices, where decision making for adopting instructional tools often involves defining an instructional goal. When an instructional goal is considered, certain affordances become more relevant than others (Krauskop, Zahn, & Hesse, 2012). An approach that incorporates teachers' pedagogical knowledge may be especially effective, given that identifying pedagogical affordances and constraints of tools is considered part of teachers' Technological Pedagogical Content Knowledge (TPACK) (Dickey, 2003; Krauskop et al., 2012). To date, no systematic method that analyzes pedagogical knowledge in eliciting pedagogical affordances and constraints is available.

## Pedagogical Affordance Analysis (PAA)

In this paper, we present *Pedagogical Affordance Analysis (PAA)*, a systematic, action-oriented, and human-centered method for eliciting pedagogical affordances and constraints of an instructional tool through leveraging teachers' Pedagogical Content Knowledge (PCK) and TPACK. PAA can be applied to both existing tools and prototypes of to-be-designed tools. PAA has three unique characteristics:

- *Goal-oriented:* In PAA, designers and teachers define a specific instructional/learning goal. PAA aims to elicit pedagogical affordances and constraints in relation to the defined goal.

- *Action-oriented*: In PAA, teachers are asked to demonstrate their PCK and/or TPACK on one or more pedagogical tasks that are relevant to the targeted instructional goal(s).
- *Comparative*: In PAA, teachers are asked to demonstrate their usual pedagogical strategies and then potential approaches using the target tool on the exact same task. PAA systematically elicits pedagogical affordances and constraints by comparing and contrasting those two types of demonstrations.

PAA comprises four steps (Figure 1), inspired partly by methods for assessing pedagogical knowledge (e.g., Krauss et al., 2008). In *Step 1*, designers and teachers work together to set an instructional/learning goal which the tool of their interest targets. In *Step 2*, designers give teachers one or more pedagogical tasks targeted at the given goal and ask them to demonstrate pedagogical strategies that they would usually choose for each task, followed by pedagogical strategies that they would choose if they were using the target tool. In *Step 3*, designers separately analyze the demonstrated PCK and/or TPACK with and without the tool using a grounded theory approach (Strauss & Corbin, 1994). They then elicit themes regarding the strategies, separately for the demonstrations with and without the target tool. Finally, in *Step 4*, designers synthesize the themes across the two demonstrations through comparison, identifying pedagogical affordances and constraints of the tool for the goal.

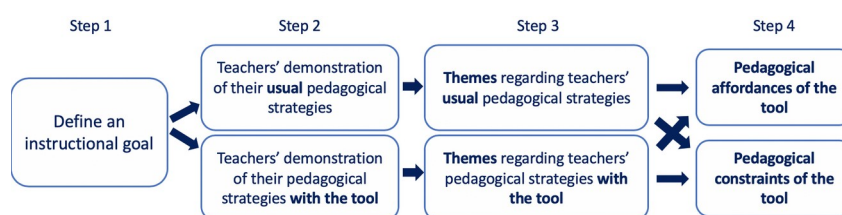


Figure 1. Procedure of Pedagogical Affordance Analysis, comprised of four steps.

## Case study: Designing a visual representation for middle school algebra

This section describes a case study in which we used PAA to elicit pedagogical affordances and constraints of visual representations called tape diagrams, which then guided our efforts to refine the design.

## Background

The use of diagrams is a promising instructional strategy to help middle-school students' learning of conceptual knowledge of algebra. Tape diagrams (TDs) are a type of diagram frequently used in countries such as Japan and Singapore where mathematics performance is considered high (Booth & Koedinger, 2012). TDs use bar-type representations to show how the different quantities are related in an equation (Figure 2). Prior studies have shown that TDs can lead to increased accuracy in problem solving and reduce conceptual errors, but they are not typically helpful for students with low prior knowledge in algebra (e.g., Booth & Koedinger, 2012). Moreover, the *learning* benefit of TDs for conceptual knowledge has never been explored; prior studies have focused only on effects on *performance*. We applied PAA to TDs to understand what core properties of TDs might help enhance students' conceptual understanding in algebra. This analysis then involved our redesign.

