PROFILING PRODUCTIVE MATHEMATICAL TEACHING MOVES IN 4TH-8TH MATHEMATICS CLASSROOMS

Kathleen Melhuish
Texas State University
melhuish@txstate.edu

Sharon K. Strickland
Texas State University
strickland@txstate.edu

Brittney Ellis
Portland State University
bme3@pdx.edu

Simon Byeonguk Han
Portland State University
byeonguk@pdx.edu

Autumn Pham
Portland State University
autpham@pdx.edu

Eva Thanheiser
Portland State University
evat@pdx.edu

As the field draws to greater consensus around components of productive mathematics classrooms, the need increases to answer questions about how we operationalize and measure these components and how we can actualize such practice in classrooms. This research report shares an analysis from a larger project aimed at describing and quantifying student and teacher components of productive classrooms at a fine-grain level. We share results from this analysis of 39 mathematics lessons with a focus on teacher moves and catalytic teaching habits that characterize these lessons. A cluster analysis identified four profiles of lessons differentiated by the existence of these catalytic teaching habits and subsequent work with student ideas. Further, these clusters appeared to account for differences in student contributions in lessons.

Keywords: Instructional activities and practices

There is general consensus that productive mathematics classrooms are ones where student thinking is integral and student discussion permeates (Jacobs & Spangler, 2017). A mathematics classroom can be conceptualized in terms of the interrelations between student(s), teachers, and content, often deemed the instructional triangle (e.g., Hawkins, 2002). The relationships between these areas can serve to position students towards engagement in meaningful mathematics. In this work, we share an analysis of residue from the instructional triangle measured via the Math Habits Tool (MHT; Melhuish, et al., 2020). The MHT was developed to capture *mathematically productive components* of classrooms in terms of both what teachers and students do in-the-moment. We conjectured that we could categorize different types of mathematics classrooms based on the existence of, and patterns within, habits of mind/interaction (student engagement in mathematics and with each other), catalytic teaching habits (teaching moves to engender student engagement with the content), and teaching routines (teaching structures that position and encourage students as contributors to mathematics).

In order to begin this discussion, we share results from an analysis of 39 lessons spanning grades 4-8. These results illustrate a partition of lessons into four clusters differentiated based on how student ideas were prompted/treated: student ideas are not prevalent and teacher-prompts are limited as well as unvaried (cluster 1); student ideas are in discourse, prompts are more prominent yet unvaried (cluster 2); student ideas are in discourse with varied prompts for student ideas with a focus on expanding ideas (cluster 3); student ideas are in discourse with varied prompts for student ideas with a focus on students engaging with each others' ideas (cluster 4).

Literature Background

Researchers and policy documents alike have identified attention to student mathematical thinking as a key component of mathematically productive classrooms (Jacobs & Spangler, 2017; National Council of Teachers of Mathematics, 2014). In Jacobs and Spangler's (2017) overview of the literature on teaching, they have unpacked this attention into two core teaching practices: noticing students thinking and orchestrating classroom discussion. Classrooms that incorporate these practices

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provide settings where student ideas serve as the grounds for moving the mathematics forward. Teachers research student ideas, provide space for student ideas to become part of the classroom discourse, and develop a shared community in which students take ownership of mathematics.

These core teaching practices have one commonality: teaching centered on students' mathematical ideas. We have identified four essential purposes for teacher moves related to students' mathematical ideas: engaging students in idea generation, researching student ideas, engaging students in expanding their ideas, and orienting students to each other's ideas. Productive idea generation can be supported through mechanisms such as allowing for private reasoning time (e.g., Kelemanik, et al., 2016) and providing and maintaining high cognitive demand tasks (e.g., Stein, et al., 1996). Researching student thinking occurs when teachers press for sharing reasoning and meaning behind student ideas to allow teachers to attend to, interpret, and decide how to respond to student thinking (e.g., Jacobs, et al., 2010). Expanding ideas then includes prompts for students to justify their responses (e.g., Boaler & Staples, 2008) or reflect on their thinking (e.g., Schneider & Artelt, 2010). The last essential component to a student idea-driven classroom is orienting students' to each other's ideas. This involves bringing student ideas into the public discourse to establish common ground (e.g., Staples, 2007), and asking students to interpret and compare each other's ideas (e.g., Stein, et al., 2008).

