

Experimental shot-by-shot estimation of quantum measurement accuracy

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Abstract: We show the direct correspondence between Bayesian probabilities obtained by the adaptive quantum measurement and experimentally observed Kholmogorov probabilities. We demonstrate the single-“shot” accuracy estimation for every individual quantum measurement outcome using these Bayesian probabilities. © 2021 The Author(s)

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Quantum measurements have enabled heretofore unforeseen accuracy and sensitivity. Quantum measurements therefore are being successfully used for real-life applications to surpass inherent deficiencies of classical measurements. Quantum measurements are a natural enhancement technique when applied to faint light characterization. Thus, optical quantum measurements are applied to important problems such as quantitative biology, efficient communication, and metrology. A large class of quantum measurements uses Bayesian inference, and estimates accuracy of each single measurement independently. Here we experimentally retrieve these accuracy estimations and demonstrate that the accuracy estimations correspond to observed accuracy of the measurement. In other words, the Bayesian probabilities that the measurement is correct estimated for each individual measurement correspond to average (Kolmogorov) probabilities to be correct. Thus, we demonstrate the physical significance of the Bayesian probabilities.

Here we consider a problem of evaluating a coherent pulse of faint light that can be prepared in any of M previously defined states. The task is to discriminate which of the M states has been prepared. A similar problem arises in digital optical communications, when such signal encodes $\log_2 M$ bits of information [1]. In this work, the quantum discriminator uses the adaptive displacement technique with a feedback provided by a single photon detector. During the measurement on one input signal, photon detection times are used to update Bayesian probabilities for each of the M possible states to determine the most probable signal state. Once the measurement on the signal is finished, the discrimination decision is made. Typically the input pulse is considered to be in the state with the highest Bayesian probability. However, the probability that the input pulse was in fact in another state due to an inherent uncertainty of the measurement is not zero. The values of these probabilities are unique to each “shot” - or a full measurement of one input signal - due to the stochastic nature of photon detection. We experimentally demonstrate that we can successfully use Bayesian probabilities to find the accuracy of the quantum measurement outcome separately for each “shot”.

We use the time resolving quantum receiver and $M = 4$ non-orthogonal input coherent states that differ by frequency and initial phase, [2,3], Fig. 1. Before discrimination of each input signal the vector of Bayesian probabilities is set to equal probabilities $P = \{0.25, 0.25, 0.25, 0.25\}$. During an experimental measurement of one signal pulse, a few photon detections can occur; P updates after each detection and at the end of the signal pulse. We then compare the true state of the input to the received state $P = p_1, \dots, p_4$. To do such a comparison, we group components p into 5%-wide bins.

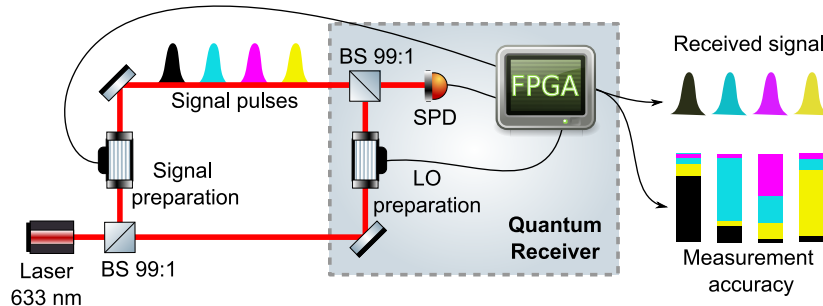


Fig. 1: Experimental setup. For each signal pulse, the quantum receiver determines the most likely input state and the accuracy of discrimination (Bayesian probability of each of the possible states).

A large number N_p of experiments with a Bayesian component value in the same bin is observed, and the probability of a successful detection q is found in each bin, where $q = N_{\text{correct}}/N_p$, N_{correct} is the number of correct detections. We then plot q as a function of p , (Fig. 2). Experimentally, we see that Kolmogorov discrimination error probabilities observed for an ensemble of single-shot measurements q are equal to the observed single-shot Bayesian probabilities p . Remarkably, this equality is true for any value of p , including the measurements with very low uncertainty $p \approx 1$ and high uncertainty $p \approx 0$. Thus, we verified that P represents the best knowledge about the input state available to a certain (here, to our) detection system. The uncertainty of each measurement can be easily seen in Fig. 3. The original 100x100 image represents the user data. Each pixel is one input symbol, and each of the 4 primary colors (cyan, magenta, yellow, and black) corresponds to a state of the communication alphabet, Fig. 3, left. After discrimination, same image is reconstructed using the best available knowledge about the input state, P . Received pixels can have an arbitrary color, because all 4 components P may be nonzero. The "polka dots" on the right range from nearly indistinguishable from a primary color, in which case the uncertainty of the measurement is still low to high-contrast, where the uncertainty is fairly high. Finally, bright "polka dots" close to a visually incorrect primary color are discrimination errors. In our case, most of the pixels are close to the primary color, because the overall error rate (symbol error rate) in this experiment is fairly low: $\text{SER} \approx 1\%$.