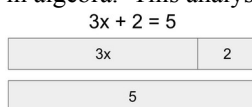


Figure 2. Example tape diagrams.

## Applying Pedagogical Affordance Analysis

We (researchers and designers) conducted a PAA with eight middle school mathematics teachers in the United States who participated either in-person or remotely. On average, participants had been teaching for 15.5 years. Only two of the teachers reported having seen TDs in the past, and none reported ever using TDs in their teaching.

In Step 1, we defined *enhancing conceptual knowledge in equation solving among middle school students with low prior knowledge using TDs* as our target instructional goal. In Step 2, we asked teachers to explain student errors in equation solving, which is an important part of their PCK (Krauss et al., 2008). We first asked them to generate a few examples of common errors and to demonstrate their usual pedagogical approaches to helping students correctly and conceptually understand the errors. We then introduced the

simplest-possible TDs together with algebraic equations, with the tapes corresponding to the two sides of the equations. The TDs varied in the alignment of the tapes and in whether the lengths of the sections were proportional to the values being represented (Figure 2). We asked teachers to demonstrate the strategies they would choose on the same tasks if they were to use TDs in their conceptual explanations. In Step 3, two researchers analyzed approximately eight hours of video recordings following a grounded theory approach in which open coding, axial coding, and selective coding were performed. They discussed frequently during each phase of coding to resolve any disagreements.

## Findings

The teachers identified many common student errors, including combining unlike terms, not keeping the sides of an equation equal, and incorrect inverse operations. By analyzing how they explained these errors, we found five themes regarding teachers' usual pedagogical strategies (usual strategies: US) and eight themes regarding their strategies using TDs (strategies with tape diagrams: STD) (Table 1).

**Table 1: Themes regarding teachers' usual pedagogical strategies and strategies with tape diagrams**

Themes regarding usual pedagogical strategies	Themes regarding strategies with tape diagrams
<ul style="list-style-type: none"> <li>• US1: Teachers choose pedagogical approaches and tools that can be used for a variety of problems and operations</li> <li>• US2: Teachers use familiar real-world examples and plain numbers so that students can relate to their own prior knowledge</li> <li>• US3: Teachers want students to make a transition from concrete to abstract thinkers</li> <li>• US4: Teachers want students to show their thinking process rather than the answer</li> <li>• US5: Teachers want students to visually understand the structure of equations and valid ways of transforming them</li> </ul>	<ul style="list-style-type: none"> <li>• STD1: Teachers use the lengths of tapes as a visually-intuitive representation of mathematical equivalence</li> <li>• STD2: Teachers use TDs to visually show students how equations can be represented</li> <li>• STD3: Teachers use the size of tapes to help students understand equation transformations</li> <li>• STD4: Teachers use TDs to help students avoid errors</li> <li>• STD5: Teachers feel that students need to be trained to use TDs since they might be too abstract for students</li> <li>• STD6: Teachers find it difficult to effectively illustrate unlike terms with TDs</li> <li>• STD7: Teachers do not want students to guess the value of variables without solving</li> <li>• STD8: Teachers are frustrated with the inability of TDs in representing certain equation types and operations</li> </ul>

We then (Step 4) identified pedagogical affordances and constraints of TDs for enhancing conceptual knowledge of equation solving by combining similar themes (e.g., US5 and STD2, generating A1 in Table 2) or contrasting themes across the columns (e.g., US1 vs. STD8, generating C3 in Table 2). When no more pairs of similar or opposite US themes could be found, STD themes were classified either as affordances or constraints, depending on whether the theme focused on helping or limiting achieving the goal (e.g., A2, A3, A4, and C4).