Theoretical Orientation and Analytic Framework

The theory underlying our work is that of the instructional triangle (Hawkins, 2002). Hawkins posited that instruction can be viewed through the relationships between teachers, students, and content. Lampert (2001) expanded on this work in analyzing her own practice, noting how instruction occurs through the arrows and introduced that the teacher plays a mediating role on the arrow reflecting the relationship between student and content. Further, Cohen et al. (2003) brought attention to not just "student" but the additional layers of interactions between students within a classroom.

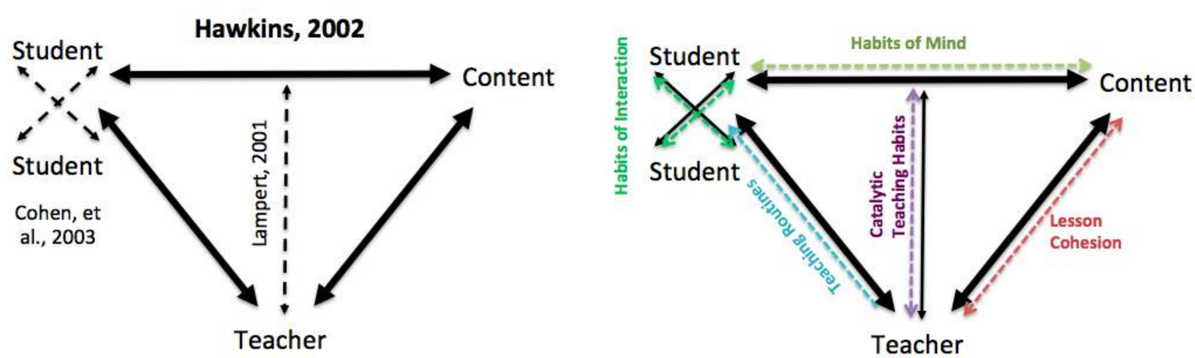

Figure 1: Instructional Triangle and the MHT Framework

In our operationalization of this triangle, we focus on key relationships within those triangles: productive ways students engage in mathematics (habits of mind), with each other around mathematics (habits of interaction), productive teaching structures (teaching routines), and individual teaching moves that can serve to catalyze student productive engagement with mathematics (catalytic teaching habits.) In this report, we focus on the category of catalytic teaching habits (CTHs) as they are both highly observable and play a key role in mediating the relationship between students and content. Recall, CTHs are prompts by the teacher to bring about students' engagement within the lesson and are represented in the instructional triangle by the line, added by Lampert, originating from the Teacher to the line connecting Students to Content. The CTHs can be found in Figure 2 in the Results section. We will also briefly analyze students' habits of interation (HoIs) which relate to communicating mathematical ideas primarly mapped to the Student-to-Student interactions within the instructional triangle. Examples of HoIs include explaining mathematical ideas, critiquing and debating, comparing ideas, exploring multiple pathways, and asking genuine questions.

Methods

For the scope of this project, we are analyzing lessons from two school districts: elementary schools (grades 4-5) from a large urban district and middle schools (grades 6-8) from a mid-size urban district, both in the United States. We are analyzing a lesson at the end of the year from all of the teachers from the middle school group, and a stratified random sample, according to Mathematical Quality of Instruction (Hill, 2014) scores, from the larger set of elementary teachers. Currently, we have coded 39 lessons (19 from 4-5 and 20 from 6-8).