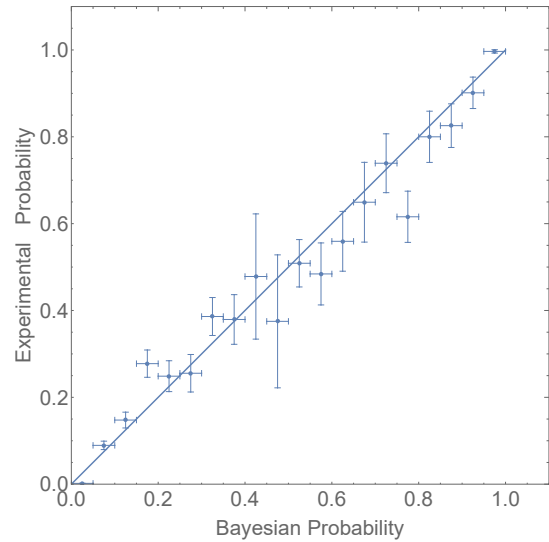


Fig. 2: Experimentally measured single shot Bayesian probability vs. the experimental average (Kolmogorov) probability of a successful state discrimination obtained for faint input ($\langle n \rangle = 3$ photons per signal pulse) prepared in one of $M = 4$ states. Vertical error bars correspond to one standard deviation, horizontal error bars show histogram bin size.

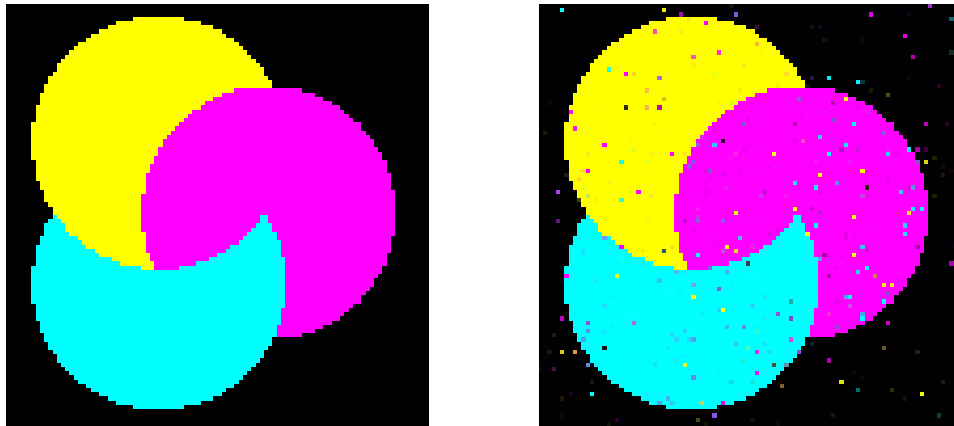


Fig. 3: Left: the original image of 100x100 pixels where 4 primary colors correspond to symbols of the communication alphabet. Right: the same image reconstructed from the experimentally obtained Bayesian probabilities after state discrimination. In later case the color for each pixel is calculated as a sum of primary colors weighted with the Bayesian probabilities of corresponding symbols.

In practice, the full Bayesian likelihood vector can be used to discard low confidence quantum measurements for unambiguous state discrimination or can supply heretofore ignored information to quantum error correction protocols. This work is the first demonstration of the experimental measurement of the Bayesian probabilities obtained by a quantum receiver providing single-"shot" quantum measurement accuracy estimation. Using this information, readily available in the quantum measurement, can help correcting communication errors. Quantum-enabled error correction could further increase the quantum measurement advantage: communication systems may become closer to the Helstrom bound using practically accessible encodings and state discrimination methods.

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