**Table 2: Pedagogical affordances and constraints of TDs in relation to the goal (relevant themes in parentheses).**

Pedagogical affordances of tape diagrams	Pedagogical constraints of tape diagrams
<ul style="list-style-type: none"> <li>• A1: Visually depict equations, relationships among quantities, and transformations (US5/STD2)</li> <li>• A2: The lengths of tapes visualize the concept of equivalence (STD1)</li> <li>• A3: The size of tapes, when proportional to the actual value of the number being represented, works as an indicator for understanding a next step (STD3)</li> <li>• A4: Help students avoid making conceptual errors by visualizing errors with tape diagrams (STD4)</li> </ul>	<ul style="list-style-type: none"> <li>• C1: Difficult to represent unlike terms (e.g., variables and constant terms) (US1/STD6)</li> <li>• C2: Students are not necessarily familiar with TDs (US2/US3/STD5)</li> <li>• C3: Not flexible in representing various operations and equations (US1/STD8)</li> <li>• C4: Students might guess the answer by measuring the length/size of tapes (STD7)</li> </ul>

## Evaluating the validity of the pedagogical affordances and constraints

Next, we worked with one of the participating teachers to *re-design* TDs and design the accompanying instruction based on the pedagogical affordances and constraints we found. We designed a novel form of diagrammatic self-explanation in an Intelligent Tutoring System (ITS), in which students are asked to *explain*

equation transformations by choosing an appropriate TD from among three options given (Figure 3). The design of the diagrams was based on A2 and A3, and aimed at overcoming C1 (Figure 4). The instructional activity was designed so that it would be aligned with A1 (visualizing equation transformations) and A4 (visualizing conceptual errors with diagrams). Equations covered in the system did not contain those with negative numbers or complex equations such as those with parentheses (C3). We conducted a classroom study with 41 students in grades 5 and 6 to test the effectiveness of this instructional strategy (Nagashima et al., 2020). We found that using TDs in a self-explanation activity helped students who had had little knowledge about solving algebra problems gain significantly more conceptual knowledge than their peers who did not use TDs. This case study illustrates that PAA can lead to effective instructional design that helps achieve a targeted goal.

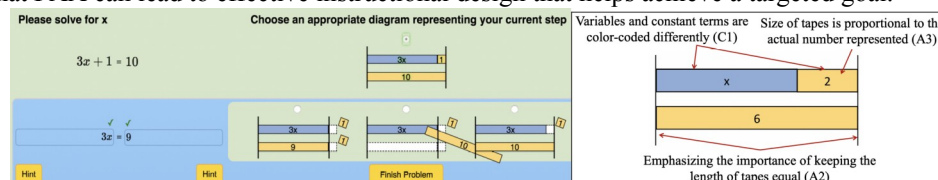


Figure 3. Our ITS with diagrammatic-self-explanation (left).  
Figure 4. The design of the tape diagrams used in the ITS (right).

## Discussion and conclusion

When designing and/or adapting an instructional tool, it is essential to understand its pedagogical affordances and constraints in relation to the specific instructional goal. This paper introduces Pedagogical Affordance Analysis, a method for eliciting pedagogical affordances and constraints of instructional tools through leveraging teachers' pedagogical knowledge. Currently, PAA is the only systematic method for this purpose. Our case study illustrates the importance of defining a goal, which allowed us to specify pedagogical tasks that helped elicit relevant themes and pedagogical affordances and constraints in the analysis. We also showed how we re-designed a tool by emphasizing pedagogical affordances and overcoming (and avoiding) pedagogical constraints, enabling us to generate novel design features that helped achieve the goal. These included the design of the TDs themselves (e.g., color-coding unlike terms) and the design of the accompanying instruction (e.g., representing conceptual errors with TDs). However, we acknowledge that PAA may not be applicable to every type of instructional tool. Specifically, PAA would not be appropriate when the definition of the to-be-designed tool is too abstract, as it could make it hard for teachers to demonstrate their pedagogical knowledge. PAA may be most effectively used when designers and/or teachers have some design ideas in mind or when evaluating existing tools.

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