Two trained coders independently watched each video-recorded lesson and qualitatively coded the classroom interaction by interpreting teacher moves and student contributions using the MHT codebook as a guide. The unit of analysis was at the contribution level so a student (or group of students) explaining one idea would be a single unit. Each substantive mathematical contribution from a student(s) would have a single HoI code. Similarly, a teacher press would be a unit. During this process, coders took detailed notes to keep a record of their rationale for assigning particular codes. Then, the two coders met to discuss and reconcile their individual interpretations until agreement was reached. In addition to student and teacher interaction, coders also noted portions of class time spent on whole class discussion (or teacher lecture), small group work, and individual work as well as rated each lesson holistically across a number of categories including *overall teacher* and *overall student*. Each lesson was rated a 1, 2, 3 or 4 within each holistic coding category where 1 represents the lowest rating and 4 represents the highest rating. The *overall teaching* captures the degree to which teacher moves reflect catalytic teaching habits, teaching structures including productive routines (such as selecting and sequencing), and ultimately if teachers prompted students towards justifying or generalizing (a requirement for a score of 4). Similarly, the *overall student* code captured whether students were engaged in math habits of mind and interaction with a scores ranging from no engagement (1), some engagement (2), engagement in many habits (3) and engagement with many habits including justifying or generalizing (4). We calculated Krippendorff (2004)'s alpha for *overall student* at 0.679 and *overall teacher* 0.764, both meeting the acceptable cutoff for reliability.

At the current stage, we focus our detailed analysis on the CTHs. To simplify this analysis, we considered a binary variable for each CTH on each lesson (occurred or did not occur.) We then conducted a two-step cluster analysis using a log-likelihood distance in order to cluster together lessons that were similar in terms of CTH profiles. Four clusters provided a fair classification with each cluster containing at least eight lessons. We further situate these clusters in relation to the summary variables: *overall student* and *overall teacher* scores.

Results

An Overview of CTHs in Lessons

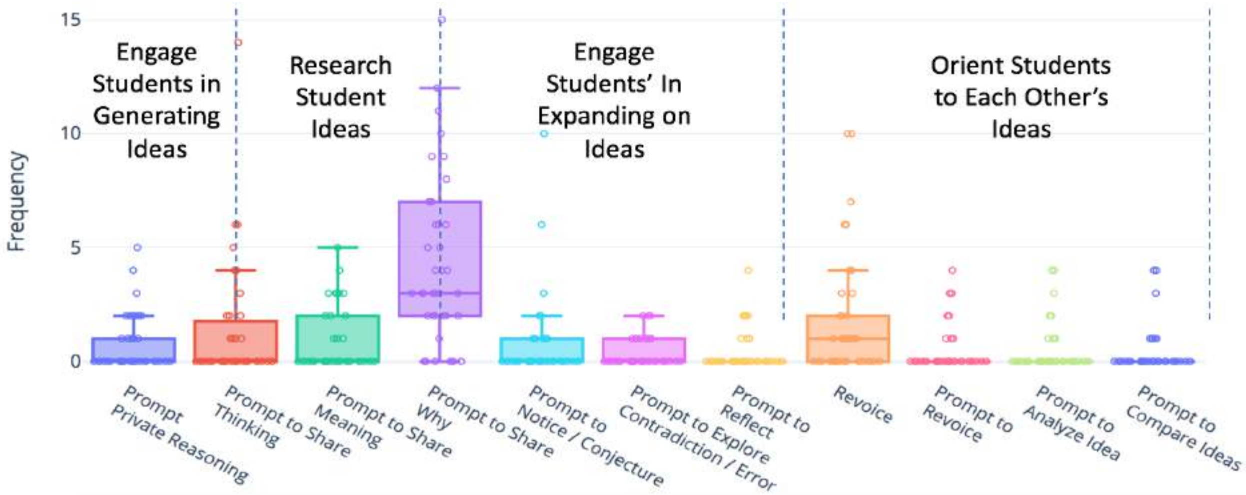

Figure 2: Boxplots Reflecting Frequency of CTHs Per Lesson

An overview of the frequency of various CTHs can be found in Figure 2. This figure contains box plots along with data points (each dot is one lesson) representing the range in frequency for each class. Notice that prompts to *share why* are most common, with well over half of the lessons including multiple occurrences. In contrast, the majority of lessons had zero prompts for students to analyze ideas (see the density of dots at 0 frequency).

Profiling Lessons Based on Types of CTHs Present

Catalytic Teaching Habit	Cluster 1 (14 lessons)	Cluster 2 (8 lessons)	Cluster 3 (8 lessons)	Cluster 4 (9 lessons)
Prompt for Private Reasoning	0%	0%	100%	77.8%
Prompt to Share Thinking	14.3%	25%	87.5%	44.4%
Prompt to Share Meaning	35.7%	0%	75%	44.4%
Prompt to Share Why	57.1%	100%	100%	100%
Prompt to Notice/Conjecture	28.6%	12.5%	25%	44.4%
Prompt to Explore Contradiction/Error	7.1%	62.5%	12.5%	44.4%
Prompt to Reflect	0%	12.5%	37.5%	55.6%
Revoices Student Idea	14.3%	87.5%	87.5%	55.6%
Prompt to Revoice Student Idea	0%	12.5%	0%	88.9%
Prompt to Analyze a Student Idea	0%	12.5%	25%	66.7%
Prompt to Compare Student Ideas	0%	0%	0%	88.9%

Table 1: Percentage of Lessons in Each Cluster with Relevant CTH Occurrence

Table 1 contains the cluster analysis results where the percentages in each cell indicate what percentage of lessons in that cluster contain the given CTH (e.g. 14.3% of lessons in cluster 1 contain a prompt to share thinking). The coloring reflects the density of lessons in that cluster containing that CTH (red: less than or equal to 25% of lessons, yellow: between 25% and 75%, green: 75% or more of lessons). Lessons in cluster 1 tended to have few CTHs with no or few variations in prompts related to students contributing ideas. Lessons in cluster 2 all included at least a basic prompt for a student to share their idea, and most included a teacher revoicing a student idea, pushing student ideas into the classroom public space. Cluster 3 reflected a greater variation in CTHs with the majority containing not just *explain why* prompts, but also general prompts to share thinking and share understanding of a mathematical idea. However, like cluster 2, student ideas were revoiced by the teacher. Cluster 4 reflected less variation in initial idea sharing mechanisms, but substantial prompts for students to engage in each other's ideas. Notice the extremely high proportion of lessons in cluster 4 that contained *prompt to revoice student idea*, *prompt to analyze a student idea*, and *prompt to compare student ideas*. These moves shift the intellectual responsibility back to the students to make sense and engage with one another's ideas.

Examples of CTHs and Corresponding Student Contributions By Cluster

In order to further situate these clusters, we share a representative exchange from a classroom in each cluster.

Cluster 1, Minimal Use of Student Ideas. The following exchange comes from a 7th grade classroom where students were guided by the teacher during a lesson about demonstrating the Pythagorean theorem using pictorial models. While displaying a visual of a right triangle, and pointing to the right angle, the teacher asked:

Teacher: What are we claiming that we have here? [pointing to right angle]
Students: Right Angle

The teacher endorsed the response, by repeating it, and continued with their explanation which included prompts for students to give short answers to teacher questions. This exchange was of a unidirectional nature (Brendefur & Frykholm, 2000) in which the teacher directed the instruction, and students were asked closed questions. Such exchanges typified lessons in cluster 1 in which few, if any, CTHs occurred. Note, an exchange such as this, with short "fill in the blank" style answers, were not categorized as any of the *Share* codes (2nd, 3rd, 4th in Table 1). The MHT codes require a request for more substantive student contributions before such thresholds are reached.

Cluster 2, Some Use of Student Ideas. In cluster 2, the lessons were characterized by *share why* CTHs and teacher revoicing of student ideas. For example, consider the following exchange from an 8th grade classroom that focused on converting between scientific and standard notation:

Teacher: It's in the tens place, but why I'm [sic] adding a zero? (CTH: Share why)
Student: In the tens, the power says two numbers. (HoI: Explain)
Teacher: Oh, very good, it says that there are two numbers. (CTH: Teacher revoice)

In this exchange, the teacher prompted the students to explain why a zero was added into the tens place based on the power. The student provided a short explanation, and then the teacher revoiced the explanation. This interaction was typical within the entirety of the lesson in which the teacher guided the exploration, but requested contributions beyond short answers from students that were then acknowledged/evaluated and revoiced.

Cluster 3, Use and Press for Expansion of Student Ideas. This next exchange comes from a 4th grade classroom during a lesson centered on understanding the definition of a pyramid. A student volunteered the definition as a solid with a polygon base. The teacher began working with this idea:

Teacher: Hmmm, what is a polygon. A square is a polygon. Triangle is a polygon. What makes those polygons? (CTH: Prompt to Share Meaning)
Student: They are closed shapes that have a straight line. (HoI: Explain)

At this point, the teacher prompted the student to repeat the idea, then the teacher revoiced the idea, "straight lines, closed shapes." The teacher then continued to prompt students to add onto this definition. In contrast to the prior example, the teacher prompt was more than just a prompt to share reasoning, but to connect a student idea to mathematical meaning.

Cluster 4, Presses for Engagement with (and Uses) Student Ideas. The subsequent exchange comes from a lesson about volumes of rectangular prisms in a 5th grade classroom. A student was presenting their solution to a problem that asked them to find volume, and the teacher asked this student to compare their contribution to a previously discussed student strategy:

> Teacher: [Student A], how could you tell [Student B] that he actually did something similar, what did he actually use? (CTH: Prompt to Compare Student Ideas)
>
> Student A: Well, he used addition... and it's basically the same thing because with this like the 3 is kind of like right here and then it would be like 7 times the width like that. (HoI: Compare)

The teacher followed up by restating Student A's multiplication strategy to explicitly connect to the addition strategy offered by Student B. Having students explain and compare each other's ideas distinguished cluster 4 from the other clusters.

Situating the CTH Profiles in Terms of Student Contributions

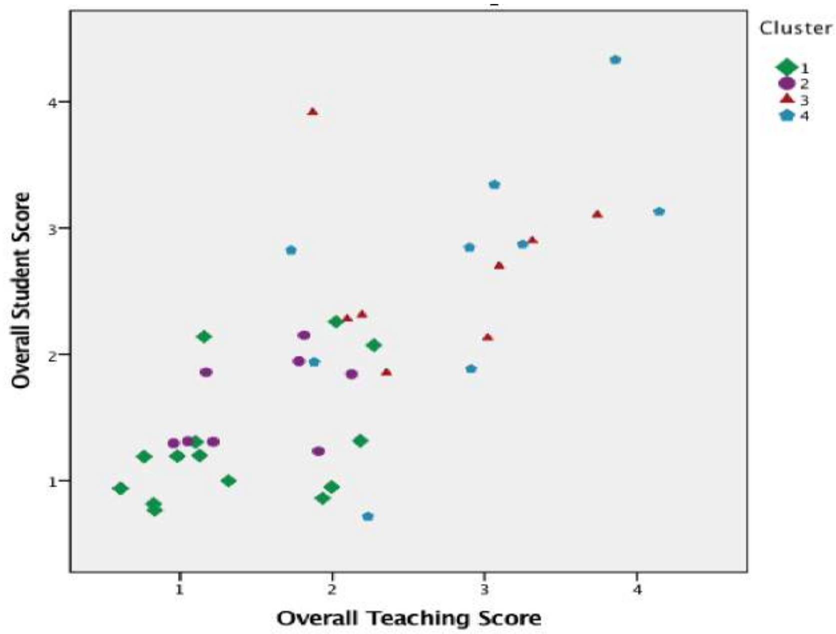

Figure 3: Scatterplot Representing Lessons on Overall Student and Overall Teaching Scores by Cluster. Note: the dots are jittered to be visible.

In order to further situate these results within the whole lesson overall, we explored how these clusters related to *overall student* and *overall teacher* scores (see Figure 3). First, we note that there *is* a substantial positive relationship between *overall teaching* and *overall student* scores as measured via the MHT. An increase in *overall teaching* score was associated with an increase in the odds of higher *overall student* score, with an odds ratio of 2.494 (95% CI, 1.410 to 3.579), Wald $\chi^2(1) = 20.314$, $p < .001$. If we then look at our lesson clusters we can discern that cluster 1 tends to have low ratings for both teacher and student. Cluster 2 includes both low and middling scores on both, and lessons in clusters 3 and 4 tend to have higher *overall teaching* and *overall student* scores.

Figure 4: Boxplots of Frequencies of Student Contributions by Cluster (as reflected by Habit of Interaction occurrences)

Further analysis examines whether these CTH prompts for more student-to-student engagement (as evidenced in cluster 4) actually relates to increased amounts of such interactions by students. A glance at student contributions paints a similar picture (see Figure 4). When comparing the number of occurrences of habits of interaction (HoI; a proxy for student math idea contribution), a one-way ANOVA identified significant differences between clusters ($F(3)=11.1215$, $p<.001$). The average number of student HoI in a cluster 1 lesson was 3.00 (sd=2.386), cluster 2—8.25 (sd=8.892), cluster 3 – 19.875 (sd=10.789), and cluster 4 – 28.889 (sd=19.915). A Tukey HSD post hoc test identified significant differences between cluster 1 and 3, 1 and 4, and 2 and 4. This reflects that variation in catalytic teaching habits seems to correspond to the number of math contributions from students.

Discussion and Future Research Plans

This analysis serves as an initial view into profiles of different mathematics classrooms. In nearly all of our analyzed lessons, teachers pressed for students to contribute to the lesson. However, the nature of these presses and how teachers worked with student ideas varied. We identified two types of lessons that were not characterized by rich use of student ideas: lessons with minimal prompting for mathematical reasoning and lessons in which student ideas were asked for and revoiced by the teacher. We also identified two distinct, productive types of lessons: those focused on generating and expanding mathematical ideas and those focused on engaging students with each other's mathematical ideas. We conjecture that both types of lessons are essential for students to engage richly with mathematics.

The aim of the MHT is to complement existing analyses of classrooms including: qualitatively robust analyses of classrooms (e.g., Stein, et al., 2008; Staples, 2007), and quantitative analysis based on a set of overall scores (e.g., Lynch, Chin, & Blazar, 2017). This existing research base has established the productivity of particular teaching moves in case studies, and that measures of overall quality of instruction can be linked to student achievement. Our work acts as a bridge between the detail of qualitatively analyzing lessons and the power of quantitative analysis of many lessons. By identifying (literature-based productive) teacher moves and student contributions in-the-moment across many lessons, we are able to profile various types of lessons. In this phase, we are creating profiles via the types of CTHs occurring. This is paired with an initial analysis of student contributions that reflect a high degree of relationships between CTH lesson profile and student contributions.

Through the course of this project, we plan to enrich and expand these initial profiles by including more than twice the number of lessons presented here and incorporating further information about student discourse. For example, this initial analysis reveals that the presence of CTHs can statistically cluster lesson types; that these lesson types are related to correlations between higher *overall teacher* scores and higher *overall student* scores. For example, lessons in cluster 4 are associated with high *overall student* scores as well as included more CTHs prompting student-to-student interaction, and nearly 10 times as many student HoI codes when compared to cluster 1 lessons. Furthermore, the student codes for habits of mind (HoMs), of particular interest to many educators, could be further linked to specific CTHs as well as to larger teaching routines. Focusing primarily on HoIs, we have not discussed HoMs in detail in this report. Where HoIs focus on the existence of student contributions and interaction, HoMs focus on the mathematical practices embedded within those interactions. For example, a student can share their thinking (an HoI) and within that exchange can make reference to a graphical *representation* as well as generate a *conjecture* (HoMs). The skeleton of such a relationship is building—CTHs are denser and more varied within clusters 3 and 4 and lessons in those clusters are largely associated with high *student overall* codes. Furthermore, clusters 3 and 4 also reveal far greater frequencies of student HoI codes.

In our next phase of analysis, we plan to expand out the components in the cluster analysis, move towards a distance metric that is non-binary to account for frequency, and begin cluster analysis leveraging the timing of the teacher and student contributions. We look forward to further exploration of the mechanisms that may promote more student interactions and deeper student engagement with mathematics.

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